Key

UNIT 5

HONORS

ALGEBRA 2

SLT 8

THE GENERAL MULTIPLICATION RULE

Name:

The General Multiplication Rule

Date: Period:

A cereal company put one of the following toys in its cereal boxes: a block (B), a toy watch (W), a toy ring (R), and a toy airplane (A). A machine that placed the toy in the box was programmed to select a toy by drawing a random number of 1 to 4. If a 1 was selected, the block (or B) was placed in the box; if a 2 was selected, a watch (or W) was placed in the box; if a 3 was selected, a ring (or R) was placed in the box; and if a 4 was selected, an airplane (or A) was placed in the box. When this promotion was launched, young children were especially interested in getting the toy airplane.

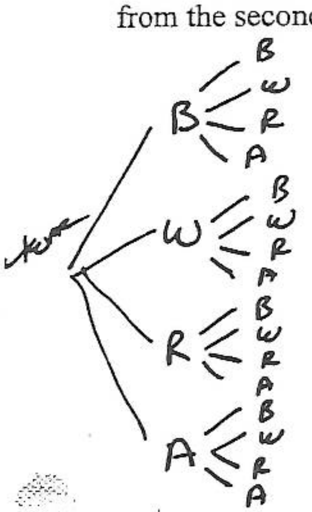
1. If you bought one box of cereal, what is your estimate of the probability of getting the toy airplane? Explain how you got your answer.

$P(\text{A toy airplane}) = \frac{1}{4}$ SAMPLE SPACE $\{B, W, R, \underline{A}\}$
ASSUMING EACH OUTCOME likely TO OCCUR

2. If you bought a second box of cereal, what is your estimate of the probability of getting the toy airplane in the second box? Explain how you got your answer.

PROBABILITY AGAIN IS $\frac{1}{4}$ – SINCE machine picks AT RANDOM every BOX IS AN OUTCOME

3. List all of the possibilities of getting two toys from two boxes of cereals. (Hint: Think of the possible outcomes as ordered pairs. For example, BA would represent a block from the first box and an airplane from the second box.)



BB	WB	RB	AB
BW	WW	RW	AW
BR	WR	RR	AR
BA	WA	RA	AA

$\underbrace{\hspace{6em}}_{16 \text{ outcomes}}$

B = Block
W = Toy watch
A = Airplane
R = Ring

4.

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4. Based on the list you created, what do you think is the probability of each of the following outcomes if two cereal boxes are purchased?
   a. One (and only one) airplane

      $\frac{6}{16} = \boxed{\frac{3}{8}}$

   b. At least one airplane

      $\boxed{\frac{7}{16}}$

   c. No airplanes

      $\boxed{\frac{9}{16}}$

   d. An airplane in the first cereal box? Explain your answer.

      $\boxed{\frac{1}{4}}$ – There are 4 possible toys, and each is likely to be in the box

   e. An airplane in the second cereal box?

      $\boxed{\frac{1}{4}}$ → same reason as above

   f. Airplanes in both cereal boxes?

      $\boxed{\frac{1}{16}}$ → 1 pair in list of outcomes (AA) out of 16 total outcomes

      $\left(\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}\right)$

5. Are the events independent or dependent?

   Independent – the toy found in 1 box doesn't affect which toy will be found in the second box

6. Which probability rule applies? Explain your answer.

   multiplication rule for independent events applies

   $$P(A \cap B) = P(A) \cdot P(B)$$

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The General Multiplication Rule

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Do you remember the famous line, "Life is like a box of chocolates," from the movie *Forrest Gump*? When you take a piece of chocolate from a box, you never quite know what the chocolate will be filled with. Suppose a box of chocolates contains 15 identical-looking pieces. The 15 are filled in this manner: 3 caramel, 2 cherry cream, 2 coconut, 4 chocolate whip, and 4 fudge.

7. If you randomly select one of the pieces of chocolate from the box, what is the probability that the piece will be filled with fudge?

$\boxed{\frac{4}{15} \approx 0.2667}$

why would you Take it in the First place?

8. Suppose the type of filling is named underneath each piece, and since you don't like fudge filling, you put back the fudge-filled piece without biting in and then randomly select a piece. What is the probability that the second piece you selected is filled with caramel?

$\frac{3}{15} = \boxed{\frac{1}{5} = 0.2 = 20\%}$

9. If A_1 is the event of picking a fudge-filled piece on the first selection and B_2 is the event of picking a caramel-filled piece on the second selection, determine $P(A_1 \cap B_2)$ with replacement.

$p(A_1 \cap B_2) = \frac{4}{15} \cdot \frac{3}{15} \approx \frac{12}{225} \approx \boxed{0.053 \text{ or } 5.3\%}$ $p(A_1) \cdot p(B_2 | A_1) = p(A_1) \cdot p(B$

10. Are the two events independent or dependent?

INDEPENDENT – I replaced the fudge filled piece, so I still have the same number of chocolates to choose from and $p(B_2) = p(B_2 | A)$

11. If instead you randomly select a second piece of chocolate after you have eaten the first fudge-filled one, because you love fudge filling, what is the probability that the second piece will be filled with caramel?

$\boxed{\frac{3}{14} \approx 0.2143 \text{ or } 21.4\%}$ (1 event already occured before this)

12. If A_1 is the event picking a fudge-filled piece on the first selection and B_2 is the event picking a caramel-filled piece on the second selection, determine $P(A_1 \cap B_2)$ without replacement.

$p(A_1 \cap B_2) = \frac{4}{15} \cdot \frac{3}{14} = \frac{12}{210} = \boxed{\frac{2}{35} \approx .057 = 5.7\%}$ $p(A_1) \cdot p(B_2 | A_1)$

13. Are the events independent or dependent?

Dependent – B/c once I eat a chocolate, I have reduced the number of chocolates to choose from, so I have changed the probability of picking a caramel – filled chocolate on 2nd selection and $p(B_2) \neq p(B_2 | A_1)$

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14. Does the rule you cited in #6 apply in the scenario in which you replace the first chocolate you select or in the scenario in which you eat the first chocolate you select? Explain your answer.

The Rule in #6 requires independent events.
So it only applies when the first chocolate chosen is replaced

$p(A \cap B) = p(A) \cdot p(B) \Rightarrow$ independent $\quad * \, p(A \cap B) = p(A) \cdot p(B|A) \Rightarrow$ dependent

Your mom has a rule that whatever you take, you eat. So, whenever you take a chocolate from the box, you eat it rather than replace it.

15. What does $P(B_1 \cap A_2)$ represent? Calculate this probability.

$p(B_1 \cap A_2)$ represents the probability of picking a caramel-filled piece first and a fudge-filled piece second

$$p(B_1 \cap A_2) = \frac{4}{15} \cdot \frac{3}{14} \approx \boxed{0.057 = 5.7\% = \frac{2}{35}}$$

16. If C represents selecting a coconut-filled piece of chocolate, what does $P(A_1 \cap C_2)$ represent? Find this probability.

$p(A_1 \cap C_2)$ represents the probability of picking a fudge-filled piece first and a coconut-filled piece second

$$p(A_1 \cap C_2) = \frac{4}{15} \cdot \frac{2}{14} = \boxed{\frac{4}{105} \approx .038 = 3.8\%}$$

17. Find the probability that both the first and second pieces selected are filled with chocolate whip.

$$\frac{4}{15} \cdot \frac{3}{14} = \boxed{\frac{2}{35} \approx 0.057 = 5.7\%}$$

18. Can you write a more general probability rule that applies in the case of replacement? Can you prove your conjecture and explain how it connects to the rule in #6?

Since $p(B|A) = \dfrac{p(A \cap B)}{p(A)}$ and $p(A|B) = \dfrac{p(A \cap B)}{p(B)}$ whether events are independent or dependent

$$p(A \cap B) = p(A) \cdot p(B|A) = p(B) \cdot p(A|B)$$

If A and B are independent, then $p(B|A) = p(B)$ and $p(A|B) = p(A)$, so

$$p(A \cap B) = p(A) \cdot p(B)$$

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Independent or Dependent?

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1. For each of the following, write the probability as the intersection (AND) of two events. Then, indicate whether the two events are independent or dependent, and calculate the probability of the intersection of the two events occurring.

   a. The probability of selecting a 6 from the first draw and a 7 on the second draw when two balls are selected without replacement from a container with 10 balls numbered 1 to 10.

   Dependent $\frac{1}{10} \cdot \frac{1}{9} = \boxed{\frac{1}{90} \approx 0.011 = 1.1\%}$

   b. The probability of selecting a 6 on the first draw and a 7 on the second draw when two balls are selected with replacement from a container with 10 balls numbered 1 to 10.

   independent $\frac{1}{10} \cdot \frac{1}{10} = \boxed{\frac{1}{100} = .01 = 1\%}$

   c. The probability that two people selected at random in a shopping mall on a very busy Saturday both have a birthday in the month of June (30 DAYS). Assume that all 365 birthdays are equally likely and ignore the possibility of a February 29 leap-year birthday.

   Independent $\frac{30}{365} \cdot \frac{30}{365} \approx \boxed{.0068 = .68\%}$

   d. The probability that two socks selected at random from a drawer containing 10 black socks and 6 white socks will both be black.

   Dependent $\frac{10}{16} \cdot \frac{9}{15} \approx \boxed{0.375 = 37.5\%}$

A

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2. A gumball machine has gumballs of 4 different flavors: sour apple (A), grape (G), orange (O), and cherry (C). There are six gumballs of each flavor. When 50¢ is put into the machine, two random gumballs come out. The event C_1 means a cherry gumball came out first, the event C_2 means a cherry gumball came out second, the event A_1 means sour apple gumball came out first, and the event G_2 means a grape gumball came out second.

a. What does $P(C_2|C_1)$ mean in this context?

the probability of a second gumball being cherry given that the first gumball was cherry

b. Find $P(C_1 \cap C_2)$.

$\frac{6}{24} \cdot \frac{5}{23} \approx \boxed{0.054 = 5.4\%}$

c. Find $P(A_1 \cap G_2)$.

$\frac{6}{24} \cdot \frac{6}{23} \approx \boxed{0.0652 = 6.5\%}$

3. Below are the approximate percentages of the different blood types for people in the United States.

Type O 44%

Type A 42%

Type B 10%

Type AB 4%

Consider a group of 100 people with a distribution of blood types consistent with these percentages. If two people are randomly selected with replacement from this group, what is the probability that

a. both people have type O blood?

$(.44)(.44) = \boxed{0.1936 = 19.4\%}$

b. the first person has type A blood and the second person has type AB blood?

$(.42)(.04) = \boxed{0.0168 = 1.7\%}$

SLT 8H HOMEWORK

Curriculum 2.0 Algebra 2: Unit 5-Topic 1, SLT 8H

Name:

The General Multiplication Rule Problem Set

Date: Period:

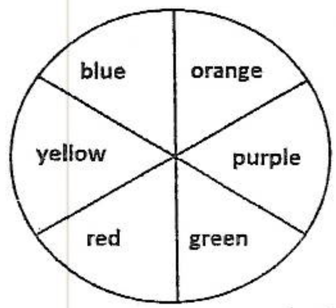
Lesson Summary

- Two events are independent if knowing that one occurs does not change the probability that the other occurs.
- Two events are dependent if knowing that one occurs changes the probability that the other occurs.
- **GENERAL MULTIPLICATION RULE:**

  $P(A \cap B) = P(A) \cdot P(B|A)$

  If A and B are independent events then $P(B|A) = P(B)$.

1. In a game using the spinner below, a participant spins the spinner twice. If the spinner lands on red both times, the participant is a winner.



a. The event participant is a winner can be thought of as the intersection of two events. List the two events.

FIRST SPIN LANDS ON RED AND SECOND SPIN LANDS ON RED

b. Are the two events independent? Explain.

INDEPENDENT – having 1st spin landing on RED DOES not AFFECT the 2nd SPIN LANDING ON RED

c. Find the probability that a participant wins the game.

$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36} \approx 0.0278 = 2.8\%$

2. The overall probability of winning a prize in a weekly lottery is $\frac{1}{32}$. What is the probability of winning a prize in this lottery three weeks in a row?

$\frac{1}{32} \cdot \frac{1}{32} \cdot \frac{1}{32} \approx \boxed{0.00003}$

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3. A Gallup poll reported that 28% of adults (age 18 and older) eat at a fast food restaurant about once a week. Find the probability that two randomly selected adults would both say they eat at a fast food restaurant about once a week.

$(.28)(.28) \approx \boxed{.0784 = 7.8\%}$

4. In the game *Scrabble*, there are a total of 100 tiles. Of the 100 tiles, 42 tiles have the vowels A, E, I, O, and U printed on them, 56 tiles have the consonants printed on them, and 2 tiles are left blank.

a. If tiles are selected at random, what is the probability that the first tile drawn from the pile of 100 tiles is a vowel? $\boxed{\frac{42}{100} = .42 = 42\%}$

b. If tiles drawn are not replaced, what is the probability that the first two tiles selected are both vowels?

$\frac{42}{100} \cdot \frac{41}{99} \approx \boxed{0.174 = 17.4\%}$

c. Event A is drawing a vowel, event B is drawing a consonant, and event C is drawing a blank tile. A_1 means a vowel is drawn on the first selection, B_2 means a consonant is drawn on the second selection, and C_2 means a blank tile is drawn on the second selection. Tiles are selected at random and without replacement.

vowel con.

i. Find $P(A_1 \cap B_2)$ $\frac{42}{100} \cdot \frac{56}{99} \approx \boxed{0.238 = 23.8\%}$

vowel blank

ii. Find $P(A_1 \cap C_2)$ $\frac{42}{100} \cdot \frac{2}{99} \approx \boxed{0.008 = .8\%}$

con blank

iii. Find $P(B_1 \cap C_2)$ $\frac{56}{100} \cdot \frac{2}{99} \approx \boxed{0.011 = 1.1\%}$

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5. To prevent a flooded basement, a homeowner has installed two special pumps that work automatically and independently to pump water if the water level gets too high. One pump is rather old and does not work 28% of the time, and the second pump is newer and does not work 9% of the time. Find the probability that both pumps will fail to work at the same time.

$(.28)(.09) \approx \boxed{0.025 = 2.5\%}$

6. According to a recent survey, approximately 77% of Americans get to work by driving alone. Other methods for getting to work are listed in the table below.

Method of getting to work	Percent of Americans using this method
Taxi	0.1%
Motorcycle	0.2%
Bicycle	0.4%
Walk	2.5%
Public Transportation	4.7%
Car Pool	10.7%
Drive Alone	77%
Work at Home	3.7%
Other	0.7%

a. What is the probability that a randomly selected worker drives to work alone?

$\boxed{0.77 = 77\%}$

b. What is the probability that two workers selected at random with replacement both drive to work alone?

$(.77)(.77) \approx \boxed{0.593 = 59.3\%}$

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7. A bag of M&Ms contains the following distribution of colors:

9 blue, 6 orange, 5 brown, 5 green, 4 red, 3 yellow – 32 TOTAL

Three M&Ms are randomly selected <u>without replacement</u>. Find the probabilities of the following events.

a. All three are blue.

$\frac{9}{32} \cdot \frac{8}{31} \cdot \frac{7}{30} \approx \boxed{0.017 = 1.7\%}$

b. The first one selected is blue, the second one selected is orange, and the third one selected is red.

$\frac{9}{32} \cdot \frac{6}{31} \cdot \frac{4}{30} \approx \boxed{0.007 = .7\%}$

c. The first two selected are red, and the third one selected is yellow.

$\frac{4}{32} \cdot \frac{3}{31} \cdot \frac{3}{30} \approx \boxed{0.001 = .1\%}$

8. Suppose in a certain breed of dog, the color of fur can either be tan or black. Eighty-five percent of the time, a puppy will be born with tan fur, while 15% of the time, the puppy will have black fur. Suppose in a future litter, six puppies will be born.

a. Are the events having tan fur and having black fur independent? Explain.

INDEPENDENT – KNOWING THE COLOR OF FUR FOR 1 PUPPY DOESN'T AFFECT THE PROBABILITY OF FUR COLOR FOR ANOTHER PUPPY

b. What is the probability that one puppy in the litter will have black fur and another puppy will have tan fur?

$(.15)(.85) \approx \boxed{0.1275 = 12.75\%}$

c. What is the probability that all six puppies will have tan fur?

$(0.85)^6 \approx \boxed{0.377 = 37.7\%}$

d. Is it likely for three out of the six puppies to be born with black fur? Justify mathematically.

NO – THE PROBABILITY OF 3 PUPPIES BEING BORN WITH BLACK FUR IS $(.15)(.15)(.15) \approx 0.003375$ – NOT LIKELY TO HAPPEN

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?. Suppose that in the litter of six puppies from #11, five puppies are born with tan fur, and one puppy is born with black fur.

a. You randomly pick up one puppy. What is the probability that puppy will have black fur?

$\boxed{\frac{1}{6} \approx 0.167}$

b. You randomly pick up one puppy, put it down, and randomly pick up a puppy again. What is the probability that both puppies will have black fur?

$\frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36} \approx 0.028 = 2.8\%}$

c. You randomly pick up two puppies, one in each hand. What is the probability that both puppies will have black fur?

0% chance - this outcome can never happen - since there is only 1 black puppy

d. You randomly pick up two puppies, one in each hand. What is the probability that both puppies will have tan fur?

$\frac{\cancel{5}^{1}}{\cancel{6}_{3}} \cdot \frac{\cancel{4}^{2}}{\cancel{5}_{1}} = \boxed{\frac{2}{3} \approx 0.667 = 66\frac{2}{3}\%}$