Normal Distributions

Student Outcomes

- Students calculate z-scores.
- Students use technology and tables to estimate the area under a normal curve.
- Students interpret probabilities in context.

Lesson Notes

In this lesson, students calculate z-scores and use technology and tables to estimate the area under a normal curve. Depending on technology resources available to students, teachers may need to have students work with a partner or in small groups.

Classwork

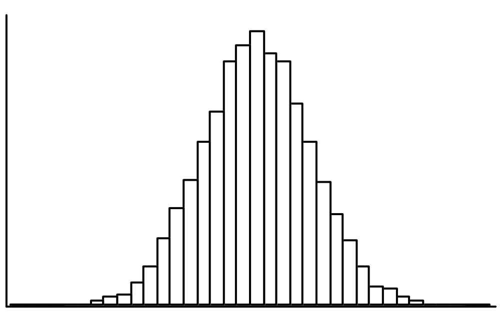
Exercise 1 (3 minutes)

This first exercise is a review of what is meant by a normal distribution. Use this as an opportunity to informally assess students' understanding of different distribution types by having students attempt the exercises independently.

Exercise 1

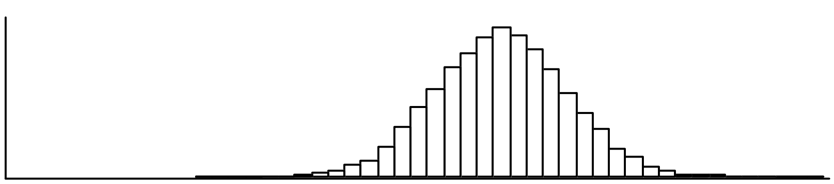
Consider the following data distributions. In the previous lesson, you distinguished between distributions that were approximately normal and those that were not. For each of the following distributions, indicate if it is approximately normal, skewed, or neither, and explain your choice:

a.



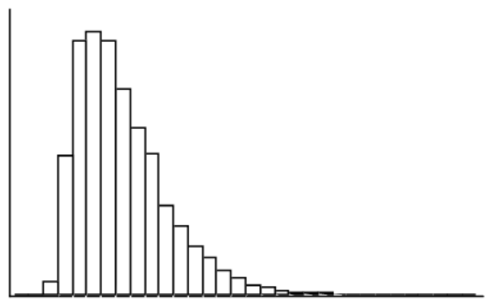
This distribution is approximately normal. It is approximately symmetric and mound shaped.

b.



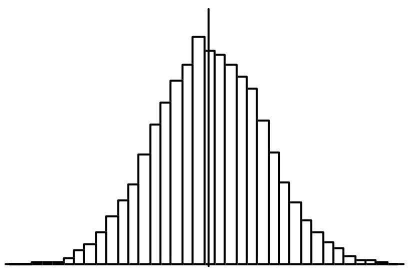
This distribution is approximately normal. It is approximately symmetric and mound shaped.

c.



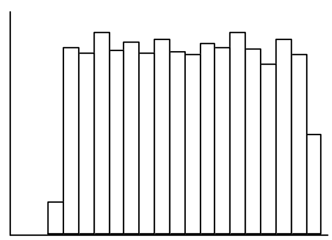
This distribution is not symmetric; therefore, it is not approximately normal. This distribution is skewed to the right as it has most of the data values in the beginning and then tapers off. It has a longer tail on the right.

d.



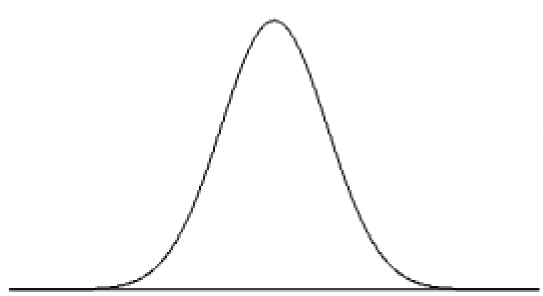
This distribution is approximately normal. It is symmetric and mound shaped.

e.



This distribution is not approximately normal. It is symmetric but not mound shaped. This distribution, however, is also not skewed. It does not have a longer tail on one side. It would be described as approximately a uniform distribution. Note that this distribution was discussed in earlier grades. It is important to emphasize that there are other types of data distributions and that not all are either approximately normal or skewed.

A normal distribution is a distribution that has a particular symmetric mound shape, as shown below.



Exercise 2 (5 minutes)

Prior to having students tackle this introduction to z-scores (or after they have completed it), it might be helpful to familiarize them with the reason why z-scores are important. Open with the following discussion:

MP.3

- Suppose that you took a math test and a Spanish test. The mean score for both tests was 80. You got an 86 in math and a 90 in Spanish. Did you necessarily do better in Spanish relative to your fellow students?
  - *No. For example, suppose the standard deviation of the math scores was 4 and the standard deviation of the Spanish scores was 8. Then, my score of 86 in math is $1\frac{1}{2}$ standard deviations above the mean, and my score in Spanish is only $1\frac{1}{4}$ standard deviations above the mean. Relative to the other students, I did better in math.*

In the above example, the z-score for math is 1.5, and the z-score for Spanish is 1.25. Negative z-scores indicate values that are below the mean.

Ask students to state in their own words what a z-score is. Essentially, look for an early understanding that a z-score measures distance in units of the standard deviation. For example, a z-score of 1 represents an observation that is at a distance of 1 standard deviation above the mean.

Scaffolding:

- For students working below grade level, consider framing the question this way: "Suppose that you took a math test and a Spanish test. The mean for both tests was 80; the standard deviation of the math scores was 4, and the standard deviation of the Spanish scores was 8. You got an 86 in math and a 90 in Spanish. On which test did you do better than most of your classmates?" Consider showing a visual representation of sample data distributions for each to aid understanding.
- For students working above grade level, consider posing the question this way: "Suppose that you took a math test and a Spanish test. The mean score for both was 80. You got an 86 in math and a 90 in Spanish. On which test did you do better than most of your classmates? Explain your reasoning."

Exercise 2

When calculating probabilities associated with normal distributions, z-scores are used. A z-score for a particular value measures the number of standard deviations away from the mean. A positive z-score corresponds to a value that is above the mean, and a negative z-score corresponds to a value that is below the mean. The letter z is used to represent a variable that has a standard normal distribution where the mean is 0 and standard deviation is 1. This distribution was used to define a z-score. A z-score is calculated by

$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}.$$

a. **The prices of the printers in a store have a mean of $\$240$ and a standard deviation of $\$50$. The printer that you eventually choose costs $\$340$.**

   i. **What is the z-score for the price of your printer?**

   $$z = \frac{340 - 240}{50} = 2$$

   ii. **How many standard deviations above the mean was the price of your printer?**

   The price of my printer was 2 standard deviations above the mean price.

b. **Ashish's height is 63 inches. The mean height for boys at his school is 68.1 inches, and the standard deviation of the boys' heights is 2.8 inches.**

i. **What is the z-score for Ashish's height? (Round your answer to the nearest hundredth.)**

$$z = \frac{63 - 68.1}{2.8} \approx -1.82$$

ii. **What is the meaning of this value?**

Ashish's height is 1.82 standard deviations below the mean height for boys at his school.

c. **Explain how a z-score is useful in describing data.**

A z-score is useful in describing how far a particular point is from the mean.

Example 1 (10 minutes): Use of z-Scores and a Graphing Calculator to Find Normal Probabilities

In this example, students are introduced to the process of calculating normal probabilities, and in this example and the two exercises that follow, z-scores are used along with a graphing calculator. (The use of tables of normal probabilities is introduced later in this lesson, and the use of spreadsheets is introduced in the next lesson.) Encourage students to always draw normal distribution curves and to show their work on the graph when working problems that involve a normal distribution.

Work through Example 3 with the class showing students how to calculate the relevant z-scores, and use a graphing calculator* to find the probability of interest. Students new to the curriculum may need additional support with the graphing calculator.

*Calculator note: The general form of this is *Normalcdf([left z bound],[right z bound])*. The *Normalcdf* function is accessed using *2nd, DISTR*. On selecting *2nd, DISTR*, some students using the more recent TI-84 operating systems might be presented with a menu asking for left bound, right bound, mean, and standard deviation. This can be avoided by having these students do the following: Press *2nd, QUIT* (to return to the home screen); press *MODE*; scroll down to the *NEXT* screen; set *STAT WIZARDS* to *OFF*.

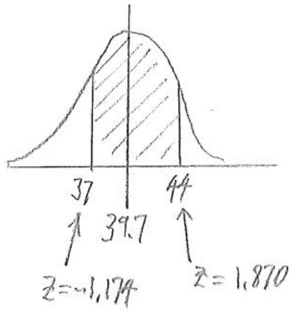
Example 1: Use of z-Scores and a Graphing Calculator to Find Normal Probabilities

A swimmer named Amy specializes in the 50-meter backstroke. In competition, her mean time for the event is 39.7 seconds, and the standard deviation of her times is 2.3 seconds. Assume that Amy's times are approximately normally distributed.

a. **Estimate the probability that Amy's time is between 37 and 44 seconds.**

The first time is a little less than 2 standard deviations from her mean time of 39.7 seconds. The second time is nearly 2 standard deviations above her mean time. As a result, the probability of a time between the two values covers nearly 4 standard deviations and would be rather large. I estimate 0.9, or 90%.

b. Using z-scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is between 37 and 44 seconds.



The z-score for 44 is $z = \frac{44 - 39.7}{2.3} \approx 1.870$, and the z-score for 37 is $z = \frac{37 - 39.7}{2.3} \approx -1.174$.

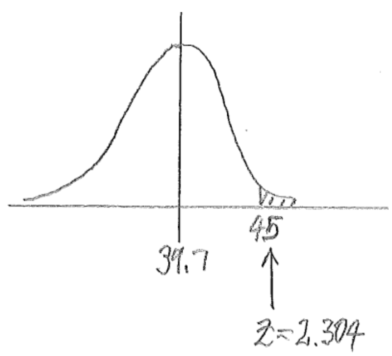
The probability that Amy's time is between 37 and 44 seconds is then found to be 0.849.

Note: If students are using TI-83 or TI-84 calculators, this result is found by entering Normalcdf$(-1.174, 1.870)$.

c. Estimate the probability that Amy's time is more than 45 seconds.

Amy's time of 45 seconds is more than 2 standard deviations from the mean of 39.7 seconds. There is a small probability that her time will be greater than 45 seconds. I estimate 0.03, or 3%.

d. Using z-scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is more than 45 seconds.



The z-score for 45 is $z = \frac{45 - 39.7}{2.3} \approx 2.304$.

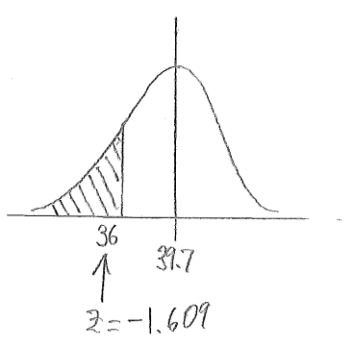
The probability that Amy's time is more than 45 seconds is then found to be 0.011.

Note: This result is found by entering Normalcdf$(2.304, 999)$ or the equivalent of this for other brands of calculators. The keystroke for positive infinity on some calculators is 1EE99. The number 999 can be replaced by any large positive number. Strictly speaking, the aim here is to find the area under the normal curve between $z = 2.304$ and positive infinity. However, it is impossible to enter positive infinity into the calculator, so any large positive number can be used in its place.

e. What is the probability that Amy's time would be at least 45 seconds?

Since Amy's times have a continuous distribution, the probability of "more than 45 seconds" and the probability of "at least 45 seconds" are the same. So, the answer is the same as part (d), 0.011.
It is worth pointing out to students that this applies to any question about the normal distribution.

f. Using z-scores and a graphing calculator and rounding your answers to the nearest thousandth, find the probability that Amy's time in her next race is less than 36 seconds.



The z-score for 36 is $z = \frac{36 - 39.7}{2.3} \approx -1.609$.

The probability that Amy's time is less than 36 seconds is then found to be 0.054.

Note: This result is found by entering Normalcdf$(-999, -1.609)$ or the equivalent of this command for other brands of calculators. Here, -999 is being used in place of negative infinity; any large negative number can be used in its place. The keystroke on some calculators for negative infinity is –1EE99.

Exercise 3 (8 minutes)

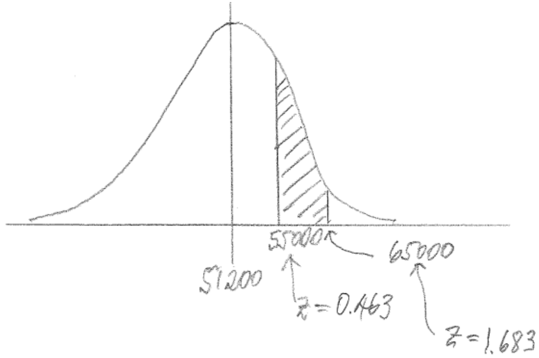
MP.4 This exercise provides practice with the skills students have learned in Example 2. Again, encourage students to draw a normal distribution curve for each part of the exercise showing work on their graphs in order to answer questions about the distribution. Let students work independently (if technology resources allow) and confirm answers with a neighbor.

Exercise 3

The distribution of lifetimes of a particular brand of car tires has a mean of $51,200$ miles and a standard deviation of $8,200$ miles.

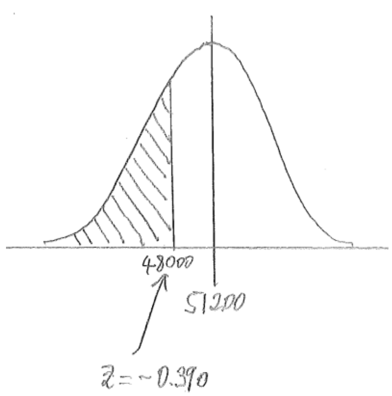
a. Assuming that the distribution of lifetimes is approximately normally distributed and rounding your answers to the nearest thousandth, find the probability of each event.

i. A randomly selected tire lasts between $55,000$ and $65,000$ miles.



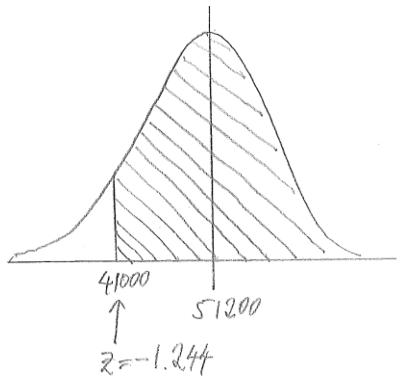
$\boldsymbol{P(\text{between } 55000 \text{ and } 65000) = 0.275}$

ii. A randomly selected tire lasts less than $48,000$ miles.



$\boldsymbol{P(\text{less than } 48000) = 0.348}$

iii. A randomly selected tire lasts at least $41,000$ miles.



$P(\text{greater than } 41000) = 0.893$

b. Explain the meaning of the probability that you found in part (a)(iii).

If a large number of tires of this brand were to be randomly selected, then you would expect about 89.3% of them to last more than $41,000$ miles.

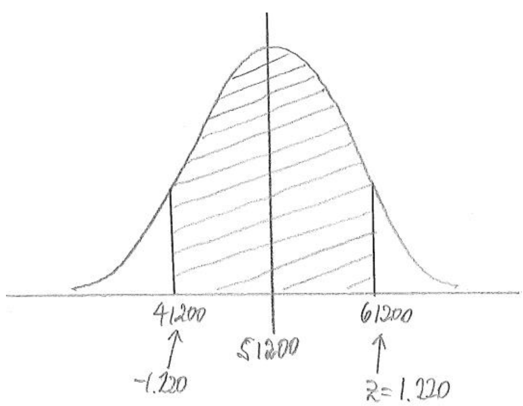
Exercise 4 (5 minutes)

MP.3

Here, students have to understand the idea of the lifetime of a tire being "within 10,000 miles of the mean." Some discussion of this idea might be necessary. Ask students what they think the statement is indicating about the lifetime of a tire. As they share their interpretations, record their summaries. Encourage students to use pictures, possibly involving a normal distribution, in their explanations. In this exercise, students can practice constructing arguments and critiquing the reasoning of others based on their understanding of a normal distribution.

Exercise 4

Think again about the brand of tires described in Exercise 3. What is the probability that the lifetime of a randomly selected tire is within $10,000$ miles of the mean lifetime for tires of this brand?



$P(\text{within } 10000 \text{ of mean}) = 0.778$

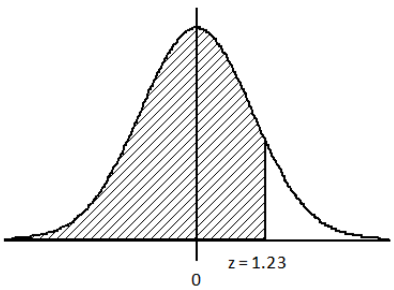
Example 2 (7 minutes): Using Table of Standard Normal Curve Areas

The last part of this lesson is devoted to the process of using tables of normal probabilities in place of the *Normalcdf* (or equivalent) function on the calculator. Completion of this example enables students who do not have access to a graphing calculator at home to complete the Problem Set. (However, if time is short, then this example could be reserved for Lesson 11, in which case Problem Set Problem 4 should be omitted from the assignment, and Lesson 11 should begin with this example.) Work through this example as a class showing students how to use the table to estimate the area under a normal curve.

Example 2: Using Table of Standard Normal Curve Areas

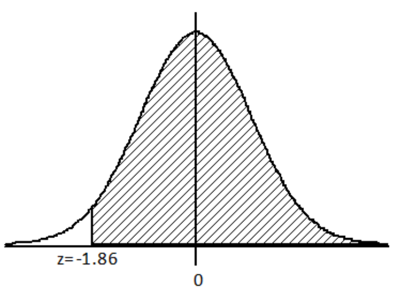
The standard normal distribution is the normal distribution with a mean of 0 and a standard deviation of 1. The diagrams below show standard normal distribution curves. Use a table of standard normal curve areas to determine the shaded areas.

a.



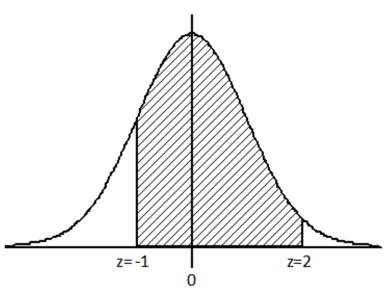
The provided table of normal areas gives the area to the <u>left</u> of the selected z-score. So, here, you find 1.2 in the column on the very left of the table, and then move horizontally to the column labeled 0.03. The table gives the required area (probability) to be 0.8907.

b.



In the left-hand column of the table, find -1.8, and move horizontally to the column labeled 0.06. The table gives a probability of 0.0314. Note that the table always supplies the area <u>to the left</u> of the chosen z-score. Also, note that the total area under any normal curve is 1. (This is the case for any probability distribution curve.) So, the required area, which is the area to the <u>right</u> of -1.86, is $1 - 0.0314 = 0.9686$.

c.



The approach here is to find the area to the left of $z = 2$ and to subtract the area to the left of $z = -1$. The table gives the area to the left of $z = 2.00$ to be 0.9772 and the area to the left of $z = -1.00$ to be 0.1587. So, the required area is $0.9772 - 0.1587 = 0.8185$.

Closing (2 minutes)

Remind students to be aware of order when calculating z-scores. Refer back to Exercise 2.

- The prices of printers in a store have a mean of $\$240$ and a standard deviation of $\$50$. The printer that you eventually choose costs $\$340$. What is wrong with the following z-score? How do you know?

$$z = \frac{240 - 340}{50} = \frac{-100}{50} = -2$$

  - *The z-score is negative, when it should be positive. The printer I chose is greater than the mean, which should result in a positive z-score.*

Have students interpret a probability in their own words. For example, refer back to Example 1.

- A swimmer named Amy specializes in the 50-meter backstroke. You found that the probability that Amy's time is between 37 and 44 seconds is 0.849. How would you interpret this?
  - *Approximately 84.9% of Amy's finish times are between 37 and 44 seconds.*

Ask students to summarize the main ideas of the lesson in writing or with a neighbor. Use this as an opportunity to informally assess comprehension of the lesson. The Lesson Summary below offers some important ideas that should be included.

> **Lesson Summary**
>
> **A *normal distribution* is a continuous distribution that has the particular symmetric mound-shaped curve that is shown at the beginning of the lesson.**
>
> **Probabilities associated with normal distributions are determined using z-scores and can be found using a graphing calculator or tables of standard normal curve areas.**

Exit Ticket (5 minutes)

Name ____ Date ____________________

Normal Distributions

Exit Ticket

The weights of cars passing over a bridge have a mean of $3,550$ pounds and standard deviation of 870 pounds. Assume that the weights of the cars passing over the bridge are normally distributed. Determine the probability of each instance, and explain how you found each answer.

a. The weight of a randomly selected car is more than $4,000$ pounds.

b. The weight of a randomly selected car is less than $3,000$ pounds.

c. The weight of a randomly selected car is between $2,800$ and $4,500$ pounds.

Exit Ticket Sample Solutions

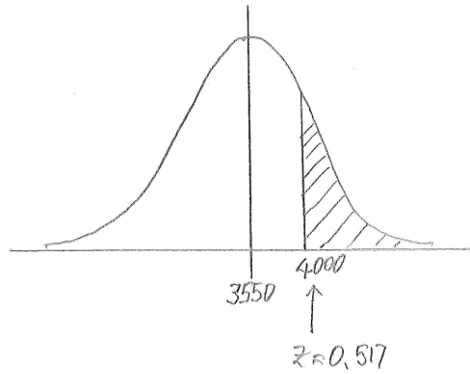
Student answers may vary if using a table versus a graphing calculator to determine the area under the normal curve.

The weights of cars passing over a bridge have a mean of $3,550$ pounds and standard deviation of 870 pounds. Assume that the weights of the cars passing over the bridge are normally distributed. Determine the probability of each instance, and explain how you found each answer.

a. **The weight of a randomly selected car is more than $4,000$ pounds.**

$P(\text{greater than } 4000) = 0.303$

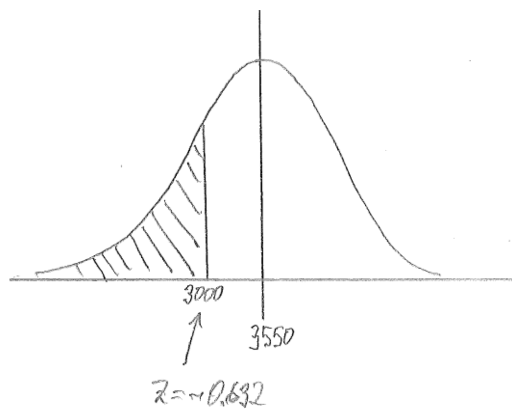
I calculated a z-score and then used my graphing calculator to find the area under the normal curve (above $z = 0.517$).



b. **The weight of a randomly selected car is less than $3,000$ pounds.**

$P(\text{less than } 3000) = 0.264$

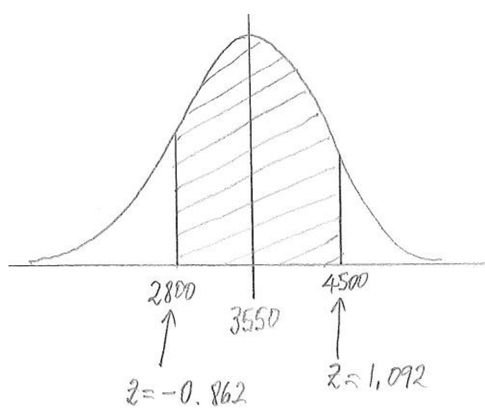
I calculated a z-score and then used my graphing calculator to find the area under the normal curve (below $z = -0.632$).



c. **The weight of a randomly selected car is between $2,800$ and $4,500$ pounds.**

$P(\text{between } 2800 \text{ and } 4500) = 0.668$

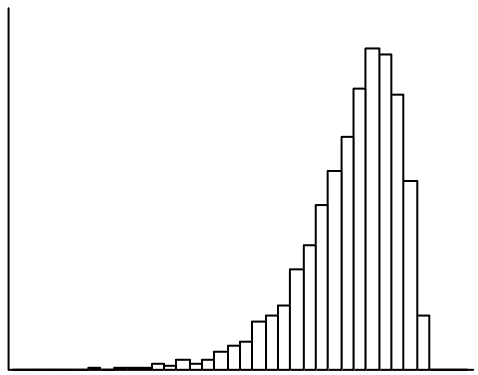
I calculated two z-scores and then used my graphing calculator to find the area under the normal curve (between $z = -0.862$ and $z = 1.092$).



Problem Set Sample Solutions

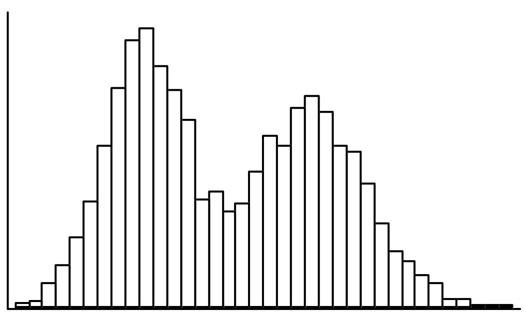
1. **Which of the following histograms show distributions that are approximately normal?**

   a.



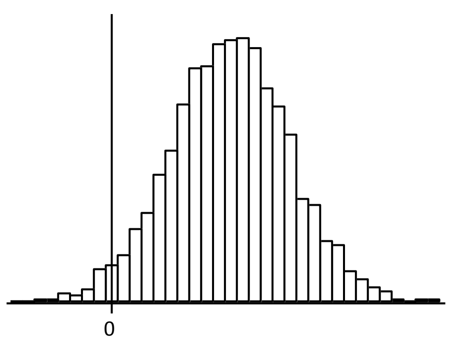
No, this distribution is not approximately normal.

   b.



No, this distribution is not approximately normal.

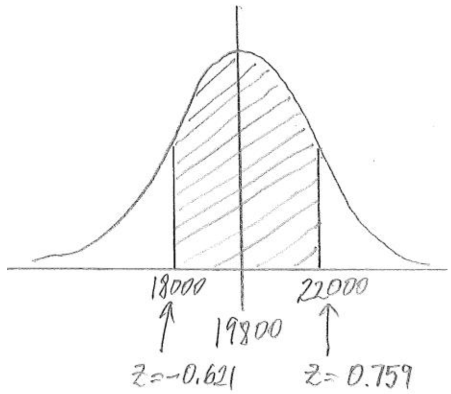
   c.



Yes, this distribution is approximately normal.

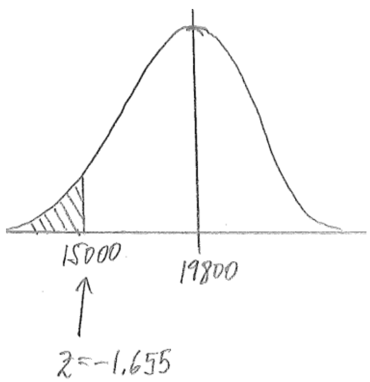
2. Suppose that a particular medical procedure has a cost that is approximately normally distributed with a mean of $\$19,800$ and a standard deviation of $\$2,900$. For a randomly selected patient, find the probabilities of the following events. (Round your answers to the nearest thousandth.)

a. The procedure costs between $\$18,000$ and $\$22,000$.



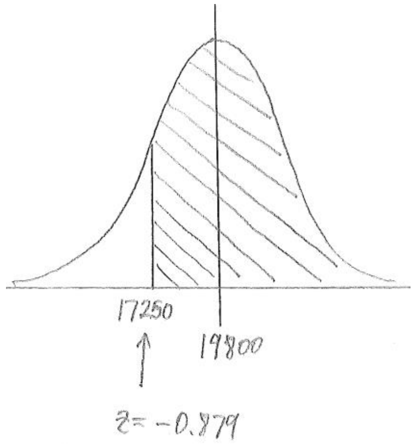
$P(\text{between 18000 and 22000}) = 0.509$

b. The procedure costs less than $\$15,000$.



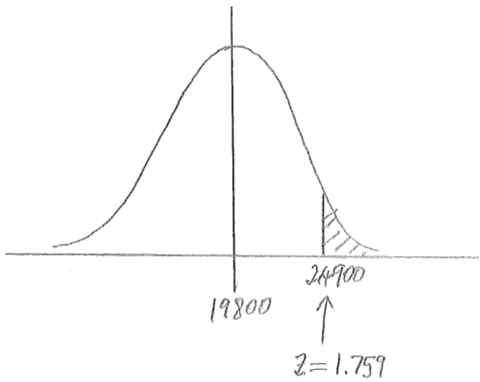
$P(\text{less than 15000}) = 0.049$

c. The procedure costs more than $\$17,250$.



$P(\text{greater than 17250}) = 0.810$

3. **Consider the medical procedure described in the previous question, and suppose a patient is charged $\$24,900$ for the procedure. The patient is reported as saying, "I've been charged an outrageous amount!" How justified is this comment? Use probability to support your answer.**



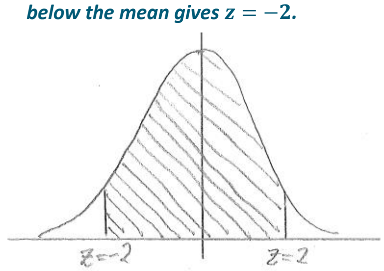
The probability that the procedure will cost at least $\$24,900$ is 0.039. So, this charge places the patient's bill in the top 4% of bills for this procedure. While the procedure turned out to be very expensive for this patient, use of the word "outrageous" could be considered a little extreme.

4. **Think again about the medical procedure described in Problem 2.**

   a. **Rounding your answers to the nearest thousandth, find the probability of each instance for a randomly selected patient.**

      i. **The cost of the procedure is within two standard deviations of the mean cost.**

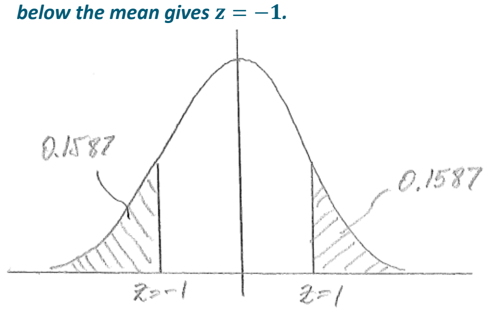
      The cost two standard deviations above the mean gives $z = 2$, and the price two standard deviations below the mean gives $z = -2$.



      The probability that the price is within 2 standard deviations of the mean is 0.954.

      ii. **The cost of the procedure is more than one standard deviation from the mean cost.**

      The cost one standard deviation above the mean gives $z = 1$, and the cost one standard deviation below the mean gives $z = -1$.



      $2(0.1587) = 0.317$

      The probability that the cost of the procedure is more than one standard deviation from the mean is 0.317.

b. **If the mean or the standard deviation were to be changed, would your answers to part (a) be affected? Explain.**

No. For example, looking at part (a)(i), the value two standard deviations above the mean will always have a z-score of 2, and the value two standard deviations below the mean will always have a z-score of -2. So, the answer will always be the same whatever the mean and the standard deviation. Similarly, in part (a)(ii), the z-scores will always be 1 and -1; therefore, the answer will always be the same, regardless of the mean and the standard deviation.

5. **Use a table of standard normal curve areas to find the following:**

a. **The area to the left of $z = 0.56$**

0.7123

b. **The area to the right of $z = 1.20$**

$1 - 0.8849 = 0.1151$

c. **The area to the left of $z = -1.47$**

0.0708

d. **The area to the right of $z = -0.35$**

$1 - 0.3632 = 0.6368$

e. **The area between $z = -1.39$ and $z = 0.80$**

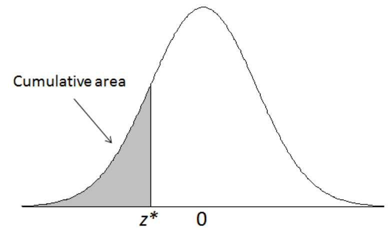
$0.7881 - 0.0823 = 0.7058$

f. **Choose a response from parts (a) through (f), and explain how you determined your answer.**

Answers will vary.

Sample response: To find the area to the right of $z = -0.35$ in part (d), I used the table to find the cumulative area up to (which is to the left of) $z = -0.35$. Then, to find the area to the right of the z-score, I had to subtract the cumulative area from 1.

Standard Normal Curve Areas



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
–3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
–3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
–3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
–3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
–3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
–3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0160	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0599
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
–0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
–0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
–0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
–0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
–0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
–0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
–0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
–0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
–0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999