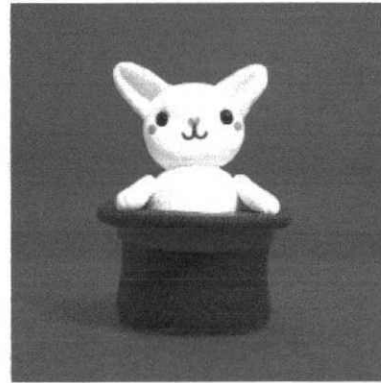


Pulling a Rabbit Out of a Hat

A Solidify Understanding Task



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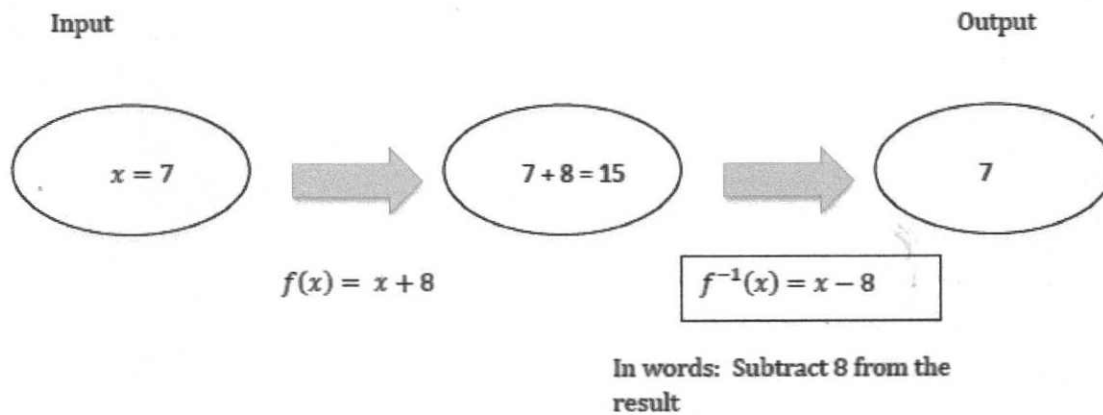
I have a trick for you:

- Pick a number, any number.
- Add 6
- Multiply by the result by 2
- Subtract 12
- Divide by 2
- The answer is the number you started with!

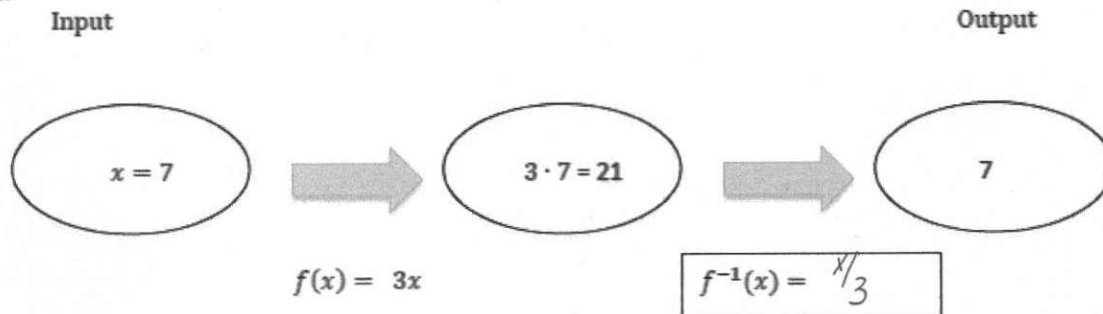
People are often mystified by such tricks but those of us who have studied inverse operations and inverse functions can easily figure out how they work and even create our own number tricks. Let's get started by figuring out how inverse functions work together.

For each of the following function machines, decide what function can be used to make the output the same as the input number. Describe the operation in words and then write it symbolically.

Here's an example:



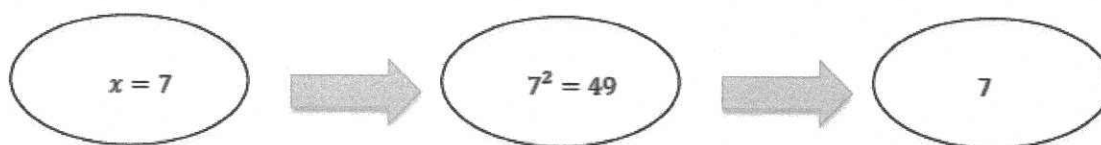
1.



2.

In words: divide the result by 3

2. Input



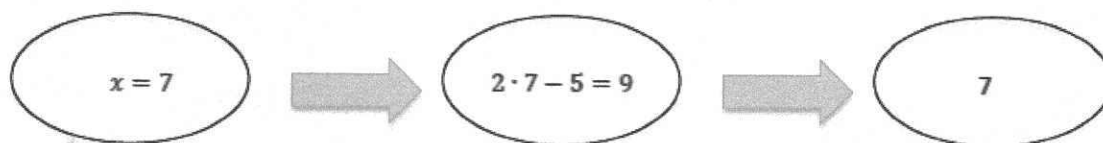
$$f(x) = x^2$$

$$f^{-1}(x) = +\sqrt{x}$$

In words: take the square root of the result

3.

Input



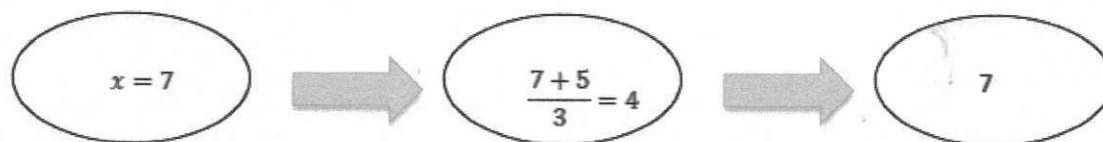
$$f(x) = 2x - 5$$

$$f^{-1}(x) = \frac{x+5}{2}$$

In words: Add 5 to the result then divide that quantity by 2

4.

Input



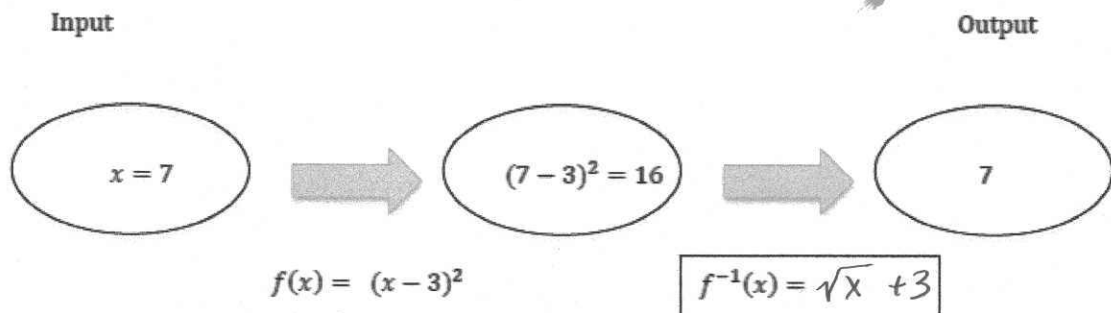
$$f(x) = \frac{x+5}{3}$$

$$f^{-1}(x) = 3x - 5$$

In words: multiply the result by 3 then subtract 5

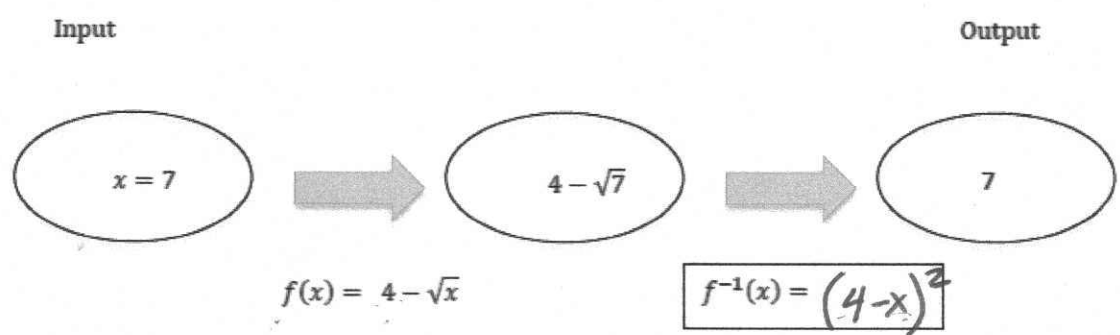


5.



In words: take the square root of the result, then add 3

6.



In words: subtract the result from 4 then square that quantity.

7. Each of these problems began with $x = 7$. What is the difference between the x used in $f(x)$ and the x used in $f^{-1}(x)$? The " x " in $f(x)$ is the independent variable, where the " x " used in $f^{-1}(x)$ was the "output" or dependent value from the original function $f(x)$.
8. In #4, could any value of x be used in $f(x)$ and still give the same output from $f^{-1}(x)$? Explain. Yes the input for $f(x)$ is \mathbb{R} and the output of $f^{-1}(x)$ is \mathbb{R} , therefore there isn't any restricting you from getting the same output.
9. Based on your work in this task and the other tasks in this module what relationships do you see between functions and their inverses?

They Undo each other



For each function, decide what function can be used to undo the given function.

Write the inverse rule verbally and algebraically.

Function	Undoing Words	Inverse Function
Ex. $f(x) = x - 3$	Add 3	$f^{-1}(x) = x + 3$
1. $f(x) = 5x$	divide by 5 result	$f^{-1}(x) = \frac{x}{5}$
2. $f(x) = x^3$	cube root result	$f^{-1}(x) = \sqrt[3]{x}$
3. $f(x) = 4x - 1$	add 1 divide result by 4	$f^{-1}(x) = \frac{x+1}{4}$
4. $f(x) = \frac{x+2}{5}$	multiply by 5 subtract 2 from result	$f^{-1}(x) = 5x - 2$
5. $f(x) = (x - 1)^3$	cube root result add 1 to result	$f^{-1}(x) = \sqrt[3]{x} + 1$
6. $f(x) = 5 + \sqrt{x}$	subtract 5 square result	$f^{-1}(x) = (x - 5)^2$
7. $f(x) = \frac{-2}{3}x + 5$	subtract 5 multiply by $-\frac{3}{2}$	$f^{-1}(x) = \frac{-3}{2}(x - 5)$
8. $f(x) = \sqrt[3]{x - 4} + 6$	subtract 6 cube result add 4 to result	$f^{-1}(x) = \sqrt[3]{x - 6} + 4$
9. $f(x) = 3(x - 4) + 2$	subtract 2 divide by 3 add 4	$f^{-1}(x) = \frac{x - 2}{3} + 4$
10. $f(x) = \frac{x+1}{4} - 7$	add 7 multiply by 4 subtract 1	$f^{-1}(x) = 4(x + 7) - 1$