ALGEBRA 2
UNIT 4B
MODELING
PACKET
ANSWERS

Name: ______________________

Date: ____________ Period: ____________

On December 23rd, Ja'Rae decides to buy a new High-Definition television set. The total price of the television, tax included, is exactly \$3,000. He decides to use the store's credit card and make equal monthly payments.

1) What factors will determine how many monthly payments it will take to pay off the credit card?

The factors are the amount of his monthly payment and the rate of interest of the credit card.

Here are the terms of the credit card.

> On the 1st of each month, you will be sent a statement regarding your account.
>
> By the 15th of that month you must make at least the minimum payment listed on your statement.
>
> If you have not paid off your entire balance, you will be charged interest of 2% of the unpaid balance, which will then be added to what you owe.

2) To make sure you understand how this works, assume the following.

You have a \$3,000 balance as of the 1st of January.

You make a \$500 payment on the 15th of January.

What will be the new balance on the statement sent on the 1st of February? If you have done this correctly, you should have obtained an answer of \$2,550.

January 1st Balance = \$3,000

Minus a \$500 payment = \$2,500

Interest Charged: \$2,500 times 0.02 = \$50

February 1st Balance = \$2,500 + \$50 = \$2,550

Name:

Date: Period:

3) Assume that Ja'Rae continues to make $500 payments on the 15th of each month. When will he have paid off his television? Show how you determined your answer.

Month	*Beginning of Month Balance*	*Payment*	*Unpaid Balance*	*Interest*	*End of Month Balance*
January	*$3,000*	*$500*	*$2,500*	*$50*	*$2,550*
February	*$2,550*	*$500*	*$2,050*	*$41*	*$2,091*
March	*$2,091*	*$500*	*$1,591*	*$31.82*	*$1,622.82*
April	*$1,622.82*	*$500*	*$1,122.82*	*$22.46*	*$1,145.28*
May	*$1,145.28*	*$500*	*$645.28*	*$12.91*	*$658.19*
June	*$658.19*	*$500*	*$158.19*	*$3.16*	*$161.35*
July	*$161.35*	*$161.35*	*$0*	*$0*	*$0*

Ja'Rae will have paid off the balance on July 15th.

$P(0) = 3000$

$P(n) = 1.02(P(n-1) - 500)$

Name: ______________________

Date: ____________ Period: ____________

4) How much more than the $3,000 total price did he pay for his television by making monthly payments? Show how you determined your answer.

Ja'Rae made 6 payments of $500 and one payment of $161.35 for a total of $3,161.35.

He paid $161.35 in interest.

5) Complete the recursive formula for the balance that Ja'Rae owes at the end of *n* months.

$$B(0)=3{,}000$$
$$B(n)=\left[B(n-1)-500\right]\times .02+\left[B(n-1)-500\right] \text{ or } 1.02\left[B(n-1)-500\right]$$

6) Create a model that will work for any Initial Balance, *B*, making a payment of *d* dollars per month, with a monthly interest rate of *r*.

$$B(0)=B_0$$
$$B(n)=\left[B(n-1)-d\right]\times r+\left[B(n-1)-d\right] \text{ or } (1+r)\left[B(n-1)-d\right]$$

7) Create an example that allows you to test your model. Have your table partner check your model.

Answers will vary

Curriculum 2.0 Algebra 2: Unit 4-Topic1, SLT 6

Modeling Medication Absorption Sample Answers

Name: ______________________

Date: ____________ Period: ____________

A patient is prescribed a medication. Here is some information about the medication.

The patient is to take a dose of 200 mg every day at 6:00 a.m. The dose is taken by injection so that its effect is immediate.

The body processes the medication such that every 24 hours, 40% of the medication is eliminated by the body, meaning that 60% of the medication remains.

1) Determine the amount of medication in the body for the first 4 days immediately following the injection. Complete the table.

Day	Amount of medication.(in mg) immediately after the injection.
1	200
2	*320*
3	*392*
4	*435.2*

2) Create a mathematical model that will allow you to determine the amount of medicine in the body immediately after the injection each day. Make sure that your model accurately reproduces the results from the table above.

$M(n)$ *represents the amount of medication in the body after the injection on day number n.*

$M(1)=200$

$M(n)=.6\times M(n-1)+200$

3) This medication is considered "safe" if its level in the body never exceeds 500 mg. Use your model to determine whether this medication is safe.

It appears that the level of medication will never exceed 500 mg.

Curriculum 2.0 Algebra 2: Unit 4-Topic1, SLT 6

Modeling Medication Absorption Sample Answers

Name: ______________________

Date: ____________ Period: ____________

4) This medication becomes "effective" when its level never falls below 260 mg. On which day is this medication considered effective. (Remember, the level is at its lowest immediately before the daily injection)

The amount of medication in the body is at its lowest just before the injection. Here is a table showing the amount of medication immediately before each day's injection. (60% of the amount immediately following the previous day's injection.)

Day	Amount of medication.(in mg) immediately before the injection.
1	*0*
2	*120*
3	*192*
4	~~*261.12*~~ 235.2
5	261.12

After the 3rd day's injection, there is 392 mg, just before the 4th day's injection the amount is 261.12. So after the injection on the third day, the medication is effective.

5) Assume that the patient has faithfully taken the medication every day. As you determined in #3, if the medication is taken every day, the amount of medication is 500 mg immediately after the injection.

a) What is the amount of medication in the patient immediately before each day's injection?

$$500 \times 0.6 = 300 \ mg$$

b) Assume that a patient misses a daily injection. How long will it take for the medication to become effective (remember, this means never falling below 260 mg). Explain how you determined your answer.

If the patient misses an injection, here is the result.

Day	*Amount of medication before injection*	*Amount of medication after injection.*
Day before missing injection	*300*	*500*
Day of missing injection	*300*	*300 (no injection)*
One day after missing injection (starts injections again)	*180*	*380*
Two days after missing injection	*228*	*428*
Three days after missing injection	*256.8*	*456.8*
Four days after missing injection	*274.08*	*474.08*

It will take four days before the medication becomes effective.

Curriculum 2.0 Algebra 2: Unit 4-Topic1, SLT 6

Modeling Triangular and Oblong Numbers Sample Answers

Name: ____________________

Date: ____________ Period: ____________

Triangular Numbers

Look at the pattern of dots below. The number of dots form a sequence of numbers called the triangular numbers.

1 3 6 10 15

$n=1$ $n=2$ $n=3$ $n=4$ $n=5$

1) What are the sixth and seventh triangular numbers? *21 and 28*

2) Create a mathematical model that will allow you to reproduce the sequence of triangular numbers. Show that your model works.

A recursive model is

$$T(1)=1$$
$$T(n)=T(n-1)+n$$

3) Models can be verbal as well. Describe how the sequence of dots is created.

Start with one dot, then add two dots, three dots, four dots, etc., as a row centered underneath the previous row.

4) Jesse has created a model. He says that an explicit function to generate triangular numbers is

$T(n)=n^2+n$.

Show that either:

Jesse is correct by confirming that his model works.

Jesse is incorrect and modify his model to make it work.

Jesse's model is incorrect. His model produces the sequence 2, 6, 12, 20, 30, ...

A correct model would be $T(n)=\frac{1}{2}(n^2+n)$

Curriculum 2.0 Algebra 2: Unit 4-Topic1, SLT 6

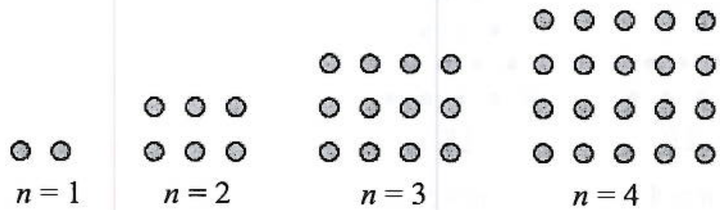
Modeling Triangular and Oblong Numbers Sample Answers

Name: ______________________

Date: ______________ Period: ______________

Oblong Numbers

Look at the pattern of dots below. The number of dots for a sequence of numbers called oblong numbers.



5) Write the fifth and sixth oblong numbers. *30, 42*

6) Create a mathematical model that will allow you to reproduce the sequence of oblong numbers. Show that your model works.

An explicit model would be

$$D(n) = n(n+1) = n^2 + n$$

7) Models can be verbal as well. Describe how the sequence of dots is created.

Create a rectangle of dots, with one more column than the number of rows, starting with one row. The number of dots can be determined by multiplying two consecutive integers, starting with 1 times 2, then 2 times 3, then 3 times 4, etc.

8) Jake has created a model. He says that a recursive model for this sequence is

$$D(1) = 2$$
$$D(n) = D(n-1) + 2n$$

Show that either:

Jake is correct by confirming that his model works.

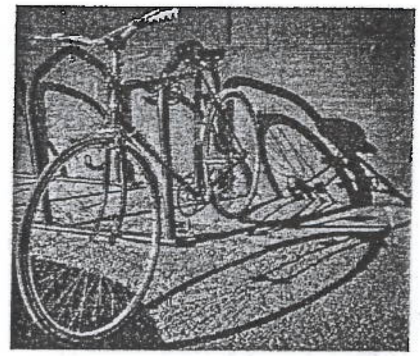
Jake is incorrect and modify his model to make it work.

Jake's model is correct. It reproduces the given sequence.

SAMPLE ANSWERS

Name . **Modeling With Functions**

Ready, Set, Go!



©2014 www.flickr.com/photos/bevgoodwin

Ready

Topic: Equations of lines

Write the equation of the line (in slope-intercept form) that is defined by the given information.

1. $A(5,9)B(7,17)$

2. $P(-3,8)Q(-4,13)$

3. $G(3,-10)H(1,-11)$

4. $L(-5,6)M(-8,8)$

5.

x	y
1	-1
6	1
11	3

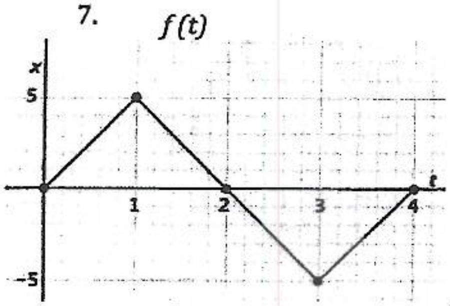
6.

x	y
1	2
5	5
9	8

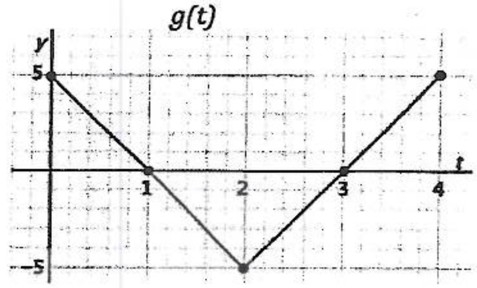
Set Topic: Parametric equations

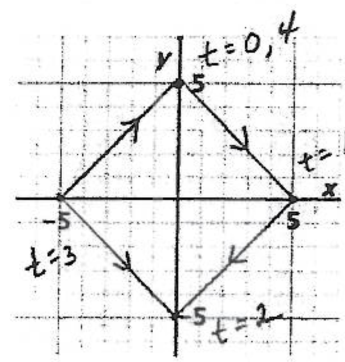
The two given graphs show the motion of a particle whose position at time *t seconds* is given by $x = f(t)$ *and* $y = g(t)$. Describe the motion of the particle. Then graph the two graphs as one graph in the xy plane. Connect the points to indicate the motion at each second.

7. $f(t)$



$g(t)$



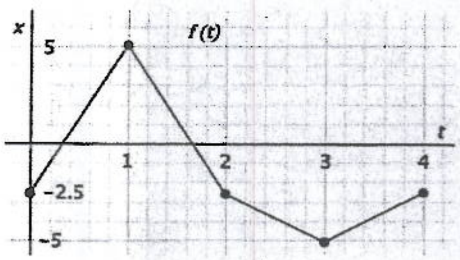


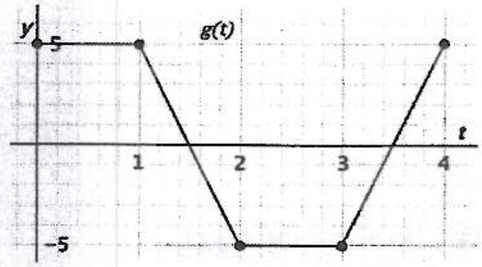
Describe the motion:

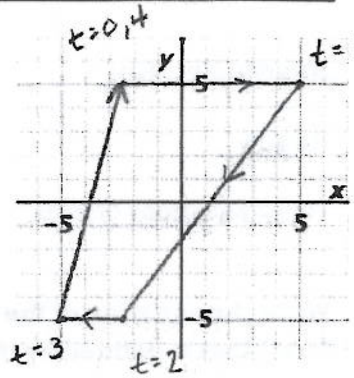
t	x	y
1	5	0
2	0	-5
3	-5	0
4	0	5

Square

8.

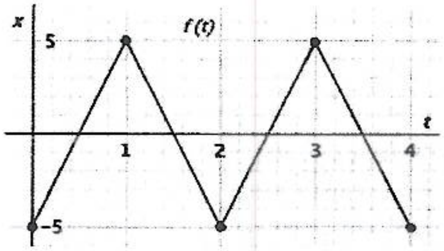


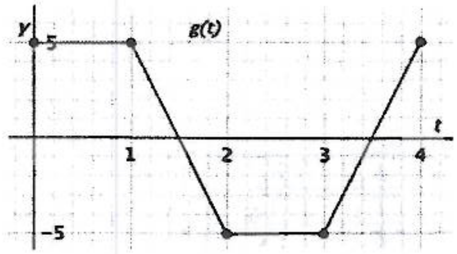


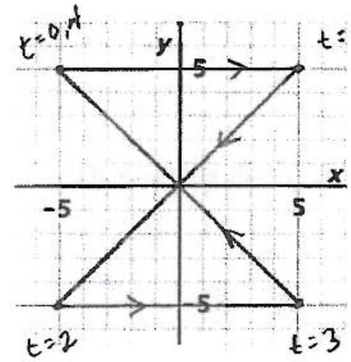


Describe the motion:

9.



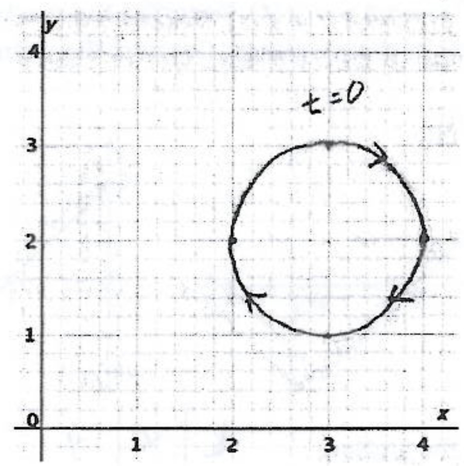




Describe the motion:

10. Fill in the table of values for the pair of parametric equations: $x = 3 + \sin t$ *and* $y = 2 + \cos t$. Sketch the graph of the two equations in the *xy* plane. Indicate the direction of the curve. Use the graph to write an equation for y as a function of x.

time (t)	$x = 3 + \sin t$	$y = 2 + \cos t$	(x, y)
0	3	3	
1	3.84	2.54	
2	3.91	1.58	
3	3.14	1.01	
4	2.24	1.34	
5	2.04	2.28	
6	2.76	2.96	
7	3.66	2.75	



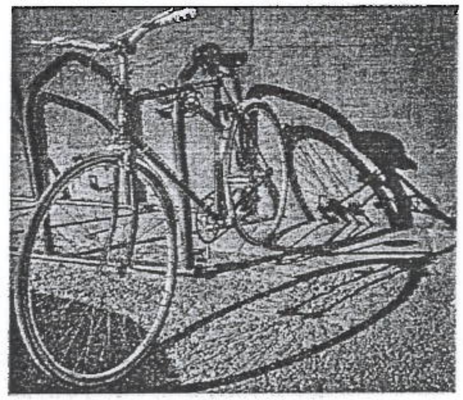
equation:

$(x-3)^2 + (y-2)^2 = 1$

SAMPLE ANSWERS

High Noon and Sunset Shadows Combined

A Develop Understanding Task



©2014 www.flickr.com/photos/bevgoodwin

In the task *High Noon and Sunset Shadows* we described the two-dimensional circular motion of a rider on a Ferris wheel by separating that motion into two components—the horizontal motion of the "high noon shadow" of the rider as it moved along the ground, and the vertical motion of the "sunset shadow" as it moved up and down along the wall of a building. Mathematicians refer to this process as *resolving the motion into its horizontal and vertical components.*

The following data was captured by filming a person's hand as she slowly traced an image in the air with the tip of a pencil. The first table captures the horizontal movement of the pencil—similar to watching the "high noon shadow" of the pencil moving across the floor. The second table captures the vertical movement of the pencil—similar to watching the "sunset shadow" of the pencil moving up or down the wall.

Table 1

time (sec)	horizontal position (inches)
0	2
1	4
2	6
3	4
4	6
5	9
6	11
7	12
8	10
9	8
10	11
11	12
12	10
12	8
14	4

Table 2

time (sec)	vertical position (inches)
0	11
1	9
2	12
3	3
4	12
5	13
6	12
7	10
8	7
9	8
10	7
11	4
12	2
12	2
14	3

1. Examine table 1 and describe what it tells you about the horizontal motion of the person's hand.

   Answers will vary.

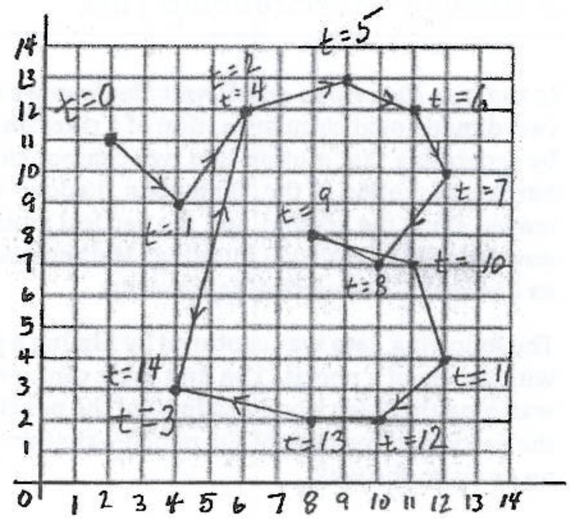
2. Examine table 2 and describe what it tells you about the vertical motion of the person's hand.

   Answers will vary.

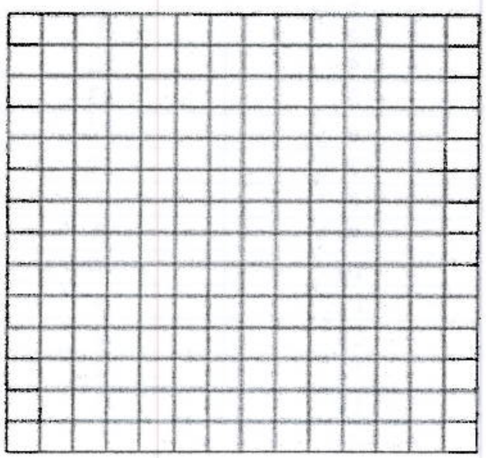
3. The person is tracing a familiar letter in the air. Can you guess what the letter is? Explain how you made your guess.

Answers will vary.

4. At each second we know the horizontal and vertical location of the tip of the pencil. Plot points on the grid at the right to indicate these locations. Connect these points in a way that would show the location of the pencil at instances in time between the seconds given. Based on this graph, what letter do you think the person was tracing?



The letter B



5. Draw a two-dimensional figure on the grid at the left. Create tables, like the ones above, to indicate where your pencil was at different moments in time as you drew your figure. Trade your tables with a partner and see if you can each replicate the figure that the other person drew based on the data you received.

Answers will vary.

Let's return to the rider on the Ferris wheel. The horizontal and vertical positions of the rider as a function of time are given by the following equations:

$$x(t) = 25\cos\left(\tfrac{\pi}{10}t\right)$$
$$y(t) = 25\sin\left(\tfrac{\pi}{10}t\right) + 30$$

6. How can you use these equations to determine the location of the rider at any instant in time? For example, how might you complete the following table?

time (seconds)	position of the rider
0	x = 25 y = 30
1	x = 23.8 y = 37.7
2	x = 20.2 y = 44.7
3	x = 14.7 y = 50.2
4	x = 7.7 y = 53.8
5	x = 0 y = 55
7.5	x = −17.7 y = 47.7
10	x = −25 y = 30
12.5	x = −17.7 y = 12.3
15	x = 0 y = 5
17.5	x = 17.7 y = 12.3
20	x = 25 y = 30

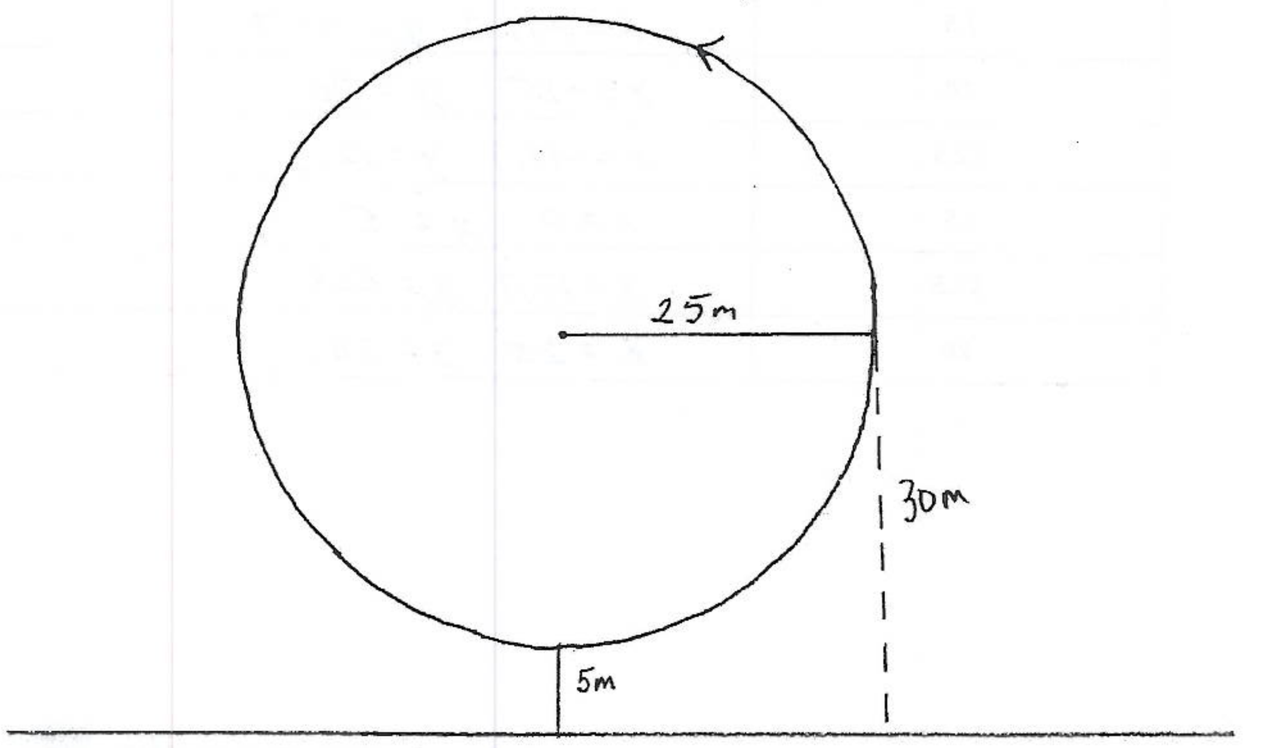
7. The horizontal and vertical position equations that describe the position of a rider on the Ferris wheel are given by the following parametric equations.

$$x(t) = 25\cos\left(\frac{\pi}{10}t\right)$$
$$y(t) = 25\sin\left(\frac{\pi}{10}t\right) + 30$$

Describe the physical characteristics of the Ferris wheel and its motion modeled by the equations.

The Ferris wheel has a radius of 25 meters.
The center of the Ferris wheel is 30 meters above the ground.
It takes 20 seconds for the Ferris wheel to make one revolution.

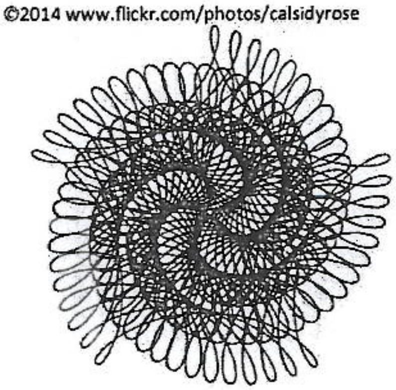
1 revolution every 20 seconds



Sample Answers

Name Modeling With Functions | 7.8H

Ready, Set, Go!

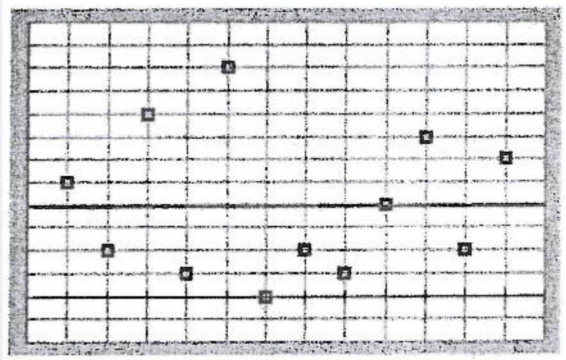


Ready

Topic: Measures of central tendency

1. Find the mean, median, and mode of the following test scores:
98, 74, 70, 68, 85, 82, 85, 94, 90, 91, 99, 85, 88, 79, 96, 98, 85, 82, 80, 86

2. The graph to the right shows 12 points whose y-values add up to 48. The y = 4 line is graphed. Use the position of the points on the graph to explain why 4 is the average of the 12 numbers.



Set Topic: Parametric equations

For each set of parametric equations,
a) Create a table of x- and y-values
b) Plot the points (x, y) in your table and sketch the graph. (Indicate the direction of the curve.)
c) Find the rectangular equation.

3. $x = 3t - 3$
$y = 2t + 1$

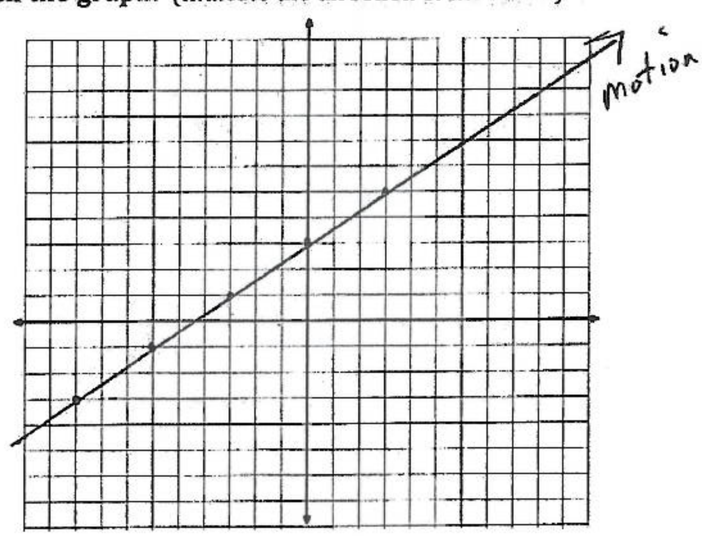
Rectangular equation:

$m = \frac{2}{3}$

$b = 3$

$y = \frac{2}{3}x + 3$

t	x	y
−2	−9	−3
−1	−6	−1
0	−3	1
1	0	3
2	3	5
3	6	7
4	9	9



Name

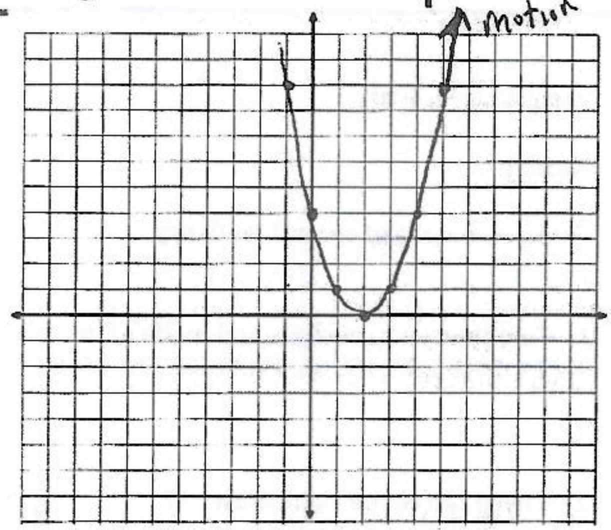
Modeling With Functions | 7.8H

4. $x = t + 2$
   $y = t^2$

t	x	y
−3	−1	9
−2	0	4
−1	1	1
0	2	0
1	3	1
2	4	4
3	5	9

Rectangular equation:

$y = (x-2)^2$



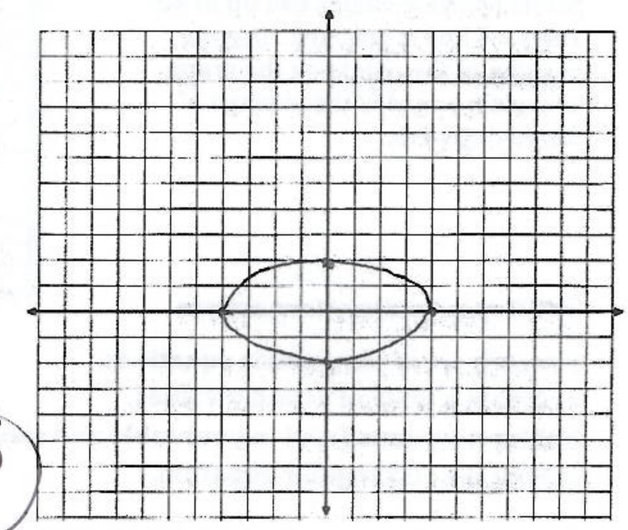
5. $x = 4\sin 2\theta$
   $y = 2\cos 2\theta$

t	x	y
0	0	2
$\pi/4$	4	0
$\pi/2$	0	−2
$3\pi/4$	−4	0
π	0	2
$5\pi/4$	4	0
$3\pi/2$	0	−2

Rectangular equation:

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

?



Match the parametric equations with their graphs.
(You may use a calculator.)

Which direction?

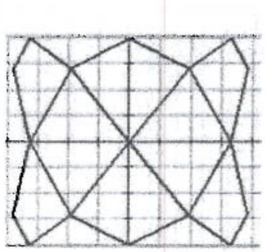
d 6. $x = 5\cos t$ *and* $y = \sin 3t$

a 7. $x = 6\sin(4t)$ *and* $y = 4\sin(6t)$

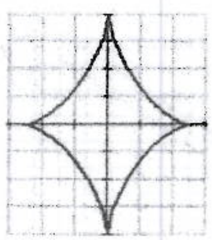
b 8. $x = 4\cos^3 t$ *and* $y = 4\sin^3 t$

C 9. $x = \frac{1}{2}(\cos t + t\sin t)$ *and* $y = \frac{1}{2}(\sin t - t\cos t)$

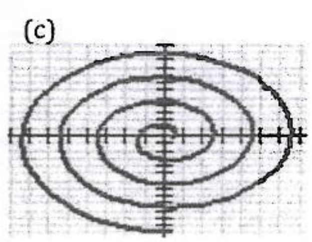
(a)



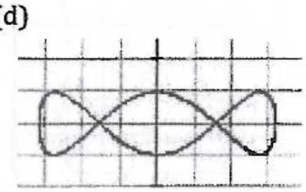
(b)



(c)



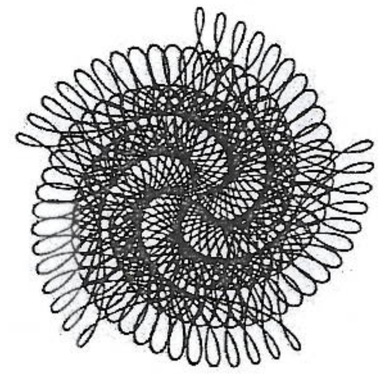
(d)



SAMPLE ANSWERS

7.8H Parametrically-Defined Curves

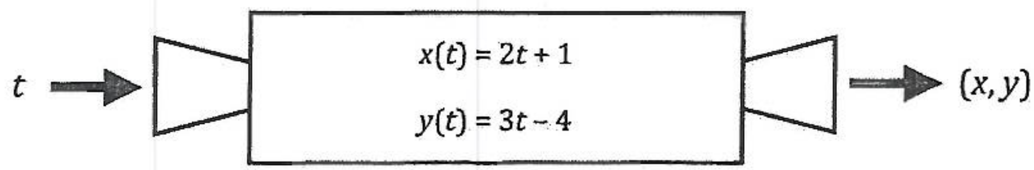
A Solidify and Practice Understanding Task



©2014 www.flickr.com/photos/calsidyrose

A **parametric curve** is defined by a set of equations that give the coordinates of points on the curve in terms of another variable called a *parameter*.

The following diagram suggests that a parametric curve can be defined as a function in which the input is a parameter variable t and the output is an ordered-pair (x, y). The rule that defines the output is a set of equations defined by $x(t)$ and $y(t)$.



1. The parametric curve defined in the diagram is a line.

   a. Find the slope of the line. $\frac{3}{2}$

   b. How does the slope of the line show up in its parametric representation?
   The numerator is the coefficient of t in the $y(t)$ function.
   The denominator is the coefficient of t in the $x(t)$ function.

   c. Find the y-intercept of the line.
   $-\frac{11}{2}$

   d. How does the y-intercept of the line relate to the parametric representation?
   The y-intercept occurs when $x = 0$. If $x = 0$, $t = -\frac{1}{2}$
   When $t = -\frac{1}{2}$, $y = -\frac{11}{2}$

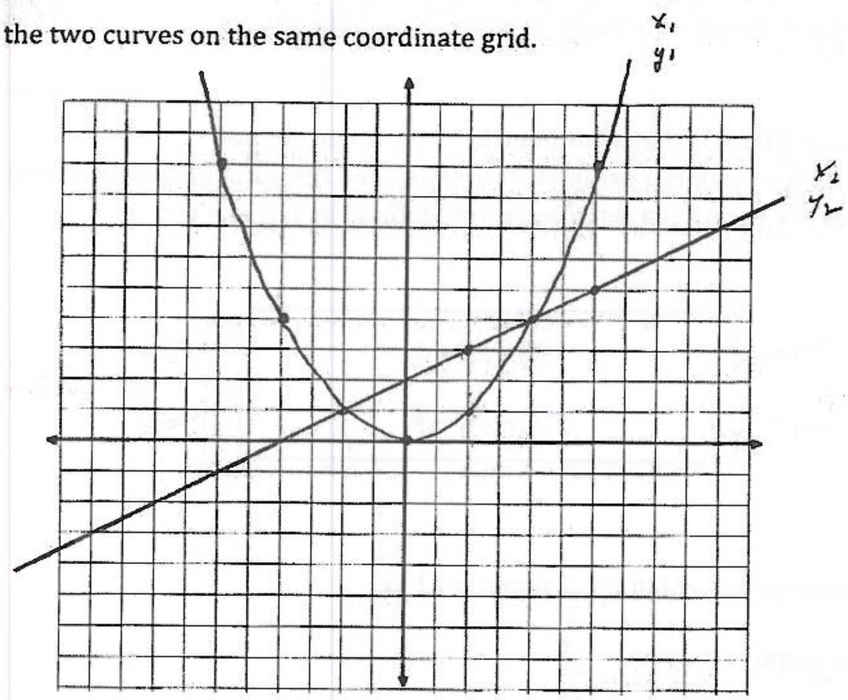
   e. Write the equation of the line $y = 2x + 3$ in parametric form.
   Answers will vary. $x = t$, $y = 2t + 3$ $\quad$ $x = t + 1$, $y = 2t + 5$

   f. Write the equation of the line $y = ½ (x - 2) + 4$ in parametric form.
   Answers will vary $x = t$, $y = \frac{1}{2}t + 3$ $\quad$ $x = 2t + 2$, $y = t + 4$

2. Given the following two parametrically-defined curves defined on the interval $-3 \le t \le 3$:

$$x_1(t) = 2t \qquad\qquad x_2(t) = 2t + 2$$
$$y_1(t) = t^2 \quad \text{and} \quad y_2(t) = t + 3$$

a. Graph the two curves on the same coordinate grid.



b. Each of these two curves is an easily recognizable function. Write the standard function equation for each of the curves.

$y_1 = \frac{1}{4} x_1^2$ $\qquad$ $y_2 = \frac{1}{2}x + 2$

c. Examine the relationship between the parametric form for each of these curves written in terms of the parameter t and its standard function form. Can you suggest a strategy for eliminating the parameter t in the parametric form of each curve to obtain the more familiar function form?

Solve the $x(t)$ equation for t, then substitute that express into the $y(t)$ equation.

d. Find the points of intersection of these two curves.

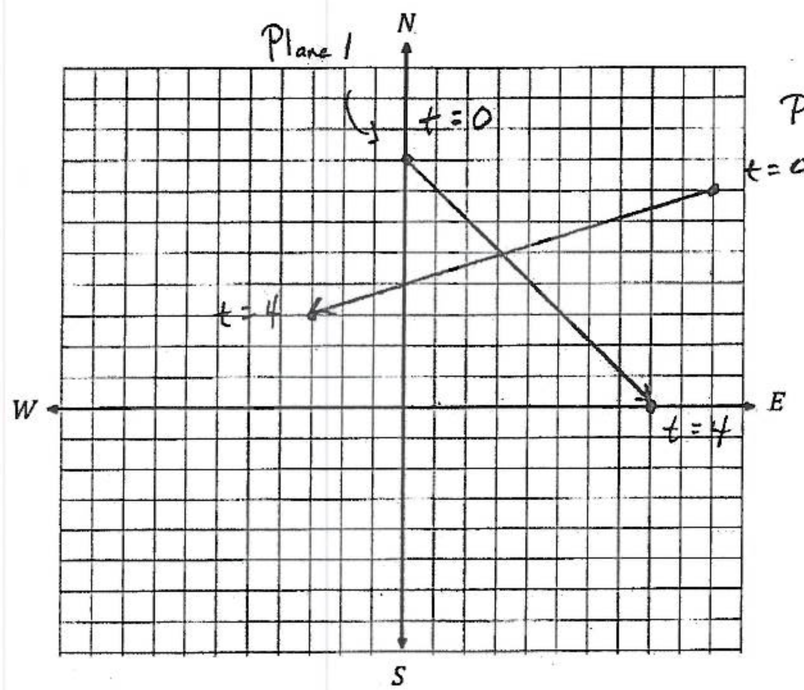
$(-2, 1)$ and $(4, 4)$

3. A flight controller is monitoring the flight paths of two planes, as given by the following two parametrically defined functions. The control tower is located at the origin and horizontal and vertical distances are measured in miles.

$$x_1(t) = 200t \qquad x_2(t) = -300t + 900$$
$$y_1(t) = -200t + 800 \quad \text{and} \quad y_2(t) = -100t + 700$$

   a. Graph the projected paths for each plane if they continue on their same course for the next four hours (that is, on the interval of time $0 \le t \le 4$).

Each square = 100



   b. Should the flight controller be concerned about a possible midair collision? Explain your reasoning.

No the paths of the planes cross at (300, 500)
However they reach that point at different times.
Plane 1 reaches that point when $t = 1\frac{1}{2}$.
Plane 2 reaches that point at $t = 2$.