Name: _____

Chapter 4 Section 4
Optimization WS 1

Date: _____ Pd: ____

Solve the following problems.
Make sure to show all work: pictures, etc.

1. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 16 cm by 20 cm sheet of cardboard. The sides will then be bent up. How large should the squares be to make the box hold as much as possible? What is that resulting volume?

2. A rectangular plot of land will be bounded on all four sides by a fence. With 1200 feet of fencing, what is the largest area you can enclose and what are those dimensions?

3. You are designing a rectangular poster to contain 60 square inches of printing with a 3 inch margin at the top and bottom, and also a 2 inch margin at each side. What overall dimensions will minimize the amount of paper used?

4. Find two numbers whose sum is 50 and whose product is as large as possible.

5. What are the dimensions of the lightest open-top, right circular cylindrical can that will hold a volume of 1000 cm^3?

HOMEWORK Name: _____

Chapter 4 Section 4
Optimization WS 2

Date: _____ Pd: ____

Solve the following problems.
Make sure to show all work: pictures, formulas, etc.

1. Your boss has asked you to design and build a 250 ft^3, square-based, open-top, rectangular plastic tank for an aquarium. The tank will be made by putting together thin sheets of clear plastic along their edges. As the production engineer, it is your job to find the dimensions for the base and height that will make the tank weigh as little as possible.

2. Find two numbers whose sum is 36 and whose product is as large as possible.

3. What are the dimensions of the lightest right circular cylindrical can that will hold a volume of 2000 cm^3?
NOTE: THE WORDING ON THIS QUESTION IS DIFFERENT THAN PROBLEM DONE IN NOTES.

4. A 225 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?

5. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 30" by 21" sheet of cardboard. The sides will then be bent up. How large should the squares be to make the box hold as much as possible? What is that resulting volume?

6. You are designing a rectangular poster to contain 45 square inches of printing with a 4 inch margin at the top and bottom, and also a 3 inch margin at each side. What overall dimensions will minimize the amount of paper used?

Name: Key

Chapter 4 Section 4
Optimization WS 1

Date: _____ Pd: ____

Solve the following problems.
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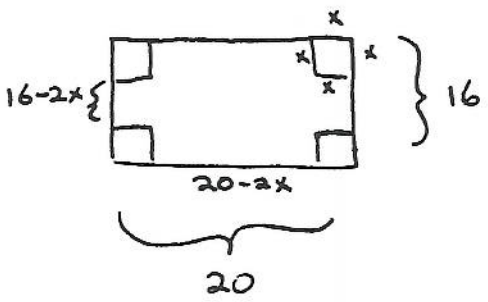
1. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 16 cm by 20 cm sheet of cardboard. The sides will then be bent up. How large should the squares be to make the box hold as much as possible? What is that resulting volume?



$V = l \cdot w \cdot h$

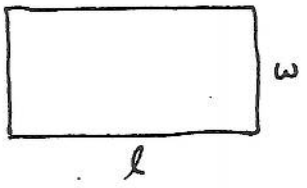
$V = (16-2x)(20-2x)(x)$

$x = 2.945\text{cm}$ will give you a maximum volume of $420.110\ \text{cm}^3$

A Square 2.945cm x 2.945cm

$x\ [0, 30, 2]$
$y\ [0, 500, 5]$

2. A rectangular plot of land will be bounded on all four sides by a fence. With 1200 feet of fencing, what is the largest area you can enclose and what are those dimensions?



$l = 300$
$w = 300$

A 300Ft x 300Ft Fence in lot will produce the maximum area (90,000 Ft²)

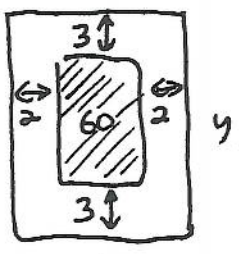
$P = 2l + 2w$

$1200 = 2l + 2w \rightarrow l + w = 600 \rightarrow w = 600 - l$

$A = lw \qquad A = l(600 - l)$

$x\ [0, 600, 25]$
$y\ [0, 95000, 25000]$

3. You are designing a rectangular poster to contain 60 square inches of printing with a 3 inch margin at the top and bottom, and also a 2 inch margin at each side. What overall dimensions will minimize the amount of paper used?



$A = (x+4)(y+6)$

$A = (x+4)\left(\frac{60}{x}+6\right)$

$x = 6.325\text{in}$
$y = 9.486\text{in}$
} will minimize to $159.895\ \text{in}^2$

$xy = 60$

$y = \frac{60}{x}$

$x\ [0, 30, 5]$
$y\ [0, 500, 25]$

4. Find two numbers whose sum is 50 and whose product is as large as possible.

$x + y = 50$

$y = 50 - x$

$y = x(50 - x)$

$x = 25$, $y = 25$ } will give maximum product of 625

$x\ [0, 50, 5]$

$y\ [0, 750, 50]$

5. What are the dimensions of the lightest open-top, right circular cylindrical can that will hold a volume of 1000 cm^3?

$V = Bh$

$V = \pi r^2 h$

$1000 = \pi r^2 h$

$h = \frac{1000}{\pi r^2}$

$S.A = 2\pi r^2 + 2\pi rh$ But open top so 1 BASE only

$S.A = \pi r^2 + 2\pi rh$

$S.A = \pi r^2 + 2\pi r\left(\frac{1000}{\pi r^2}\right)$

$S.A = \pi r^2 + \frac{2000}{r}$

$y = \pi x^2 + \frac{2000}{x}$

$x = 6.828\ cm$

$h = 6.828\ cm$

$h = \frac{1000}{\pi (6.828)^2}$

$x\ [0, 50, 1]$

$y\ [0, 750, 50]$

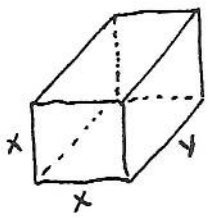
HOMEWORK Name: Key

Chapter 4 Section 4
Optimization WS 2

Date: _____ Pd: ____

Solve the following problems.
Make sure to show all work: pictures, formulas, etc.

1. Your boss has asked you to design and build a 250 ft^3, square-based, open-top, rectangular plastic tank for an aquarium. The tank will be made by putting together thin sheets of clear plastic along their edges. As the production engineer, it is your job to find the dimensions for the base and height that will make the tank weigh as little as possible.



$V = x^2y$

$250 = x^2y$

$y = \frac{250}{x^2}$

S.A. $= 2B + ph$ BUT NO TOP SO

$S.A = B + ph$

$S.A = x^2 + 4xy$

$S.A = x^2 + 4x\left(\frac{250}{x^2}\right)$

$S.A = x^2 + \frac{1000}{x}$

$x = 7.979\ ft, y = 3.927\ ft$ MIN VOLUME $188.993\ ft^3$

2. Find two numbers whose sum is 36 and whose product is as large as possible.

$x + y = 36$

$y = 36 - x$

$x(36 - x) = MAX$

$y = x(36 - x)$ FIND MAX OF EQN.

MAX = 18

18 × 18 will give maximum product (324)

3. What are the dimensions of the lightest right circular cylindrical can that will hold a volume of 2000 cm^3?
NOTE: THE WORDING ON THIS QUESTION IS DIFFERENT THAN PROBLEM DONE IN NOTES.

$V_{cyL} = \pi r^2 h$

$2000 = \pi r^2 h$

$h = \frac{2000}{\pi r^2}$

$h = \frac{2000}{\pi(6.828)^2}$

$h = 13.655$

$S.A = 2\pi r^2 + 2\pi rh$

$(2B) + ph$

$S.A = 2\pi r^2 + 2\pi r\left(\frac{2000}{\pi r^2}\right)$

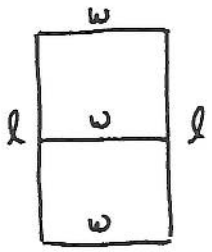
$S.A = 2\pi r^2 + \frac{4000}{r}$

$r = 6.828$

$S.A = 878.755$

A RADIUS OF 6.828 cm AND A HEIGHT OF 13.655 cm WILL GIVE MINIMUM SURFACE AREA

4. A 225 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of the sides. What dimensions for the outer rectangle will require the smallest total length of fence? How much fence will be needed?



$P = 2l + 3w$

$P = 2l + 3\left(\frac{225}{l}\right)$

$P = 2l + \frac{675}{l}$

$l = 18.511$

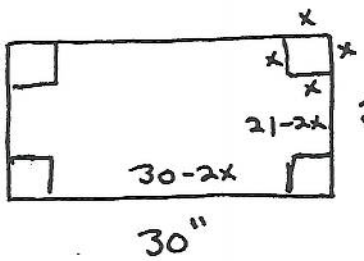
$w = 12.155$

$225 = lw$

$w = \frac{225}{l}$

A length of 18.511 m and width of 12.155 m will give perimeter of 73.487 m

5. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 30" by 21" sheet of cardboard. The sides will then be bent up. How large should the squares be to make the box hold as much as possible? What is that resulting volume?



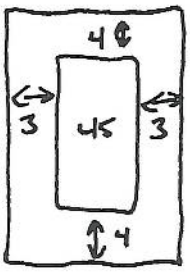
$V = x(30-2x)(21-2x)$

$x = 4.056"$ × $4.056"$ squares } max vol of $1144.167\ in^3$

WIN $x\ [0, 30, 2]$

$y\ [-500, 1500, 25]$

6. You are designing a rectangular poster to contain 45 square inches of printing with a 4 inch margin at the top and bottom, and also a 3 inch margin at each side. What overall dimensions will minimize the amount of paper used?



$xy = 45$

$y = \frac{45}{x}$

$(x+6)(y+8) = A$

$x + 6\left(\frac{45}{x} + 8\right) = A$

$x = 5.745$ in

$y = 7.833$ in