

Rational Functions

Day 1 . Day 2

Worksheet A

Steps to Manually Sketch:

1. factor
2. find domain
3. reduce/simplify
 - if term cancels - hole (find pt)
4. determine VA, HA, SA-graph
5. determine xint and yint
6. determine if graph crosses HA
7. plot extra pts if needed

ex. graph $y = \frac{x+2}{x-2}$

D: \mathbb{R} except $x=2$

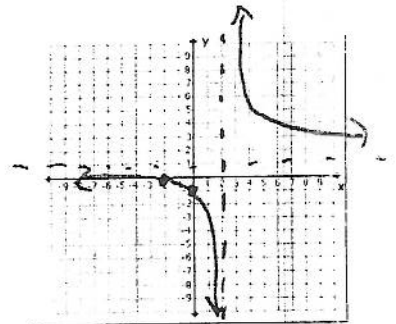
VA: $x=2$

HA: $y=1$

SA: —

x int: -2

y int: -1



end day 2

$$\frac{x+2}{x-2} = 1$$

$$x-2 = x+2$$

$$-2 = 2$$

\therefore NO

ex. graph $y = \frac{1}{x}$ (hyperbola)

D: \mathbb{R} except $x=0$

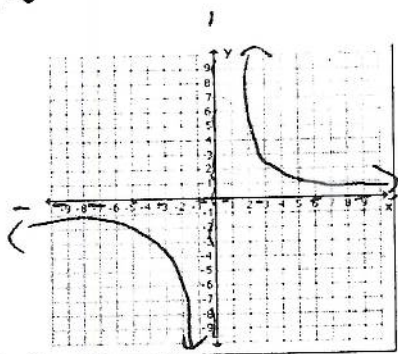
VA: $x=0$

HA: $y=0$

SA: None

x int: None

y int: None



$$\frac{1}{x} = 0$$
$$1 = 0$$
$$\emptyset$$

*

ex. graph $y = \frac{5}{x+3} - 2 = \frac{5-2x-6}{x+3} = \frac{-2x-1}{x+3}$

VA: $x = -3$

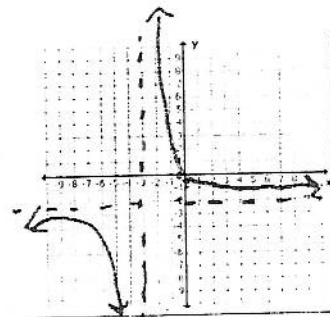
HA: $y = -2$

SA: None

D: \mathbb{R} , except $x = -3$

x int: $-\frac{1}{2}$

y int: $-\frac{1}{3}$



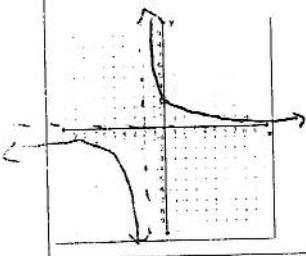
~~graph~~

$$\frac{x}{y} = -4.5$$
$$-5 \frac{1}{2} = -4.5$$
$$4 \frac{1}{2}$$

Day 2 practice problems

ex. $y = \frac{5}{x+2}$

D: $\mathbb{R}, x \neq -2$
 VA: $x = -2$
 HA: $y = 0$



x int: none
 y int: $5/2$

cross HA?

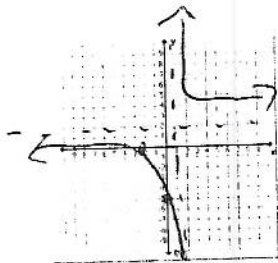
$$\frac{5}{x+2} = 0$$

0/5

x	y
8	5/10 = 1/2
-1	5
-7	-1

$y = \frac{2x+5}{x-1}$ D: $\mathbb{R}, x \neq 1$

VA: $x = 1$
 HA: $y = 2$



x int: $-5/2$
 y int: -5

cross HA?

$$\frac{2x+5}{x-1} = 2$$

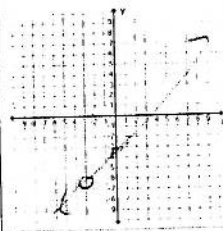
$$2x - 2 = 2x + 5$$

$$-2 + 5$$

x	y
-6	1
-1	-1.5
2	4
3	5.5

$y = \frac{x^2-9}{x+3}$

$(x-3)(x+3)$
 $x+3$
 $y = x-3$



D: $\mathbb{R}, x \neq -3$
 hole @ $(-3, -6)$

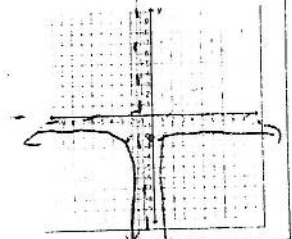
VA: none
 HA: none

SA: $x = -3$
 R: $\mathbb{R}, x \neq -3$

lim $x \rightarrow -3$

$y = \frac{-2}{(x+1)^2}$

VA: $x = -1$
 HA: $y = 0$



x int: none
 y int: -2

D: $\mathbb{R}, x \neq -1$
 R: $\mathbb{R}, x \neq -1$

x	y
-2	-2
-3	-1/2
1	-2

$\frac{-2}{(x+1)(x+1)}$

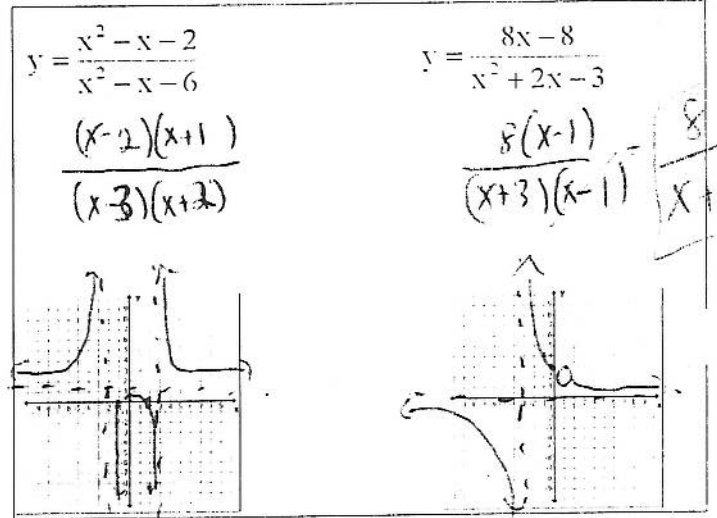
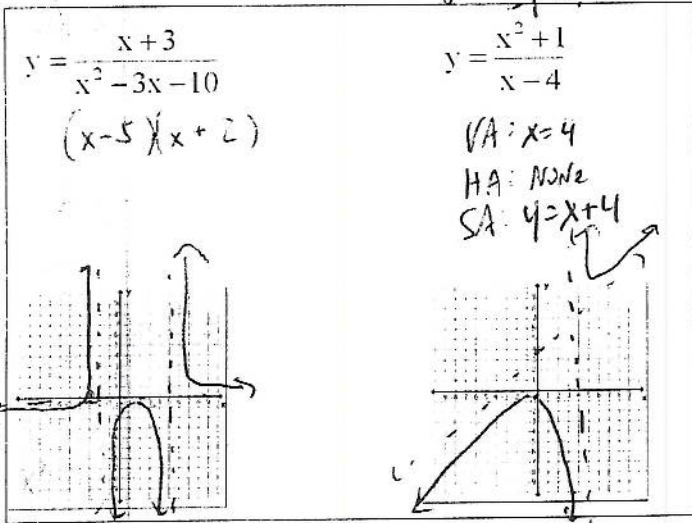
not really rational

$$\begin{array}{r|l} -3 & 0 \\ 3 & -6 \\ \hline -4 & -0.534 \\ -8 & -0.064 \end{array}$$

$$x^2 + 1 = 0$$

$$x^2 = -1$$

$$\begin{array}{r|l} x & 4 \\ 3 & 10 \\ \hline 5 & 26 \\ 6 & 18.5 \end{array}$$



VA: $x=5$
 $x=-2$
 HA: $y=0$
 x int: -3
 y int: $-3/10$

x int: none
 y int: $-1/4$
 no range by hand

VA: $x=3$ $x=-2$
 HA: $y=1$
 x int: $(2,0)$ $(-1,0)$
 y int: $(0, 1/3)$

hole @ $(1, 2)$
 VA: $x=-3$
 HA: $y=0$
 D: $\mathbb{R}, \text{exc } -3, 1$

case 3A?

$$\frac{x+3}{x^2-3x-10} = 0$$

$$x+3=0$$

$$x=-3 \text{ yep}$$

$$\begin{array}{r} x+4 \\ x-4 \overline{) x^2+0x+1} \\ \underline{-x^2+4x} \\ 4x+1 \\ \underline{-4x+16} \\ 17 \end{array}$$

$$\begin{array}{r|l} x & 4 \\ -4 & 1.3 \\ 4 & 1.66 \\ 5 & 1.3 \end{array}$$

LEB: as $x \rightarrow -\infty, f(x) \rightarrow -\infty$
 REB: as $x \rightarrow \infty, f(x) \rightarrow \infty$

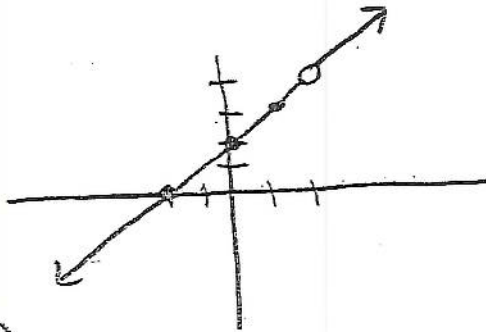
x int: none
 y int: $8/3$

$$\begin{array}{r|l} x & 4 \\ -1 & 4 \\ -2 & 8 \\ -4 & -8 \\ -5 & -4 \\ 5 & 1 \end{array}$$

D: $\mathbb{R}, \text{exc } 0, 2$

Worksheet A answers

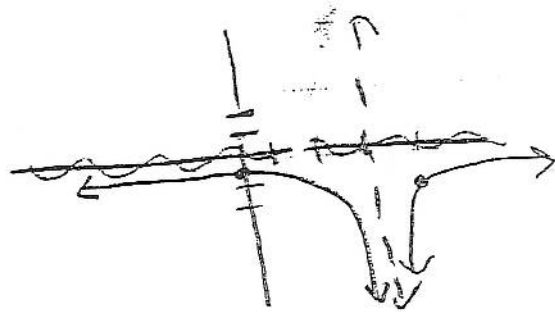
1) $f(x) = \frac{x^2 - 4}{x - 2}$ $f(x) = \frac{(x+2)(x-2)}{(x-2)}$ hole at $x = 2$
 $f(2) = 2 + 2 = 4$
 $(2, 4)$



$f(x) = x + 2$

x int
 $x + 2 = 0$
 $-2 = -2$
 $x = -2$ $(-2, 0)$

2) $f(x) = \frac{-2}{(x-3)^2}$



VA $x - 3 = 0$
 $x = 3$

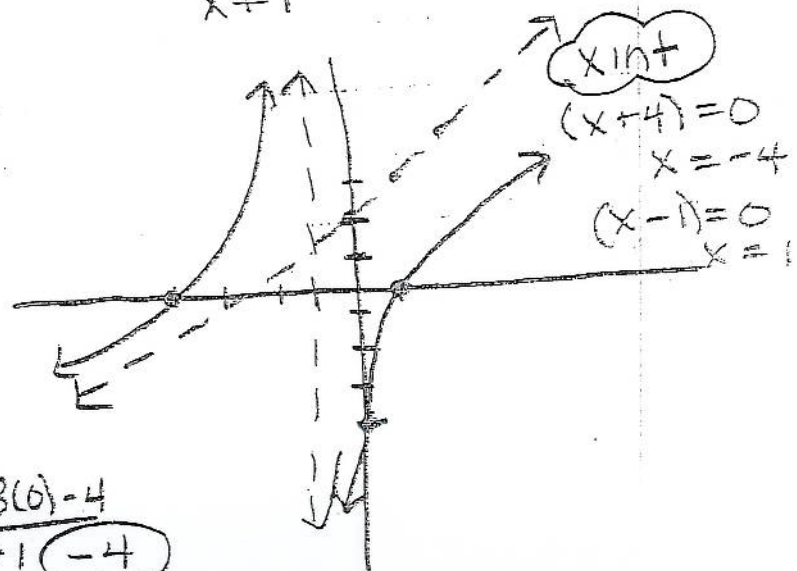
y int $\frac{-2}{(0-3)^2} = -\frac{2}{9}$ x int - none

HA $y = 0$

$f(4) = \frac{-2}{(4-3)^2} = -2$

3) $f(x) = \frac{x^2 + 3x - 4}{x + 1}$ $f(x) = \frac{(x+4)(x-1)}{x+1}$

SA - $\frac{x+2}{x+1} \overline{) x^2 + 3x - 4}$
 $-x^2 + x$
 $2x - 4$
 $-2x + 2$
 -6



VA $x + 1 = 0$
 $x = -1$

y int $\frac{0^2 + 3(0) - 4}{0 + 1} = -4$

x int
 $(x+4) = 0$
 $x = -4$
 $(x-1) = 0$
 $x = 1$

$$\textcircled{4} f(x) = \frac{x^2}{x^2 - x}$$

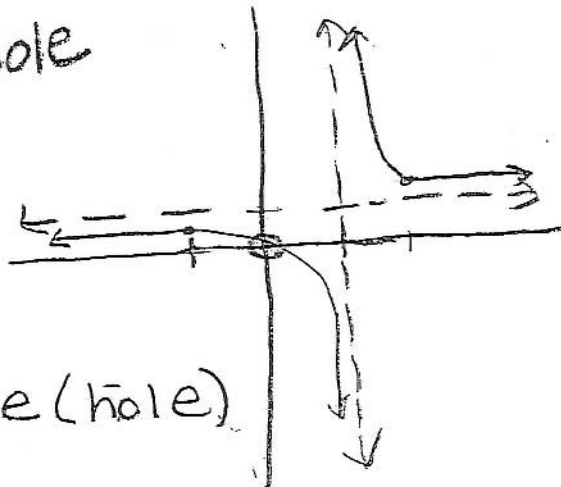
$$f(x) = \frac{x}{x(x-1)}$$

hole

VA $x - 1 = 0$
 $x = 1$

$$f(x) = \frac{x}{x-1}$$

HA $y = 1$ $\frac{0}{0-1} = 0$ x_{int} None (hole)
 Hole $x = 0$ $(0, 0)$



y_{int} none (hole)

$$f(-1) = \frac{(-1)^2}{(-1)^2 - 1} = \frac{1}{2}$$

$(-1, 1/2)$

$$f(2) = \frac{2^2}{2^2 - 2} = \frac{4}{2} = 2$$

$$\textcircled{5} f(x) = \frac{-x}{x^2 - 4}$$

$$f(x) = \frac{-x}{(x+2)(x-2)}$$

VA $x+2=0$ $x-2=0$
 $x=-2$ $x=2$

VA $y=0$ $f(-3) = \frac{-(-3)}{(-3)^2 - 4}$
 $= \frac{3}{5}$

$y_{int} = \frac{-0}{0^2 - 4} = 0$

$$f(3) = \frac{-3}{(3)^2 - 4} = \frac{-3}{5}$$

x_{int} $-x=0$
 $x=0, 0$

$$f(-1) = \frac{-(-1)}{(-1)^2 - 4} = \frac{1}{3}$$

$$f(1) = \frac{-1}{(1)^2 - 4} = \frac{1}{3}$$

