The semester A examination for Honors Algebra 2 will consist of two parts. Part 1 will be selected response on which a calculator is NOT allowed. Part 2 will be grid-in and short answer on which a calculator will be allowed.

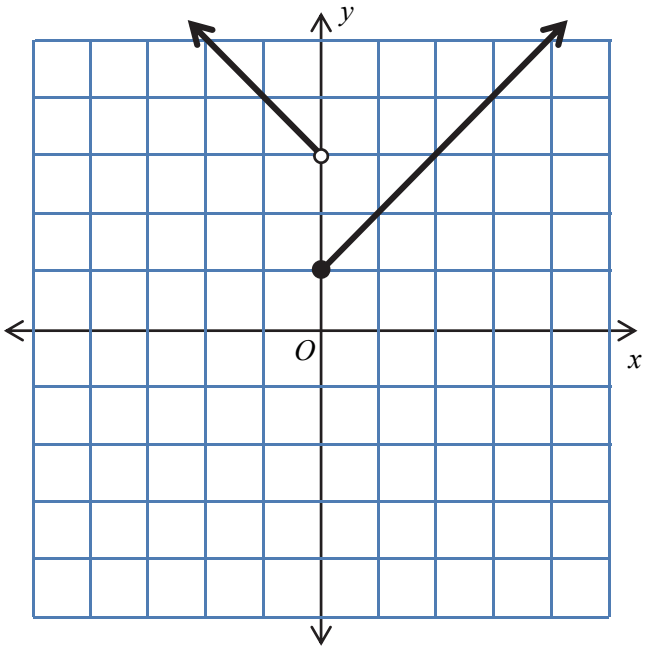
The symbol indicates that a student should be able to complete this item without a calculator.

- If a calculator is used to find points on a graph, the appropriate calculator function (i.e., zero, intersect, minimum, or maximum) should be used. The trace function should not be used.
- Unless otherwise stated, decimal approximations must be correct to three places after the decimal point.
- Unless otherwise specified, the domain of any function is assumed to be the set of all real numbers for which the function rule returns a real number.
- The symbol $[x]$ represents the greatest integer function.
- No formulas will be provided in the examination booklet.

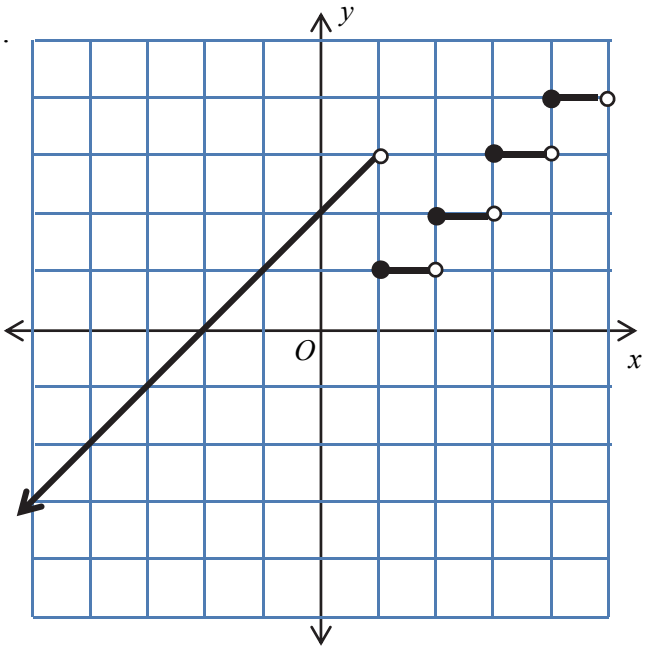
1. Sketch the graph of $f(x)=\begin{cases} x+1, & \text{if } x \le 0 \\ 3-4x, & \text{if } x > 0 \end{cases}$

For items 2 and 3, write the functions represented by the graphs below.

2.



3.



4. When ordering items from a catalog, the buyer has the option of having items gift-wrapped. The shipping charge for every order is \$12.00. In addition, if 5 items or less are gift-wrapped, the charge is \$5.00 per item gift-wrapped. If more than 5 items but less than 10 items are gift-wrapped, the charge is \$4.50 per item gift-wrapped. If 10 or more items are gift-wrapped, the charge is \$3.50 for each item gift-wrapped.

   a. Write a piece-wise function for the total charge of gift-wrapping x items, including the shipping charge.

   b. If the total charge, including shipping was \$48.00, how many items were gift-wrapped?

5. Let $g(x) = 3|x+2|-4$

   a. Sketch the graph of $g(x)$.

   b. What is the domain of $g(x)$?

   c. What is the range of $g(x)$?

   d. What is the vertex of the graph of $g(x)$?

   e. What is the equation of the axis of symmetry of the graph of $g(x)$?

   f. What is the minimum value of $g(x)$?

   g. Describe the transformations of the graph of $f(x) = |x|$ that will result in the graph of $g(x)$.

   h. Is the function $g(x)$ continuous?

For items 6 through 13, use the following functions.

$f(x) = x-3$ $\quad$ $g(x) = 2x-8$ $\quad$ $h(x) = x^2-2$

For items 6 and 7, evaluate.

6. $f(g(3))$

7. $h(f(-7))$

For items 8 through 13, perform the indicated operations.

$f(x) = x - 3$ $g(x) = 2x - 8$ $h(x) = x^2 - 2$

8. $f(x) + g(x)$

9. $f(x) - g(x)$

10. $f(x) \cdot g(x)$

11a. $\dfrac{f(x)}{g(x)}$ 11b. What is the domain of $\dfrac{f(x)}{g(x)}$?

12. $g(h(x))$

13. $h(f(x))$

14. If $f(x) = \sqrt{16 - x^2}$ and $g(x) = 2x$

a. What is the domain of $f(g(x))$?

b. What is the domain of $g(f(x))$?

For items 15 through 18, state whether or not the function is one-to-one.

15. $f(x) = 2x - 5$

16. $f(x) = -x^2$

17. $f(x) = |x|$

18. $f(x) = x^3 - 9x$

19. Are $f(x) = 7x - 6$ and $g(x) = \dfrac{x + 6}{7}$ inverse functions? Verify your answer algebraically.

20. If $f(x)=\frac{1}{3}x+5$, which of the following represents the inverse function $f^{-1}(x)$?

A $f^{-1}(x)=-\frac{1}{3}x-5$ **C** $f^{-1}(x)=3x-5$

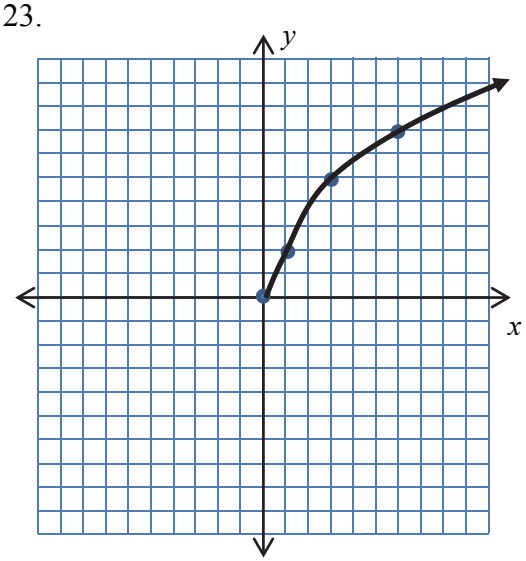
B $f^{-1}(x)=3x-\frac{1}{5}$ **D** $f^{-1}(x)=3x-15$

21. If $g(x)=9x-10$, determine the inverse function $g^{-1}(x)$.

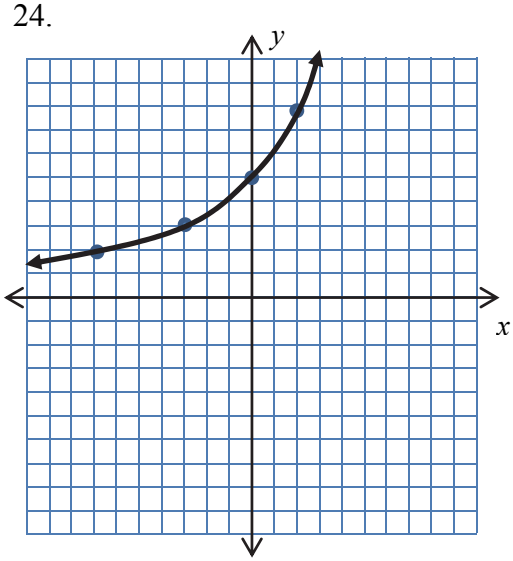
22. If (x,y) is a point of the graph of $f(x)$, what are the coordinates of a corresponding point on the graph of $f^{-1}(x)$?

For items 23 and 24, sketch the graph of the inverse function.

23.



24.



25. Let $f(x)=|x+3|+4$. What modification of the domain of $f(x)$ results in its inverse, $f^{-1}(x)$, being a function?

For items 26 and 27, let $f(x)=|x-2|$. Describe the transformations of $f(x)$ that will produce the graphs of the following functions.

26. $g(x)=|x|+1$

27. $h(x)=5|x-3|-9$

28. The graph of a function $f(x)$ is to be transformed to create three new functions. Write the rule for each function in terms of $f(x)$ for the following.

    a. The graph of $f(x)$ is reflected about the *x*-axis and stretched horizontally by a factor of three to create $g(x)$. What is the function rule for $g(x)$?

    b. The graph of $f(x)$ is reflected about the *y*-axis and translated down five units to create the graph of $h(x)$. What is the function rule for $h(x)$?

    c. The graph of $f(x)$ is stretched vertically by a factor of six, then translated to the left seven units to create the graph of $j(x)$. What is the function rule for $j(x)$?

For items 29 through 32, solve the systems of equations.

29.

$$\begin{aligned} x+2y&=7\\ y-z&=-3\\ x+3z&=17 \end{aligned}$$

30.

$$\begin{aligned} x+y-z&=-2\\ 3x-2y+z&=11\\ -5x+y+z&=-8 \end{aligned}$$

31.

$$\begin{aligned} 4x+y+3z&=17\\ 5x+2y-z&=18\\ 3x-3y+4z&=7 \end{aligned}$$

32.

$$\begin{aligned} x+2y+3z&=13\\ y&=3x+1\\ z&=2x-5 \end{aligned}$$

33. Barry's Fast Food offers three different types of items; burgers, fries, and shakes. Jill visited Barry's Fast Food three times.

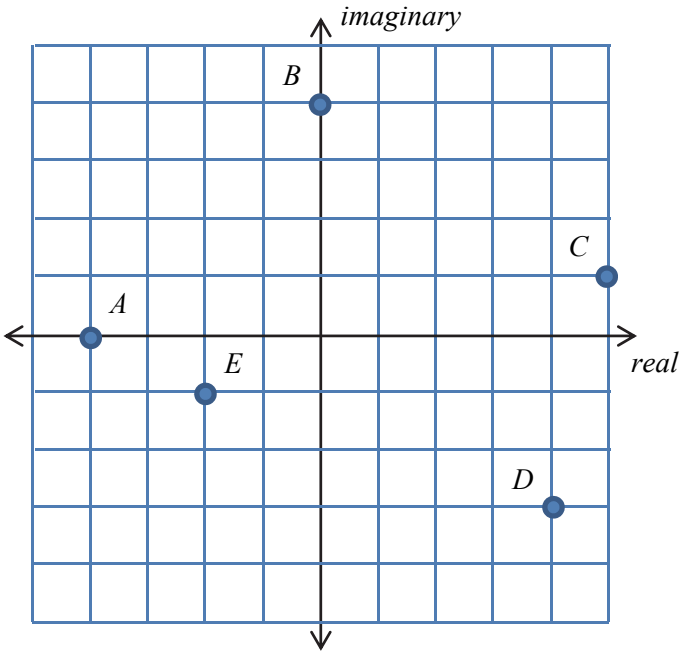
The first time, four burgers, one fry, and two shakes cost \$20. The second time, two burgers, two fries, and two shakes cost \$18. The third time, four burgers, three fries and one shake cost \$25.

a. Write a system of equations that represents this situation. Be sure to define the variables.

b. Determine the cost of a burger, a fry, and a shake.

34. The volume of a gift box is given by the function $V(h) = 4h^3 - 32h^2 + 60h$, where h represents the height of the box, with domain $0 < h \leq 3$. What is the height of the box with maximum volume? Your answer should be correct to three places after the decimal point.

35. Write the complex number represented by points *A, B, C, D,* and *E* on the graph below.



36. Identify each of the following a real, pure imaginary, and/or complex.

a. $\sqrt{3}$ b. $\sqrt{-9}$ c. $5+2i$

For items 37 through 44, perform the indicated operations.

37. $(3+2i)+(5-7i)$

38. $(5-6i)(1-i)$

39. $(8-2i)(8+2i)$

40. $(2-7i)^2$

41. $\dfrac{7i}{2-i}$

42. $\dfrac{2+7i}{1+6i}$

43. $|2-3i|$

44. $|5+12i|$

For items 45 and 46, fill in the blank with the number that will complete the trinomial square.

45. x^2-6x+____

46. x^2+11x+_____

47. Describe the relationship between the discriminant b^2-4ac and the nature of the roots (zeros) of the quadratic equation $ax^2+bx+c=0$.

For items 48 and 49, solve for all values of x.

48. $x^2-3x+11=0$

49. $3x^2+x+2=0$

For items 50a through 50c, grid in and bubble your answer in the grids provided.

50. A arrow is shot from a height of 32 feet with an initial velocity of 56 feet per second. The function for the height, in feet of the arrow at time *t*, in seconds is given by the function $h(t) = -16t^2 + 56t + 32$.

a. After how many seconds will the arrow hit the ground?

b. How many feet high will the arrow be at $t = 1.5$ seconds?

c. What is the maximum height that the arrow will reach?

50a.

	/	/	
•	•	•	•
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

50b.

	/	/	
•	•	•	•
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

50c.

	/	/	
•	•	•	•
0	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

51. If $f(x) = -x^2$, on what interval of *x*-values is $f(x)$ increasing?

52. What are the left- and right-end behaviors of the function $f(x) = x^3 - 4x - 8$?

53. What are the left- and right-end behaviors of the function $f(x) = -x^4 - 4x^3 - 4$?

54. A polynomial function has a general form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$, where n is a positive integer. Describe the left- and right-end behaviors for the following values of a_n and n.

a. a_n is positive, n is even.

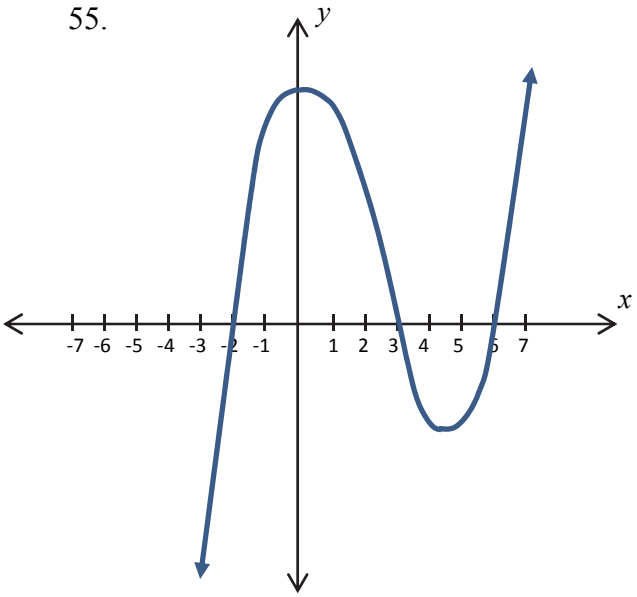
b. a_n is positive, n is odd.

c. a_n is negative, n is even.

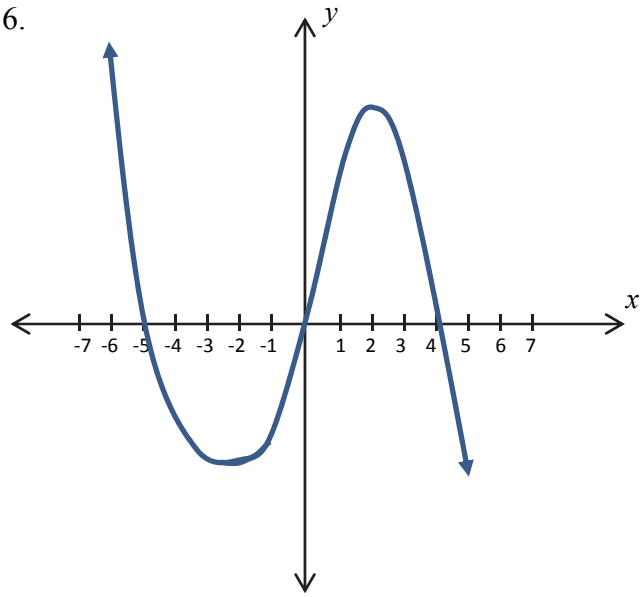
d. a_n is negative, n is odd.

For items 55 and 56, write a polynomial function in factored form for the graphs shown below.

55.



56.



For items 57 and 58, factor.

57. $343x^3 - 125$

58. $x^3 + 64$

For items 59 and 60, find all zeros of each function. Show how you determined the zeros.

59. $f(x) = 125x^3 - 8$

60. $f(x) = 64x^3 + 27$

61. Write a polynomial function with real coefficients in factored form if two of its complex zeros are 3 and $9i$.

62. Write a polynomial function in factored form with real coefficients if three of its complex zeros are 8, $7i$, and $-5i$.

63. Determine if each expression below is a factor of x^3+6x^2-x-30. Write yes or no for each expression.

a. $x-2$ b. $x+6$ c. $x-5$

d. $x+2$ e. $x+3$ f. $x+5$

64. Divide. $\left(2x^3-9x^2-8x+15\right)\div\left(x-5\right)$

65. Given $f\left(x\right)=x^3+3x^2-x-3$.

a. Sketch the graph and find the zeros of the function.

b. Describe the left-end behavior of the graph.

c. Describe the right-end behavior of the graph.

66. Let $f\left(x\right)=4x^5-7x^3+5x^2+15$. Determine if each number below is a possible rational root (zero) according to the rational root (zero) theorem. Write yes or no for each number.

a. 5 b. $-\frac{1}{4}$ c. $\frac{3}{2}$ d. 4 e. $\frac{1}{3}$

67. Using the rational root (zero) theorem, list the possible rational roots (zeros) of $f\left(x\right)=5x^4-7x^2+4$.

68. A fourth degree polynomial with real coefficients could have how many imaginary roots?

69. Let $f\left(x\right)=x^4-x-4$.

a. How many zeros does this function have?

b. How many real zeros does this function have?

70. Let $f(x) = x^5 - 3x^2 - 4$.

a. How many zeros does $f(x)$ have?

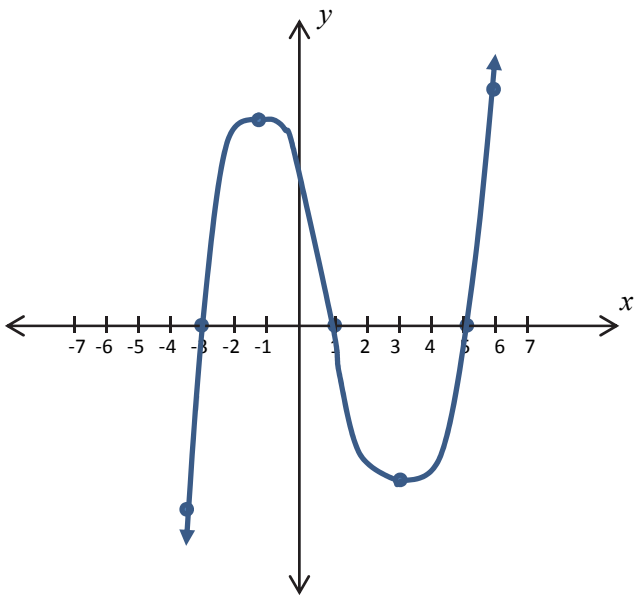
A 0 **B** 1 **C** 4 **D** 5

b. How many real zeros does $f(x)$ have?

A 0 **B** 1 **C** 4 **D** 5

71. Solve the inequality $x^4 - 5x^3 - 25x^2 + 65x + 84 \geq 0$.

72. Look at the graph of $f(x) = x^3 - 3x^2 - 13x + 15$ below.



Solve the inequality $x^3 - 3x^2 - 13x + 15 < 0$.

73. The solutions of $x^3 - x^2 - 50x - 48 = 0$ are $x = -6, -1,$ and 8.

a. Write the polynomial $x^3 - x^2 - 50x - 48$ in factored form.

b. Solve the inequality $x^3 - x^2 - 50x - 48 \leq 0$ algebraically.

74. The solutions of $x^3 - 6x^2 - 19x + 84 = 0$ are $x = -3, 4,$ and 7.

a. Write the polynomial $x^3 - 6x^2 - 19x + 84$ in factored form.

b. Solve the inequality $x^3 - 6x^2 - 19x + 84 \geq 0$ algebraically.

75. Solve the inequality $x(x-6)(x+2) > 0$

76. Complete the chart below. Any decimal approximation should be accurate to three places after the decimal point.

Function	Value(s) of any local maximums	Value(s) of any local minimums	Interval(s) where the function is increasing	Interval(s) where the function is decreasing
$f(x) = \frac{x^3}{3} + 3x^2 + 5x$				
$g(x) = \frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x + 4$				

77. Sam throws a ball from the top of a building. The table below shows the height of the ball above the ground.

Time (sec) t	Height (ft) $f(t)$
0	400
1	434
2	436
3	406

a. Write a polynomial function that best fits the data.

b. How high is the ball after 4 seconds?

c. When does the ball hit the ground?

For items 78 and 79,

a. Determine the degree of the polynomial that models the data.

b. Use the regression feature of your calculator to write a function that models the data.

78.

x	1	2	3	4	5
$f(x)$	5	14	37	80	149

79.

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

80. Write the equation in vertex form, $y = a(x-h)^2 + k$, for the parabola with vertex $(-3,-2)$ and passing through the point $(7,38)$.

81. Write the equation in vertex form, $y = a(x-h)^2 + k$, for the parabola with vertex $(1,4)$ and passing through the point $(5,52)$.