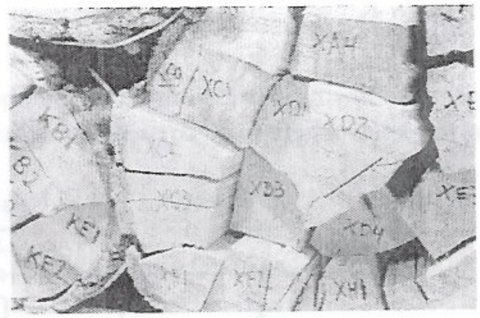
Sample Answers

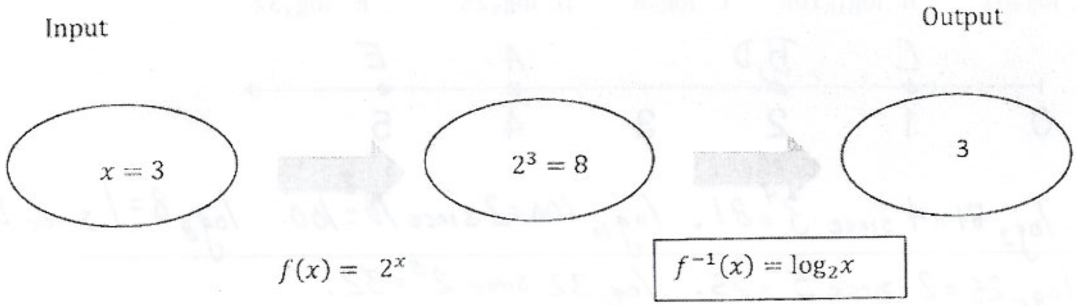
Log Logic

A Develop Understanding Task



We began thinking about logarithms as inverse functions for exponentials in Tracking the Tortoise. Logarithmic functions are interesting and useful on their own. In the next few tasks, we will be working on understanding logarithmic expressions, logarithmic functions, and logarithmic operations on equations.

We showed the inverse relationship between exponential and logarithmic functions using a diagram like the one below:



We could summarize this relationship by saying:

$2^3 = 8$ so, $\log_2 8 = 3$

Logarithms can be defined for any base used for an exponential function. Base 10 is popular. Using base 10, you can write statements like these:

$10^1 = 10$ so, $\log_{10} 10 = 1$

$10^2 = 100$ so, $\log_{10} 100 = 2$

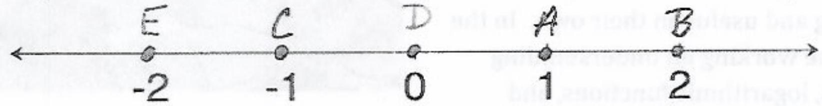
$10^3 = 1000$ so, $\log_{10} 1000 = 3$

The notation is a little strange, but you can see the inverse pattern of switching the inputs and outputs.

The next few problems will give you an opportunity to practice thinking about this pattern and possibly make a few conjectures about other patterns that you may notice with logarithms.

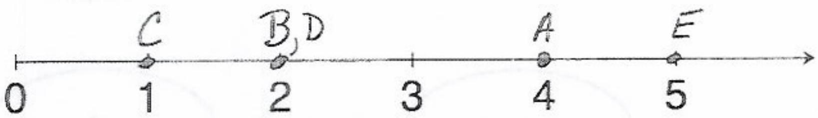
Place the following expressions on the number line. Use the space below the number line to explain how you knew where to place each expression.

1. A. $\log_3 3$ B. $\log_3 9$ C. $\log_3 \frac{1}{3}$ D. $\log_3 1$ E. $\log_3 \frac{1}{9}$



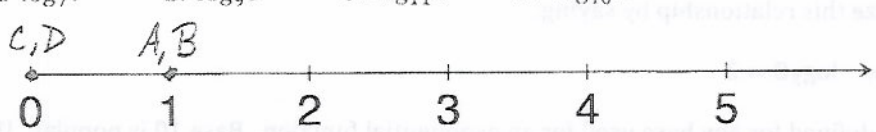
Explain: $\log_3 3 = 1$ since $3^1 = 3$. $\log_3 9 = 2$ since $3^2 = 9$. $\log_3 \frac{1}{3} = -1$ since $3^{-1} = \frac{1}{3}$.
$\log_3 1 = 0$ since $3^0 = 1$. $\log_3 \frac{1}{9} = -2$ since $3^{-2} = \frac{1}{9}$

2. A. $\log_3 81$ B. $\log_{10} 100$ C. $\log_8 8$ D. $\log_5 25$ E. $\log_2 32$



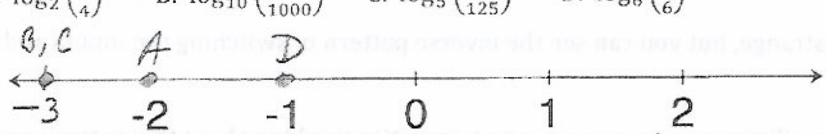
Explain: $\log_3 81 = 4$ since $3^4 = 81$. $\log_{10} 100 = 2$ since $10^2 = 100$. $\log_8 8 = 1$ since $8^1 = 8$
$\log_5 25 = 2$ since $5^2 = 25$. $\log_2 32$ since $2^5 = 32$.

3. A. $\log_7 7$ B. $\log_9 9$ C. $\log_{11} 1$ D. $\log_{10} 1$



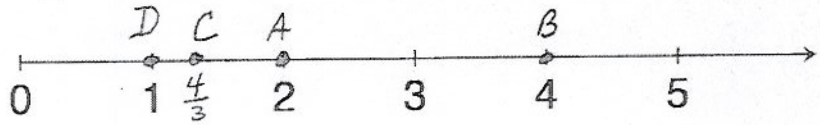
Explain: $\log_7 7 = 1$ since $7^1 = 7$. $\log_9 9 = 1$ since $9^1 = 9$
$\log_{11} 1 = 0$ since $11^0 = 1$. $\log_{10} 1 = 0$ since $10^0 = 1$

4. A. $\log_2 \left(\frac{1}{4}\right)$ B. $\log_{10} \left(\frac{1}{1000}\right)$ C. $\log_5 \left(\frac{1}{125}\right)$ D. $\log_6 \left(\frac{1}{6}\right)$



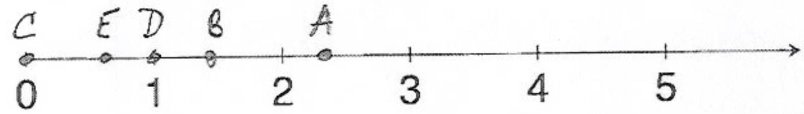
Explain: $\log_2 \frac{1}{4} = -2$ since $2^{-2} = \frac{1}{4}$. $\log_{10} \frac{1}{1000} = -3$ since $10^{-3} = \frac{1}{1000}$
$\log_5 \frac{1}{125} = -3$ since $5^{-3} = \frac{1}{125}$
$\log_6 \frac{1}{6} = -1$ since $6^{-1} = \frac{1}{6}$

5. A. $\log_4 16$ B. $\log_2 16$ C. $\log_8 16$ D. $\log_{16} 16$



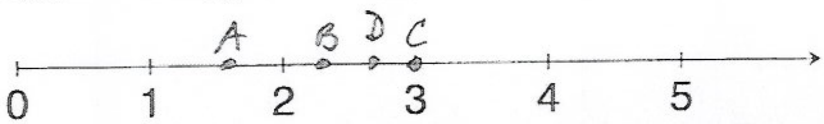
Explain: A, B, D as in # 1-4 C. $\log_8 16 = x$, $8^x = 16$, $2^{3x} = 2^4$, $3x = 4$, $x = \frac{4}{3}$

6. A. $\log_2 5$ B. $\log_5 10$ C. $\log_6 1$ D. $\log_5 5$ E. $\log_{10} 5$



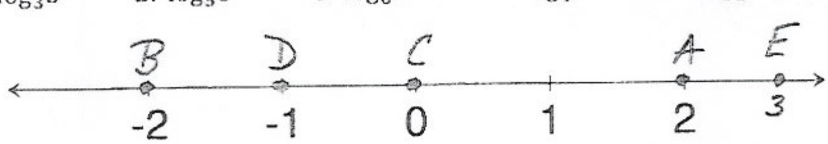
Explain: A. $\log_2 5$ is between 2 and 3. B. $\log_5 10$ is between 1 and 2.
E. $\log_{10} 5$ is between 0 and 1.

7. A. $\log_{10} 50$ B. $\log_{10} 150$ C. $\log_{10} 1000$ D. $\log_{10} 500$



Explain: The bases are equal, so the order of the points should be the same as the order of the arguments. I know that $\log_{10} 10 = 1$, $\log_{10} 100 = 2$, $\log_{10} 1000 = 3$ so the placement of A, B, D are approximate.

8. A. $\log_3 3^2$ B. $\log_5 5^{-2}$ C. $\log_6 6^0$ D. $\log_4 4^{-1}$ E. $\log_2 2^3$



Explain: A. $\log_3 9 = 2$ B. $\log_5 \frac{1}{25} = -2$ C. $\log_6 1 = 0$
D. $\log_4 \frac{1}{4} = -1$ E. $\log_2 8 = 3$.