Key

UNIT 5

HONORS

ALGEBRA 2

SLT 7

COMPOUND EVENTS

Name: **Probability**

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Ready

Topic: Independent and Dependent events

In some of the situations described below the first event effects the subsequent event (dependent events). In others each of the events is completely independent of the others (independent events). Determine which situations are dependent and which are independent.

1. A coin is flipped twice. The first event is the first flip and the second event is the next flip.

   INDEPENDENT

2. A bag of marbles contains 3 blue marbles, 6 red marbles and 2 yellow marbles. Two of the marbles are drawn out of the bag. The first event is the first marble taken out the second event is the second marble taken out. Independent, if 1st marble is put BACK IN

   Dependent, if 1st MARBLE is held out

3. An attempt to find the probability of there being a right-handed or a left-handed batter a the plate in a baseball game. The first event is the 1st batter to come to the plate. The second event is the second player to come up to the plate. Dependent (3 lefties on the team)

4. A standard die is rolled twice. The first event is the first roll and the second event is the second roll. INDEPENDENT

5. Two cards are drawn from a standard deck of cards. The first event is the first card that is drawn the second event is the second card that is drawn.

   Independent if 1st CARD is put BACK into deck

   Dependent if 1st CARD is not put BACK into deck

Name:

Compound Events

Date: Period:

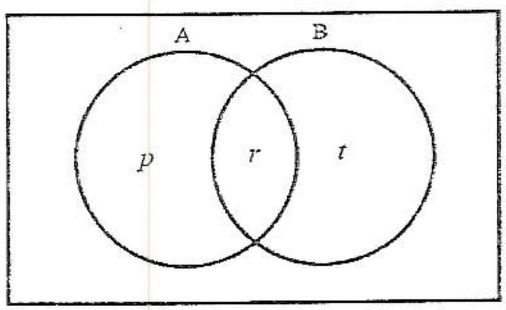
1) At Mom's Diner, everyone drinks coffee. Mom observed 100 customers:
   - 60 customers put cream in their coffee.
   - 50 customers put sugar in their coffee.
   - 40 customers put both cream and sugar in their coffee.

   How many customers put either cream or sugar in their coffee?

$$60+50-40 = \boxed{70}$$

$$P(A) \text{ or } p(B) = p(A)+p(B) - p(A \text{ AND } B)$$
$$p(A \text{ or } B)$$
$$p(A \cup B)$$

2) Let A and B be any two events. Write a formula for $P(A \text{ or } B)$ that shows its relationship with $P(A \text{ and } B)$, $P(A)$, and $P(B)$.

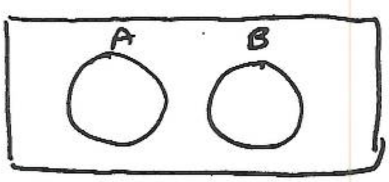


$$p(A) = p+r$$
$$p(B) = t+r$$
$$p(A \cancel{\ } B) = p+r+t$$
$$= (p+r)+(t+r)-r$$
$$\therefore p(A \text{ or } B) = p(A)+p(B) - p(A \text{ AND } B)$$

3) Is it possible that $P(A \text{ or } B) = P(A) + P(B)$? If so, when? If not, why not?

$$p(A \text{ or } B) = p(A) + p(B) \text{ iF AND only iF } p(A \text{ AND } B) = 0$$

A AND B have no common outcomes (no overlap)



Name:
Date: Period:

4) When a car is brought to a repair shop for service:

- $p(T)$ the probability that it will need the transmission fluid replaced is 0.38,
- $p(B)$ the probability that it will need the brake pads replaced is 0.28,
- $p(T \text{ and } B)$ and the probability that it will need both the transmission fluid and the brake pads replaced is 0.16.

T: a car needs the transmission fluid replaced.
B: a car needs the brake pads replaced be B.

a. What are the values of:

i. $P(T)$

$\boxed{0.38}$

ii. $P(B)$

$\boxed{0.28}$

iii. $P(T \text{ and } B)$

$\boxed{0.16}$

b. Find the probability that a randomly selected car either needs the transmission fluid replaced or needs the brake pads replaced.

$$p(T \text{ or } B) = p(T) + p(B) - p(T \text{ and } B)$$
$$p(T \cup B) = 0.38 + 0.28 - 0.16 = \boxed{0.5}$$

Name:

Compound Events AND → multiply
OR → ADD, THEN SUBTRACT OVERLAP

Date: Period:

5) Josie will soon be taking exams in math and Spanish. She estimates that the probability she passes the math exam is 0.9 and the probability that she passes the Spanish exam is 0.8. She is also willing to assume that the results of the two exams are independent of each other.

a. Using Josie's assumption of independence, calculate the probability that she passes both exams.

$p(\text{passes Both}) = (0.9)(0.8) = \boxed{0.72}$

b. Find the probability that Josie passes at least one of the exams.

$p(\text{passes math or Spanish}) = p(\text{passes math}) + p(\text{passes Spanish}) - p(\text{passes Both})$

$= 0.9 + 0.8 - 0.72 = \boxed{0.98}$

6) A set of 40 cards consists of:

- 10 black cards showing squares.
- 10 black cards showing circles.
- 10 red cards showing Xs.
- 10 red cards showing diamonds.

A card will be selected at random from the set. Find the probability that the card is black or shows a diamond.

The events "is black" AND "shows a diamond" are DISJOINT since there are no black cards that show diamonds. So

$p(\text{Black or Diamond}) = p(\text{Black}) + p(\text{Diamond})$ – no overlap – only red diamonds

$p = \frac{20}{40} + \frac{10}{40} = \frac{30}{40} = \boxed{\frac{3}{4} \text{ or } .75 \text{ or } 75\%}$

7) A red cube has faces labeled 1 through 6, and a blue cube has faces labeled in the same way. The two cubes are rolled. Find the probability that

a. both cubes show 6s.

$p(\text{Red shows 6 AND Blue shows 6}) = p(\text{Red shows 6}) \cdot p(\text{Blue shows 6})$

$p(R6 \wedge B6) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

b. the total score is at least 11.

5+6
6+5
6+6

$p(\text{Total is at least 11}) = p(\text{R AND B shows 6}) + p(\text{R shows 5, AND B shows 6}) + p(\text{R shows 6 AND B shows 5})$

$p = p(6,6) + p(5,6) + p(6,5)$

$p = \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}$

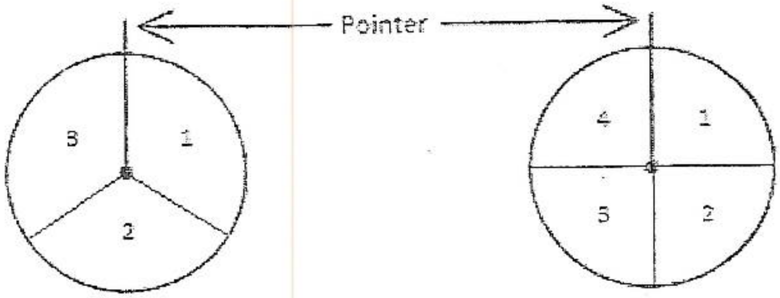
$p = \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36} = \boxed{\frac{1}{12}}$

Curriculum 2.0 Algebra 2: Unit 5-Topic 1, SLT 7

Compound Events

Name: ______________________

Date: ______________ Period: ______________



8) The diagram above shows two spinners. For the first spinner, the scores 1, 2, and 3 are equally likely, and for the second spinner, the scores 1, 2, 3, and 4 are equally likely. Both pointers will be spun. Find the probability that:

a. the total of the scores on the two spinners is 2.

$$p(\text{TOTAL}=2) = p(1,1) = \frac{1}{3}\cdot\frac{1}{4} = \boxed{\frac{1}{12}}$$

b. the total of the scores on the two spinners is 3.

$$p(\text{TOTAL}=3) = p(1,2)+p(2,1) = \frac{1}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{4} = \frac{1}{12}+\frac{1}{12} = \frac{2}{12} = \boxed{\frac{1}{6}}$$

c. the total of the scores on the two spinners is 5.

$$p(\text{TOTAL}=5) = p(1,4)+p(2,3)+p(3,2)$$
$$= \frac{1}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{4}+\frac{1}{3}\cdot\frac{1}{4}$$
$$= \frac{1}{12}+\frac{1}{12}+\frac{1}{12} = \frac{3}{12} = \boxed{\frac{1}{4} \text{ or } .25 \text{ or } 25\%}$$

d. the total of the scores on the two spinners is not 5.

$$p(\text{TOTAL is not } 5) = 1 - p(\text{TOTAL is } 5) = 1-\frac{1}{4} = \boxed{\frac{3}{4} \text{ or } .75 \text{ or } 75\%}$$

Name:

Problem Set

Date: Period:

1) Of the works of art at a large gallery, 59% are paintings (A) and 83% are for sale (B). When a work of art is selected at random, let the event that it is a painting be A and the event that it is for sale be B.

   a. What are the values of $P(A)$ and $P(B)$?

   Dependent here so $p(A) \cdot p(B) \neq p(A \cap B)$

   $P(A) = \boxed{0.59}$ $P(B) = \boxed{0.83}$

   b. Suppose you are told that $P(A \text{ and } B) = 0.51$. Find $P(A \text{ or } B)$.

   $\cancel{p(A \cap B) = p(A) \cdot p(B)}$

   $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

   $= 0.59 + 0.83 - 0.51$

   $= \boxed{0.91 = 91\%}$

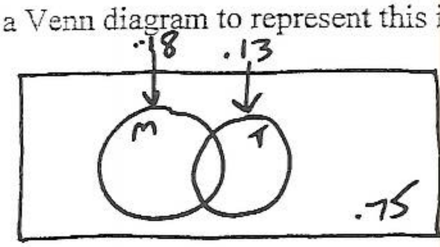
   c. Suppose now that you are not given the information in part (b), but you are told that the events A and B are independent. Find $P(A \text{ or } B)$.

   $p(A \cap B) = p(A) \cdot p(B) = (0.59)(0.83) = \boxed{0.4897}$

   $p(A \cup B) = p(A) + p(B) - p(A \cap B) = 0.59 + 0.83 - 0.4897 = \boxed{.9303}$

2) A traveler estimates that, for an upcoming trip, the probability of catching malaria is 0.18, the probability of catching typhoid is 0.13, and the probability of catching neither of the two diseases is 0.75.

   a. Draw a Venn diagram to represent this information.



   (Addition Rule)

   $p(M \cup T) = p(M) + p(T) - p(M \cap T)$

   $.25 = .18 + .13 - p(M \cap T)$

   $-.06 = -p(M \cap T)$

   $\boxed{p(M \cap T) = 0.06 = 6\%}$

   b. Calculate the probability of catching both of the diseases.

   $p(M \cup T) = 1 - .75 = 0.25$

   c. Are the events catches malaria and catches typhoid independent? Explain your answer.

   If independent then $p(M \cap T)$ must equal $p(M) \cdot p(T)$

   $p(M \cap T) = .06$

   $p(M) \cdot p(T) = (.18)(.13) = .0234$

   therefore not independent

Name: ____________________

Date: ____________ Period: ____________

3) A deck of 40 cards consists of

- 10 black cards showing squares, numbered 1–10,
- 10 black cards showing circles, numbered 1–10,
- 10 red cards showing Xs, numbered 1–10,
- 10 red cards showing diamonds, numbered 1–10.

A card will be selected at random from the deck.

a. i. Are the events *the card shows a square* and *the card is red* disjoint? Explain.

→ not a possibility

yes - there is no card that is red and a square

ii. Calculate the probability that the card will show a square or will be red.

$$p(\text{Square} \cup \text{Red}) = p(\text{Square}) + p(\text{Red}) - p(\text{Square} \cap \text{Red})$$
$$= \frac{10}{40} + \frac{20}{40} - 0 = \frac{30}{40} = \boxed{\frac{3}{4} = .75 = 75\%}$$

b. i. Are the events the card shows a 5 and the card is red disjoint? Explain.

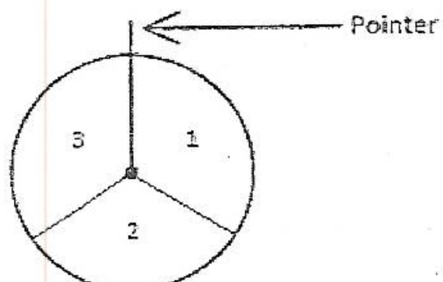
NO - there are red 5's in the deck

$$p(5 \cap \text{Red}) = p(5) \cdot p(\text{Red}) = \frac{4}{40} \cdot \frac{20}{40} = \boxed{\frac{1}{20}}$$

ii. Calculate the probability that the card will show a 5 or will be red.

$$p(5 \cup \text{Red}) = p(5) + p(\text{Red}) - p(5 \cap \text{Red}) \rightarrow$$
$$\frac{4}{40} + \frac{20}{40} - \frac{2}{40} = \frac{22}{40} = \boxed{\frac{11}{20}}$$

4) The diagram below shows a spinner. When the pointer is spun, it is equally likely to stop on 1, 2, or 3. The pointer will be spun three times. Expressing your answers as fractions in lowest terms, find the probability and explain how the answer was determined that the total of the values from all thee spins is:



a. 9 $p(\text{Total}=9) = p(3,3,3) = \frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} = \boxed{\frac{1}{27}}$

b. 8 $p(\text{Total}=8) = p(3,3,2) + p(2,3,3) + p(3,2,3)$

$$= \underbrace{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\frac{1}{27}} + \underbrace{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\frac{1}{27}} + \underbrace{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\frac{1}{27}} = \frac{3}{27} = \boxed{\frac{1}{9}}$$

c. 7 $p(\text{Total}=7) =$

$$p(2,3,2) + p(3,2,2) + p(3,3,1) + p(3,1,3) + p(2,2,3) + p(1,3,3)$$
$$6\left(\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}\right) = 6\left(\frac{1}{27}\right) = \frac{6}{27} = \boxed{\frac{2}{9}}$$

Name:

Date: Period:

5) A number cube has faces numbered 1 through 6, and a coin has two sides, "heads" and "tails". The number cube will be rolled once, and the coin will be flipped once. Find the probabilities of the following events.

a. The number cube shows a 6.

$p(6) = \boxed{\frac{1}{6}}$

b. The coin shows "heads."

$p(H) = \boxed{\frac{1}{2}}$

c. The number cube shows a 6, and the coin shows "heads."

$p(6 \cap H) = \frac{1}{6} \cdot \frac{1}{2} = \boxed{\frac{1}{12}}$

d. The number cube shows a 6, or the coin shows "heads."

$p(6 \cup H) = p(6) + p(H) - p(6 \cap H) \rightarrow \frac{1}{6} + \frac{1}{2} - \frac{1}{12} = \frac{2}{12} + \frac{6}{12} - \frac{1}{12} = \boxed{\frac{7}{12}}$

6) Kevin will soon be taking exams in math, physics, and French. He estimates the probabilities of his passing these exams to be as follows:

- Math: 0.9, → Failing math $1 - .9 = 0.1$
- Physics: 0.8, → Failing physics $1 - .8 = 0.2$
- French: 0.7. → Failing french $1 - .7 = 0.3$

Kevin is willing to assume that the results of the three exams are independent of each other. Find the probability that Kevin will

a. pass all three exams.

$p(\text{pass all 3 exams}) = p(m) \cdot p(p) \cdot p(F) = (.9)(.8)(.7) = \boxed{.504}$

b. pass math but fail the other two exams.

$p(\text{pass math, fail physics and French})$

$(.9)(.2)(.3) = \boxed{.054}$

c. pass exactly one of the three exams.

$p(\text{pass exactly 1 exam}) = p(\text{passes m, fails p, fails F}) + p(\text{passes p, fails m, fails F}) + p(\text{passes F, fails m, fails p})$

$p = (.9)(.2)(.3) + (.8)(.1)(.3) + (.7)(.1)(.2)$

$= .054 + .024 + .014 = \boxed{.092 \text{ or } 9.2\%}$