1.2 Flipping Ferraris

A Solidify Understanding Task

When people first learn to drive, they are often told that the faster they are driving, the longer it will take to stop. So, when you're driving on the freeway, you should leave more space between your car and the car in front of you than when you are driving slowly through a neighborhood. Have you ever wondered about the relationship between how fast you are driving and how far you travel before you stop, after hitting the brakes?

1. Think about it for a minute. What factors do you think might make a difference in how far a car travels after hitting the brakes?

   the car's speed

There has actually been quite a bit of experimental work done (mostly by police departments and insurance companies) to be able to mathematically model the relationship between the speed of a car and the braking distance (how far the car goes until it stops after the driver hits the brakes).

2. Imagine your dream car. Maybe it is a Ferrari 550 Maranello, a super-fast Italian car. Experiments have shown that on smooth, dry roads, the relationship between the braking distance (d) and speed (s) is given by $d(s) = 0.03s^2$. Speed is given in miles/hour and the distance is in feet.

   a) How many feet should you leave between you and the car in front of you if you are driving the Ferrari at 55 mi/hr?

   $d(55) = 0.03(55)^2 = 0.03 \times 3025$

   $d(55) = 90.75$ ft

   b) What distance should you keep between you and the car in front of you if you are driving at 100 mi/hr?

   $d(100) = 0.03(100)^2 = 0.03 \times 10{,}000$

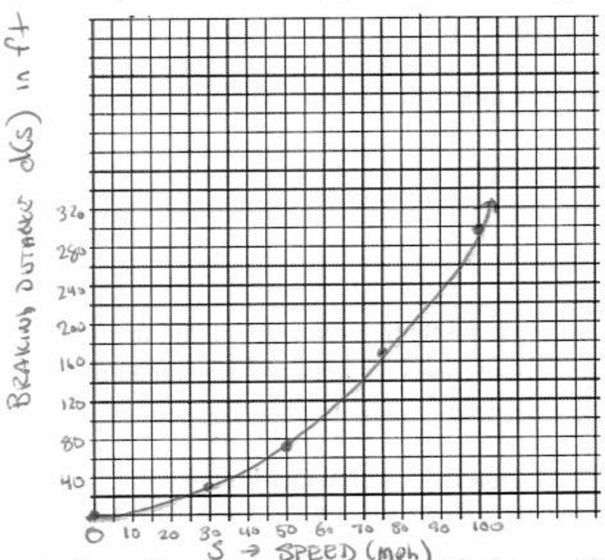
   $d(100) = 300$ ft

   c) If an average car is about 16 feet long, about how many car lengths should you have between you and that car in front of you if you are driving 100 mi/hr?

   $\frac{300}{16} = 18.75 \sim 19$ car lengths

d) It makes sense to a lot of people that if the car is moving at some speed and then goes twice as fast, the braking distance will be twice as far. Is that true? Explain why or why not. We know $d(100) = 300$ ft. If this is true, $d(50)$ should be half that distance. However, $d(50) = 0.03(50)^2 = 0.03 \times 2500 = 75$ ft, which is only 1/4 of the braking distance 300 ft.

3. Graph the relationship between braking distance *d(s),* and speed *(s)*, below.



s	d(s)
0	0
30	27
50	75
75	168.75
100	300

4. Describe all the mathematical features of the relationship between braking distance and speed for the Ferrari modeled by $d(s) = 0.03s^2$.

- the rate of change is not constant
- the relationship between braking distance and speed is quadratic in nature
- the second differences of the braking distances is constant

5. What if the driver of the Ferrari 550 was cruising along and suddenly hit the brakes to stop because she saw a cat in the road? She skidded to a stop, and fortunately, missed the cat. When she got out of the car she measured the skid marks left by the car so that she knew that her braking distance was 31 ft.

a) How fast was she going when she hit the brakes?

$d(s) = 0.03s^2$ $\qquad d(s) = 31$

$\frac{31}{0.03} = \frac{0.03s^2}{0.03}$ $\qquad 1033.33 = s^2$ $\qquad s = \sqrt{1033.33} = 32.15$ mph

c) If she didn't see the cat until she was 15 feet away, what is the fastest speed she could be traveling before she hit the brakes if she wants to avoid hitting the cat?

$$\frac{15}{0.03} = \frac{0.03s^2}{0.03}$$

$$500 = s^2$$

$$s = \sqrt{500} = 22.36 \text{ mph}$$

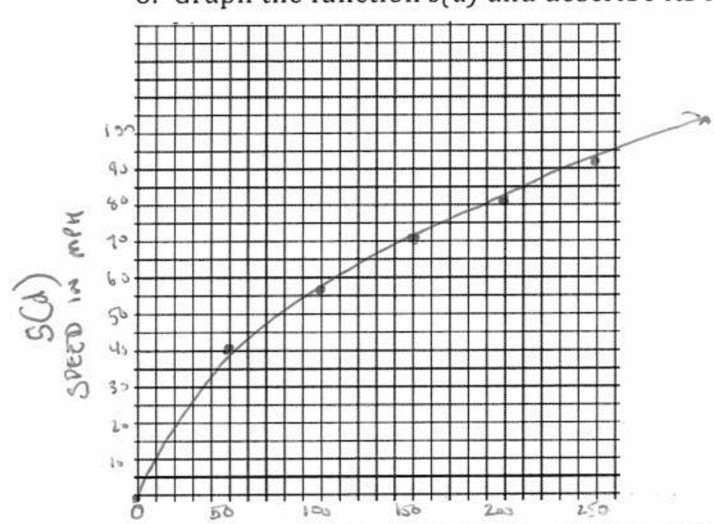
6. Part of the job of police officers is to investigate traffic accidents to determine what caused the accident and which driver was at fault. They measure the braking distance using skid marks and calculate speeds using the mathematical relationships just like we have here, although they often use different formulas to account for various factors such as road conditions. Let's go back to the Ferrari on a smooth, dry road since we know the relationship. Create a table that shows the speed the car was traveling based upon the braking distance.

d	s(d) mph
0	0
50	40.82
100	57.74
150	70.71
200	81.65
250	91.29
300	100

7. Write an equation of the function *s(d)* that gives the speed the car was traveling for a given braking distance.

$$s(d) = \sqrt{\frac{d}{0.03}}$$

8. Graph the function *s(d)* and describe its features.



- Vertex @ (0,0)
- always increasing
- increases at slower rate as x increases (rate of change is not constant)
- relationship is modeled w/ a square root function

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BRAKING DISTANCE in ft

d

9. What do you notice about the graph of *s(d)* compared to the graph of *d(s)*? What is the relationship between the functions *d(s)* and *s(d)*?

They are both increasing. However s(d) increases quickly at first and then at a slower rate. d(s) increases slowly at first, then at a higher rate. The points on each curve are symmetric with respect to y = x. That is (x, y) → (y, x).

These are inverse functions.

10. Consider the function $d(s) = 0.03s^2$ over the domain of all real numbers, not just the domain of this problem situation. How does the graph change from the graph of *d(s)* in question #3?

The graph would be a parabola with vertex at (0, 0).
It would be decreasing at $x \leq 0$.
Symmetry w/ respect to the y-axis.

11. How does changing the domain of *d(s)* change the graph of the inverse of *d(s)*?

The inverse of d(s) would no longer be a function, as each input would have two outputs.

The graph would look like a parabola that opens right.

12. Is the inverse of *d(s)* a function? Justify your answer.

No. Each input would be assigned two outputs.