

Problem Solving Practice

Exponential Equations

1. At 11:00 a.m., Sally leaves a serving of yogurt on a table. The number of bacteria in Sally's yogurt at that time is 1. The number of bacteria in her yogurt increases by a factor of 4 every minute. **TWENTY FIVE** minutes later, at **11:25**, Lupe leaves a serving of yogurt on a heater. The number of bacteria in Lupe's yogurt at that time is 1, but due to the increased temperature, the bacteria increases by a factor of 8 every minute. At what time will the number of bacteria in each yogurt be the same?

$$\begin{array}{l}
 t=0 \quad 1 \\
 t=1 \quad 4 \\
 \hline
 4^t
 \end{array}
 \qquad
 \begin{array}{l}
 t=25 \quad 1 \\
 t=26 \quad 8 \\
 \hline
 8^{t-25}
 \end{array}$$

$$4^t = 8^{t-25}$$

$$(2^2)^t = (2^3)^{t-25}$$

$$2t = 3t - 75$$

$$-t = -75$$

$$t = 75 \text{ mins} \rightarrow \text{12:15}$$

2. At noon, a computer virus infects 1 kilobyte of a computer's memory. Every minute thereafter, the number of kilobytes infected triples (is multiplied by three). At twelve minutes after noon, a much worse computer virus infects 1 kilobyte of another computer's memory. Each minute thereafter, the number of kilobytes infected in that computer is multiplied by nine. When will the number of kilobytes infected be the same in both computers?

$$\begin{array}{l}
 t=0 \quad 1 \\
 t=1 \quad 3 \\
 \hline
 3^t
 \end{array}
 \qquad
 \begin{array}{l}
 t=12 \quad 1 \\
 t=13 \quad 9 \\
 \hline
 9^{t-12}
 \end{array}$$

$$3^t = 9^{t-12}$$

$$3^t = (3^2)^{t-12}$$

$$3^t = 3^{2t-24}$$

$$t = 2t - 24$$

$$t = 24 \rightarrow \text{12:24}$$

Note: A kilobyte is a unit of computer memory or data storage capacity equal to approximately 1,000 bytes.

3. At the beginning of the year 2000 geologists discovered a rock whose weight was estimated to be one megaton. Each year, due to erosion, the mass of the rock decreased by a factor of $\frac{1}{2}$. At the beginning of 2010, a second rock was found whose mass was also estimated to be one megaton. Each year, again due to erosion, the mass of the rock decreased by a factor of $\frac{1}{4}$. Let t be the number of years since 2000. After how many years will the masses of both rocks be the same?

$$\begin{array}{l}
 t = 0 \quad 1 \text{ megaton} \\
 t = 1 \quad \frac{1}{2} \\
 \hline
 \frac{1}{2}^t
 \end{array}$$

$$\begin{array}{l}
 t = 10 \quad 1 \\
 t = 11 \quad \frac{1}{4} \\
 \hline
 \frac{1}{4}^{t-10}
 \end{array}$$

$$\begin{aligned}
 \frac{1}{2}^t &= \frac{1}{4}^{t-10} \\
 (2^{-1})^t &= (2^{-2})^{t-10} \\
 -1t &= -2t + 20 \\
 t &= 20
 \end{aligned}$$

4. The town of El Dorado was founded in 1680 with a population of 100. Every decade (10 years), the population of El Dorado doubled. The town of Las Cruces was founded 80 years later (in 1760) with a population of 100. Every decade, the population of Las Cruces multiplied by 4. In what year were the populations of the two cities the same? Hint: let t represent the number of decades since El Dorado was founded.

$$\begin{array}{l}
 t = 0 \quad 100 \\
 t = 1 \quad 200 \\
 \text{decade} \\
 100(2)^t
 \end{array}
 \begin{array}{l}
) * 2
 \end{array}$$

$$\begin{array}{l}
 t = 8 \quad 100 \\
 t = 9 \quad 400 \\
 100(4)^{t-8}
 \end{array}$$

$$\frac{100(2)^t}{100} = \frac{100(4)^{t-8}}{100}$$

$$2^t = 4^{t-8}$$

$$2^t = (2^2)^{t-8}$$

$$2^t = 2^{2t-16}$$

$$t = 2t - 16$$

$$-t = -16$$

$$t = 16 \text{ decades}$$