Problem Set Sample Solutions

The Problem Set gives students an opportunity to practice using the numerical methods for approximating solutions to exponential equations that they have established in this lesson.

1. **Solve each of the following equations for x using the same technique as was used in the Opening Exercises.**

   a. $2^x = 32$

      $x = 5$

   b. $2^{x-3} = 2^{2x+5}$

      $x = -8$

   c. $2^{x^2-3x} = 2^{-2}$

      $x = 1$ *or* $x = 2$

   d. $2^x - 2^{4x-3} = 0$

      $x = 1$

   e. $2^{3x} \cdot 2^5 = 2^7$

      $x = \frac{2}{3}$

   f. $2^{x^2-16} = 1$

      $x = 4$ *or* $x = -4$

   g. $3^{2x} = 27$

      $x = \frac{3}{2}$

   h. $3^{\frac{2}{x}} = 81$

      $x = \frac{1}{2}$

   i. $\frac{3^{x^2}}{3^{5x}} = 3^6$

      $x = 6$ *or* $x = -1$

2. **Solve the equation $\frac{2^{2x}}{2^{x+5}} = 1$ algebraically using two different initial steps as directed below.**

   a. **Write each side as a power of 2.**

      $2^{2x-(x+5)} = 2^0$

      $x - 5 = 0$

      $x = 5$

   b. **Multiply both sides by 2^{x+5}.**

      $2^{2x} = 2^{x+5}$

      $2x = x + 5$

      $x = 5$

3. Find consecutive integers that are under and over estimates of the solutions to the following exponential equations.

a. $2^x = 20$

$2^4 = 16$ *and* $2^5 = 25$, *so* $4 < x < 5$.

b. $2^x = 100$

$2^6 = 64$ *and* $2^7 = 128$, *so* $6 < x < 7$.

c. $3^x = 50$

$3^3 = 27$ *and* $3^4 = 81$, *so* $3 < x < 4$.

d. $10^x = 432,901$

$10^5 = 100,000$ *and* $10^6 = 1,000,000$, *so* $5 < x < 6$.

e. $2^{x-2} = 750$

$2^9 = 512$ *and* $2^{10} = 1,024$, *so* $9 < x - 2 < 10$; *thus,* $11 < x < 12$.

f. $2^x = 1.35$

$2^0 = 1$ *and* $2^1 = 2$, *so* $0 < x < 1$.

4. Complete the following table to approximate the solution to $10^x = 34,198$ to two decimal places.

t	$P(t)$
1	10
2	100
3	$1,000$
4	$10,000$
5	$100,000$

t	$P(t)$
4.1	$12,589.254$
4.2	$15,848.932$
4.3	$19,952.623$
4.4	$25,118.864$
4.5	$31,622.777$
4.6	$39,810.717$

t	$P(t)$
4.51	$32,359.366$
4.52	$33,113.112$
4.53	$33,884.416$
4.54	$34,673.685$

t	$P(t)$
4.531	$33,962.527$
4.532	$34,040.819$
4.533	$34,119.291$
4.534	$34,197.944$
4.535	$34,276.779$

$10^x = 34,198$

$10^{4.53} \approx 34,198$

5. Complete the following table to approximate the solution to $2^x = 18$ to two decimal places.

t	$P(t)$
1	2
2	4
3	8
4	16
5	32

t	$P(t)$
4.1	17.1484
4.2	18.3792

t	$P(t)$
4.11	17.2677
4.12	17.3878
4.13	17.5087
4.14	17.6305
4.15	17.7531
4.16	17.8766
4.17	18.0009

t	$P(t)$
4.161	17.8890
4.162	17.9014
4.163	17.9138
4.164	17.9262
4.165	17.9387
4.166	17.9511
4.167	17.9635
4.168	17.9760
4.169	17.9884
4.170	18.0009

$2^x = 18$

$2^{4.17} \approx 18$

6. Approximate the solution to $5^x = 5555$ to four decimal places.

Since $5^5 = 3125$ and $5^6 = 15,625$, we know that $5 < x < 6$.

Since $5^{5.3} \approx 5064.5519$ and $5^{5.4} \approx 5948.9186$, we know that $5.3 < x < 5.4$.

Since $5^{5.35} \approx 5488.9531$ and $5^{5.36} \approx 5578.0092$, we know that $5.35 < x < 5.36$.

Since $5^{5.357} \approx 5551.1417$ and $5^{5.358} \approx 5560.0831$, we know that $5.357 < x < 5.358$.

Since $5^{5.3574} \approx 5554.7165$ and $5^{5.3575} \approx 5555.6106$, we know that $5.3574 < x < 5.3575$.

Since $5^{5.35743} \approx 5554.9847$ and $5^{5.35744} \approx 5555.0741$, we know that $5.35743 < x < 5.35744$.

Thus, the approximate solution to this equation to two decimal places is 5.3574.

7. A dangerous bacterial compound forms in a closed environment but is immediately detected. An initial detection reading suggests the concentration of bacteria in the closed environment is one percent of the fatal exposure level. This bacteria is known to double in growth (double in concentration in a closed environment) every hour and can be modeled by the function $P(t) = 100 \cdot 2^t$, where t is measured in hours.

   a. In the function $P(t) = 100 \cdot 2^t$, what does the 100 mean? What does the 2 mean?

   The 100 represents the initial population of bacteria, which is 1% of the fatal level. This means that the fatal level occurs when $P(t) = 10,000$. The base 2 represents the growth rate of the bacteria; it doubles every hour.

   b. Doctors and toxicology professionals estimate that exposure to two-thirds of the bacteria's fatal concentration level will begin to cause sickness. Without consulting a calculator or other technology, offer a rough time limit for the inhabitants of the infected environment to evacuate in order to avoid sickness in the doctors' estimation. Note that immediate evacuation is not always practical, so offer extra evacuation time if it is affordable.

   The bacteria level is dangerous when $P(t) = 100 \cdot 2^t = \frac{2}{3}(10,000) \approx 6666.67$.

   Since $2^6 = 64$, $P(6) \approx 6400$, so inhabitants of the infected area should evacuate within 6 hours to avoid sickness.

c. **A more conservative approach is to evacuate the infected environment before bacteria concentration levels reach one-third of fatal levels. Without consulting a calculator or other technology, offer a rough time limit for evacuation in this circumstance.**

Under these guidelines, The bacteria level is dangerous when $P(t) = 100 \cdot 2^t = \frac{1}{3}(10,000) \approx 3333.33$. Since $2^5 = 32$, $P(5) \approx 3200$, so the conservative approach is to recommend evacuation within 5 hours.

d. **Use the method of the Example to approximate when the infected environment will reach fatal levels (100%) of bacteria concentration, to the nearest minute.**

We need to approximate the solution to $100 \cdot 2^t = 10,000$, which is equivalent to solving $2^t = 100$.

t	2^t
1	2
2	4
3	8
4	16
5	32
6	64
7	128

t	2^t
6.1	68.5935
6.2	73.5167
6.3	78.7932
6.4	84.4485
6.5	90.5097
6.6	97.0059
6.7	103.9683

t	2^t
6.61	97.6806
6.62	98.3600
6.63	99.0442
6.64	99.7331
6.65	100.4268

t	2^t
6.641	99.8022
6.642	99.8714
6.643	99.9407
6.644	100.0010

t	2^t
6.6436	99.9822
6.6437	99.9892
6.6438	99.9961
6.6439	100.0030

Inhabitants need to evacuate within 6.644 hours, which is approximately 6 hours and 39 minutes.