Name:

Evaluating Independence Using 2-Way Tables Sample Answers.

Date: Period:

In previous lessons, conditional probabilities were used to investigate whether or not there is a connection between two events. This lesson formalizes this idea and introduces the concept of independence.

1. Several questions are posed below. Each question is about a possible connection between two events. For each question, identify the two events and indicate whether or not you think that there would be a connection. Explain your reasoning.
   a. Are high school students whose parents or guardians set a midnight curfew less likely to have a traffic violation than students whose parents or guardians have not set such a curfew?

      Responses will vary. Accept any student conclusion based on a reasonable explanation.

   b. Are left-handed people more likely than right-handed people to be interested in the arts?

      Responses will vary. Accept any student conclusion based on a reasonable explanation.

   c. Are students who regularly listen to classical music more likely to be interested in mathematics than students who do not regularly listen to classical music?

      Responses will vary. Accept any student conclusion based on a reasonable explanation.

   d. Are people who play video games more than 10 hours per week more likely to select football as their favorite sport than people who do not play video games more than 10 hours per week?

      Responses will vary. Accept any student conclusion based on a reasonable explanation.

Two events are **independent** when knowing that one event has occurred does not change the likelihood that the second event has occurred. How can conditional probabilities be used to tell if two events are independent or not independent?

If $P(A \text{ given } B)$ *is the same as* $P(A)$, *then* A *and* B *are independent events.*

Recall the hypothetical 1000 two-way frequency table that was used to classify students at Rufus King High School according to gender and whether or not they participated in the after-school athletic program.

Participation in the after-school athletic program (Yes or No) of males and females

	Participate in the after-school athletic program	Do not participate in the after-school athletic program	Total
Females	232	348	580
Males	168	252	420
Total	400	600	1000

2. For each of the following, indicate whether the probability described is one that can be calculated using the values in the table. Also indicate whether or not it is a conditional probability.
   a. The probability that a randomly selected student participates in the after-school athletic program.

      Yes, it can be calculated. It is not a conditional probability since the target population includes all students.

   b. The probability that a randomly selected student who is female participates in the after-school athletic program.

      Yes, it can be calculated. It is a conditional probability since the target population includes female students.

   c. The probability that a randomly selected student who is male participates in the after-school athletic program.

      Yes, it can be calculated. It is a conditional probability since the target population only includes male students.

3. Use the table to calculate each of the probabilities described.
   a. The probability that a randomly selected student participates in the after-school athletic program.

      $$\frac{400}{1000} = 0.4$$

   b. The probability that a randomly selected student who is female participates in the after-school athletic program.

      $$\frac{232}{580} = 0.4$$

c. The probability that a randomly selected student who is male participates in the after-school athletic program.

$$\frac{168}{420} = 0.4$$

4. Would your prediction of whether or not a student participates in the after-school athletic program change if you knew the gender of the student? Explain your answer.

No. The conditional probabilities are the same $(\mathbf{0.40})$ *no matter which gender is chosen.*

$\frac{400}{1000} = 40\%$ $\frac{232}{580} = .4$ $\frac{168}{420} = .4$

Two events are **independent** if knowing that one event has occurred does not change the probability that the other event has occurred. For example, consider the following two events:

F: the event that a randomly selected student is female

S: the event that a randomly selected student participates in the after-school athletic program.

F and S would be independent if the probability that a randomly selected student participates in the after-school athletic program is equal to the probability that a randomly selected student who is female participates in the after-school athletic program. If this were the case, knowing that a randomly selected student is female does not change the probability that the selected student participates in the after-school athletic program. Then F and S would be independent.

5. Based on the definition of independence, are the events *randomly selected student is female* and *randomly selected student participates in the after-school athletic program* independent? Explain.

Yes, they are independent because the gender of the student has no effect on the probability that a randomly selected student participates in an after-school athletics program.

6. A randomly selected student participates in the after-school athletic program.

a. What is the probability this student is a female?

$$\frac{232}{400} = 0.58$$

b. Using only your answer from part (a), what is the probability that this student is a male? Explain how you arrived at your answer.

$1 - 0.58 = 0.42$

Consider the data below.

	No household member smokes	At least one household member smokes	Total
Student indicates he or she has asthma	69	113	182
Student indicates he or she does not have asthma	473	282	755
Total	542	395	937

7. You are asked to determine if the two events *a randomly selected student has asthma* and *a randomly selected student has a household member who smokes* are independent. What probabilities could you calculate to answer this question?

Answers may vary.

I could compare the probability that the student has asthma to the probability that she has asthma given that she has a household member who smokes. If these two probabilities are the same, then the two events must be independent.

A - Asthma
B - smoker

8. Calculate the probabilities you described in Exercise 7.

The probability that the student has asthma is $182/937 \approx 0.19$.

The probability that she has asthma given that she has a household member who smokes is $113/395 \approx 0.29$.

$P(A \text{ given } B) \stackrel{?}{=} P(A)$

9. Based on the probabilities you calculated in Exercise 8, are these two events independent or not independent? Explain.

Since the two probabilities are not the same, these are dependent events.

10. Is the probability that a randomly selected student who has asthma (B) has a household member (A) who smokes the same as the probability that a randomly selected student who does not have asthma has a household member who smokes? Explain your answer.

The probabilities are different since the events are not independent.

$\frac{113}{182} \neq \frac{282}{755}$

11. A student is selected at random. The selected student indicates that he or she has a household member who smokes. What is the probability that the selected student has asthma?

$\frac{113}{395} \approx 0.286$

Illustrative Mathematics

Two-Way Tables and Probability

Task

Each student in a random sample of seniors at a local high school participated in a survey. These students were asked to indicate their gender and their eye color. The following table summarizes the results of the survey.

		Eye color			
		Brown	Blue	Green	Total
Gender	Male	50	40	20	110
	Female	40	40	10	90
	Total	90	80	30	200

a. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male? $\frac{110}{200}$ = 55%

b. Suppose that one of these seniors is randomly selected. What is the probability that the selected student has blue eyes? $\frac{80}{200}$ = 40%

c. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male and has blue eyes?

$\frac{40}{200}$ = 20%

d. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male or has blue eyes? $\frac{150}{200}$ = 75%

e. Suppose that one of these seniors is randomly selected. What is the probability that the selected student has blue eyes, given that the student is male? $\frac{40}{110}$ = 36.3%

f. Suppose that one of these seniors is randomly selected. What is the probability that the selected student is a male, given that the student has blue eyes?

$\frac{40}{80}$ = 50%

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Lesson Summary

Data organized in a two-way frequency table can be used to calculate conditional probabilities.

Two events are independent if knowing that one event has occurred does not change the probability that the second event has occurred.

Probabilities calculated from two-way frequency tables can be used to determine if two events are independent or not independent.

1. Consider the following questions.

a. A survey of the students at a Midwest high school asked the following questions:

"Do you use a computer at least 3 times a week to complete your school work?"
"Are you taking a mathematics class?"

Do you think the events *a randomly selected student is taking a mathematics class* and *a randomly selected student uses a computer at least* 3 *times a week* are independent or not independent? Explain your reasoning.

Answers will vary. Examine student responses to make sure that students understand the concept of independent events.

b. The same survey also asked students the following:

"Do you participate in any extracurricular activities at your school?"
"Do you know what you want to do after high school?"

Do you think the events *a randomly selected student participates in extracurricular activities* and *a randomly selected student knows what he or she wants to do after completing high school* are independent or not independent? Explain your reasoning.

Answers will vary. Examine student responses to make sure that students understand the concept of independent events.

c. People attending a professional football game in 2013 completed a survey that included the following questions:

"Do you think football is too violent?"
"Is this the first time you have attended a professional football game?"

Do you think the events *a randomly selected person who completed the survey is attending a professional football game for the first time* and *a randomly selected person who completed the survey thinks football is too violent* are independent or not independent? Explain your reasoning.

Answers will vary. Examine student responses to make sure that students understand the concept of independent events.

2. Complete the table below in a way that would indicate the two events *uses a computer* and *is taking a mathematics class* are independent.

	Uses a computer at least **3** times a week for school work	Does not use a computer at least **3** times a week for school work	Total
In a mathematics class	*420*	*280*	700
Not in a mathematics class	*180*	*120*	*300*
Total	600	*400*	1000

60% of students in a math class and 60% of students not in a math class use a computer 3 or more times a week.

70% of students who use a computer 3 or more times a week are in a math class and 70% of students who do not use a computer are in a math class.

3. Complete the following hypothetical 1000 table. Are the events *participates in extracurricular activities* and *know what I want to do after high school* independent or not independent? Justify your answer.

	Participate in extracurricular activities	Do not participate in extracurricular activities	Total
Know what I want to do after high school	*550*	*250*	800
Do not know what I want to do after high school	50	*150*	*200*
Total	600	*400*	1000

The events "students participate in extra-curricular activities" and "student knows what I want to do in high school" are not independent. There are various ways to show this. For example, $\frac{50}{200} = 0.25$ *(the probability that a randomly selected student who does not know what he wants to do after high school participates in extra-curricular activities) does not equal* $\frac{550}{800} = 0.6875$ *(the probability that a randomly selected student who does know what he wants to do after high school participates in extra-curricular activities).*

4. The following hypothetical 1000 table is from Lesson 2.

	No household member smokes	At least one household member smokes	Total
Student indicates he or she has asthma	73	120	193
Student indicates he or she does not have asthma	506	301	807
Total	579	421	1000

The actual data from the entire population is given in the table below.

	No household member smokes	At least one household member smokes	Total
Student indicates he or she has asthma	69	113	182
Student indicates he or she does not have asthma	473	282	755
Total	542	395	937

a. Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has asthma has at least one household member who smokes?

$$\frac{120}{193} \approx 0.622$$

b. Based on the actual data, what is the probability that a randomly selected student who has asthma has at least one household member who smokes (round your answer to 3 decimal places)?

$$\frac{113}{182} \approx 0.621$$

c. Based on the hypothetical 1000 table, what is the probability that a randomly selected student who has no household member who smokes has asthma?

$$\frac{73}{579} \approx 0.126$$

d. Based on the actual data, what is the probability that a randomly selected student who has no household member who smokes has asthma?

$$\frac{69}{542} \approx 0.127$$

e. What do you notice about the probabilities calculated from the actual data and the probabilities calculated from the hypothetical 1000 table? *Virtually the same.*

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5. As part of the asthma research, the investigators wondered if students who have asthma are less likely to have a pet at home than students who do not have asthma. They asked the following two questions:

   "Do you have asthma?"
   "Do you have a pet at home?"

   Based on the responses to these questions, you would like to set up a two-way table that you could use to determine if the following two events are independent or not independent:

   Event 1: a randomly selected student has asthma
   Event 2: a randomly selected student has a pet at home.

   a. What would you use to label the rows of the two-way table?

      Rows could be labeled "Has Asthma" and "Does Not Have Asthma"

   b. What would you use to label the columns of the two-way table?

      Columns could be labeled "Has a Pet" and "Does Not Have a Pet"

      The rows and columns in part a) and part b) could be interchanged.

   c. What probabilities would you calculate to determine if Event 1 and Event 2 are independent?

      Answers may vary. Row conditional probabilities or column conditional probabilities would have to equal for the events to be independent. This would mean that the probability that a randomly selected student has asthma is independent of the event that the student has a pet.