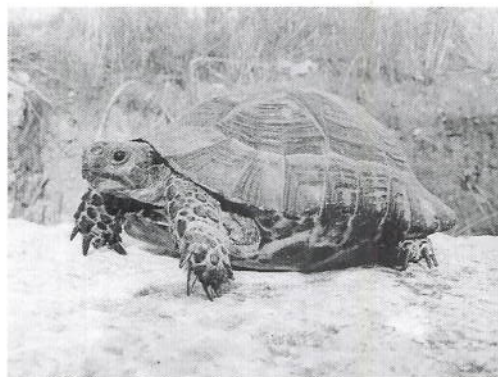


## 1.3 Tracking the Tortoise

### *A Solidify Understanding Task*

You may remember a task from last year about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race.



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In the task, we modeled the distance from the starting line that both the tortoise and the hare travelled during the race. Today we will consider only the journey of the tortoise in the race.

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

$$d(t) = 2^t \text{ (} d \text{ in meters and } t \text{ in seconds)}$$

The tortoise family decides to watch the race from the sidelines so that they can see their darling tortoise sister, Shellie, prove the value of persistence.

1. How far away from the starting line must the family be, to be located in the right place for Shellie to run by 10 seconds after the beginning of the race? After 20 seconds?

After 10 seconds:  $d(10) = 1,024$  meters

After 20 seconds:  $d(20) = 1,048,576$  meters

2. Describe the graph of  $d(t)$ , Shellie's distance at time  $t$ . What are the important features of  $d(t)$ ?

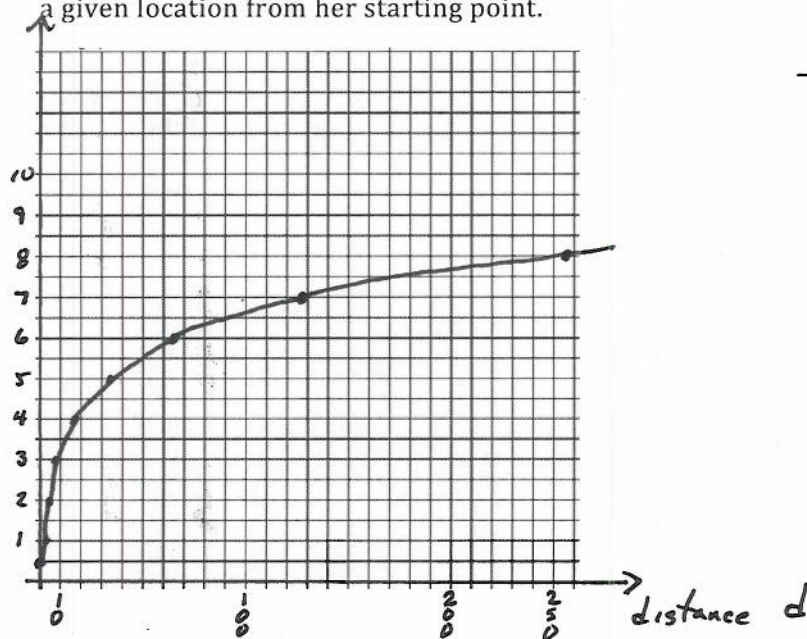
The graph is exponential. Its left endpoint is at  $(0, 1)$ . The graph of the function is increasing for all values of  $t$ .

3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past? *6 seconds,  $2^6 = 64$*
4. How long must they wait to see Shellie run by if they stand 1024 meters away from her starting point?

*10 seconds,  $2^{10} = 1024$*

5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point.

*time  
 $t(d)$*



Distance $d$	Time $d(t)$
1	0
2	1
4	2
8	3
16	4
32	5
64	6
128	7
256	8
512	9

6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point? *From the table, between 7 and 8 seconds.*

*If solved numerically or graphically, approximately 7.78 seconds.*

7. What is the relationship between  $d(t)$  and the graph that you have just drawn? How did you use  $d(t)$  to draw the graph in #5?

*The graphs of  $d(t)$  and the graph in #5 represent inverse functions. If I know the domain and range values of  $d(t)$ , I can switch those to find points*

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*On the inverse.*

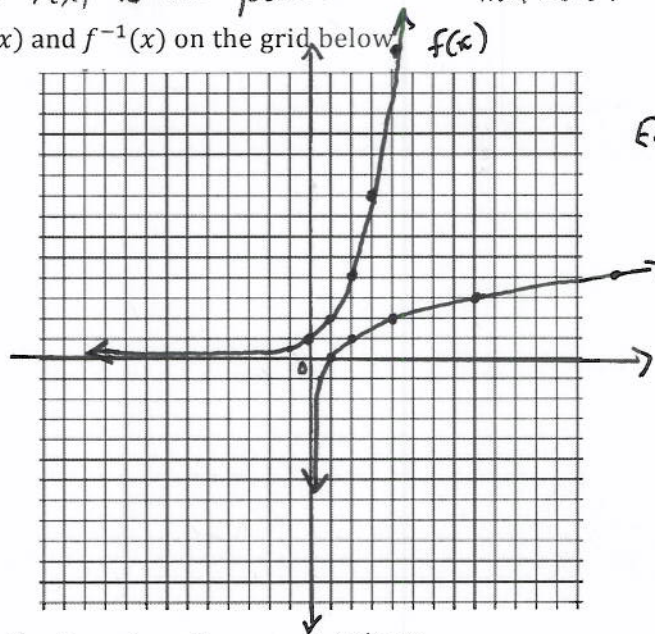
8. Consider the function  $f(x) = 2^x$ .

A) What are the domain and range of  $f(x)$ ? Is  $f(x)$  invertible?  $\rightarrow$  Yes.

Domain of  $f(x)$  is all real numbers.

Range of  $f(x)$  is all positive real numbers.

- B) Graph  $f(x)$  and  $f^{-1}(x)$  on the grid below.



Each square is one unit.

- C) What are the domain and range of  $f^{-1}(x)$ ?

Domain of  $f^{-1}(x)$  is all positive real numbers.

Range of  $f^{-1}(x)$  is all real numbers.

9. If  $f(3) = 8$ , what is  $f^{-1}(8)$ ? How do you know?

$f^{-1}(8) = 3$  The domain values of  $f$  become the range values of  $f^{-1}(x)$   
The range values of  $f$  become the domain values of  $f^{-1}(x)$

10. If  $f\left(\frac{1}{2}\right) = 1.414$ , what is  $f^{-1}(1.414)$ ? How do you know?

$$f^{-1}(1.414) = \frac{1}{2}$$

For the inverse function, the domain element and the corresponding range element are switched

11. If  $f(a) = b$  what is  $f^{-1}(b)$ ? Will your answer change if  $f(x)$  is a different function? Explain.

$$f^{-1}(b) = a.$$

This should be the same as long as  $f(x)$  has a function that is an inverse.