36. If $\alpha = -\beta$, the terms in x^2 and y^2 will vanish, giving $16\beta + -6\beta x - 4\beta y - 12\beta = 0$ or $3x + 2y = 2$. This linear equation is an equation containing the points of intersection of the circles of Problems 34 and 35.

37. If $2\alpha = \beta$, then the given equation will simplify to $3\alpha x^2 + 3\alpha y^2 - 12\alpha x - 8\alpha y - 40\alpha = 0$, or $x^2 + y^2 - 4x - 8y/3 - 40/3 = 0$. Completing the squares in x and y, we get $(x - 2)^2 + (y - 4/3)^2 = 172/9$. This is an equation of a circle through the points of intersection of the circles of Problems 34 and 35.

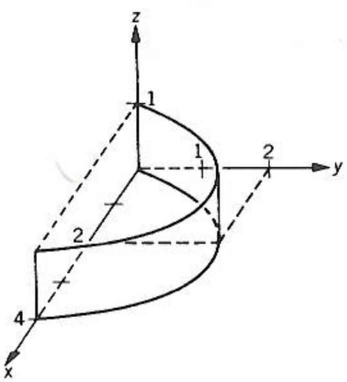
Calculator Problem

If $A = (-5.2, 1.4)$, $B = (3.7, 4.5)$, and $C = (-2.8, -6.1)$, then $\overline{AB}$ has slope 31/89 and midpoint $(-0.75, 2.95)$, $\overline{AC}$ has slope $-25/8$ and midpoint $(-4, -2.35)$. So the center is the intersection of $y - 2.95 = (-89/31)(x + 0.75)$ and $y + 2.35 = (8/25)(x + 4)$. The center is $(0.5850182, -0.8827941)$. The radius is 6.2191305 units.

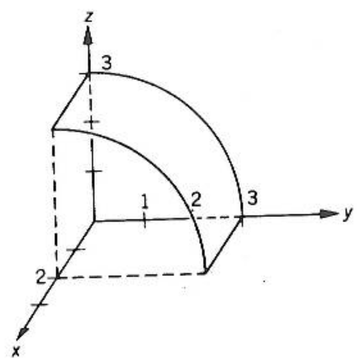
Section 17-3, page 432

Set A

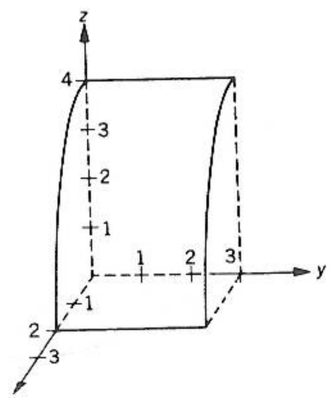
1.



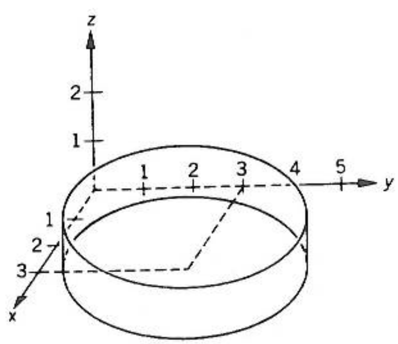
2.



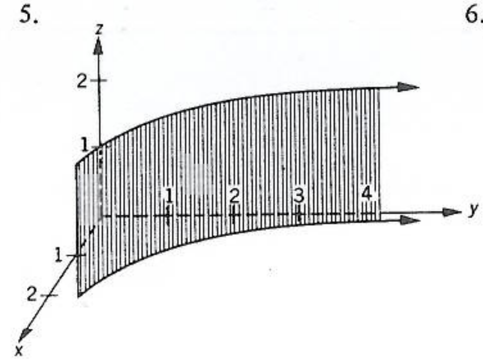
3.



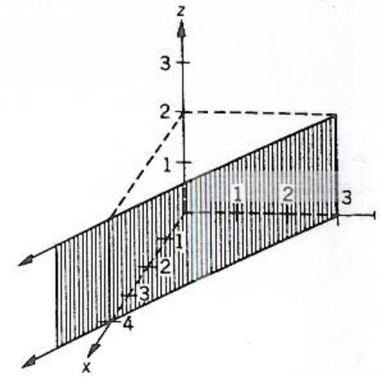
4.



5.

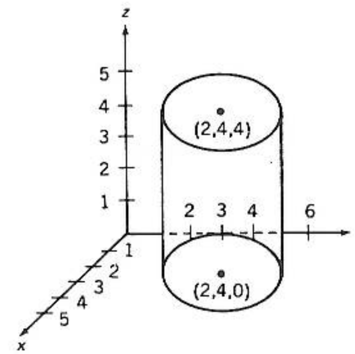


6.

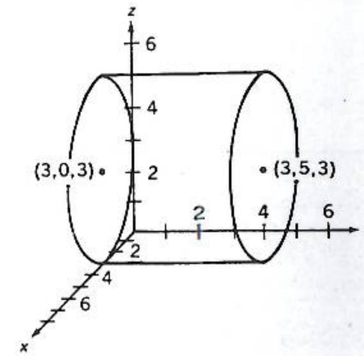


Set B

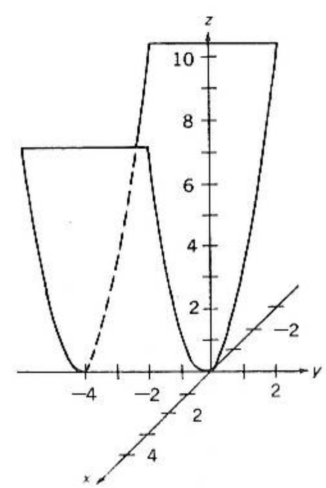
7. $(x - 2)^2 + (y - 4)^2 = 4$



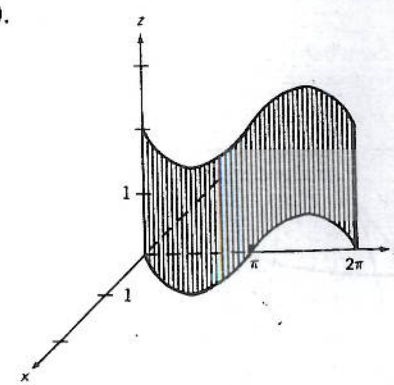
8. $(x - 3)^2 + (z - 3)^2 = 9$



9.

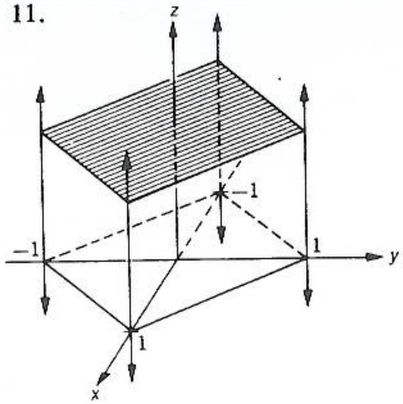


10.

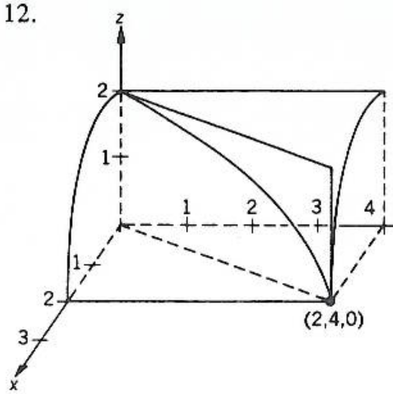


Set C

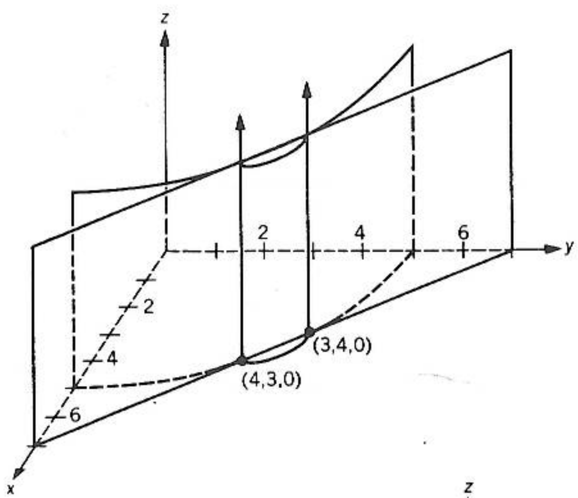
11.



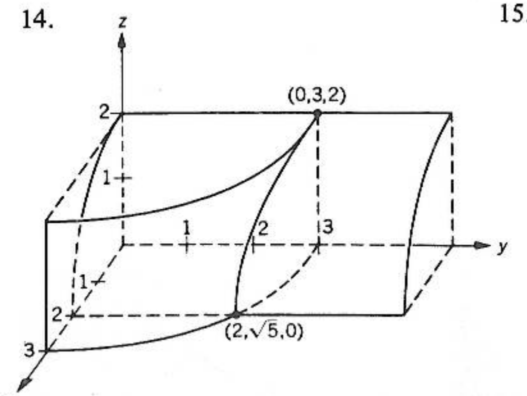
12.



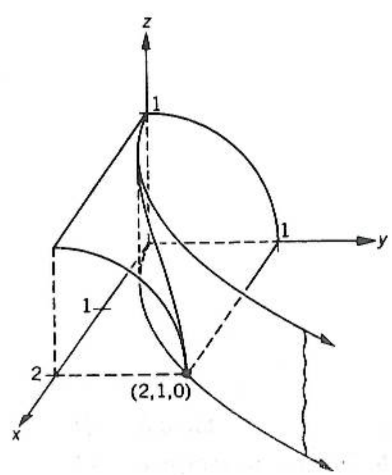
13.



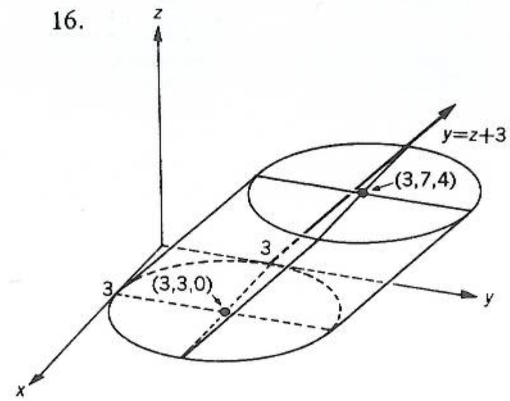
14.



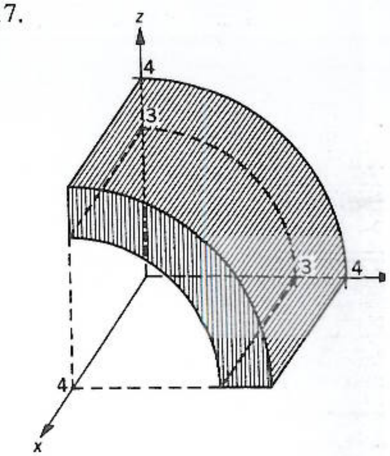
15.



16.



17.



Calculator Problem

Shortest distance from A to B along the surface of the cylinder:
$\sqrt{4\pi^2 + 8^2} = \sqrt{103.4781} = 10.172434$

Section 17-4, page 435

Set A

1. $x^2 + y^2 + z^2 - 4x - 2y + 1 = 0$
2. $4x^2 + 4y^2 + 4z^2 - 8x - 8y + 8z + 11 = 0$
3. $x^2 + y^2 + z^2 + 8x - 4y + 6z + 4 = 0$
4. $x^2 + y^2 + z^2 = a^2$
5. $x^2 + y^2 + z^2 + 2x - 4y + 4z = 0$
6. $x^2 + y^2 + z^2 - 6x - 4y - 8z + 13 = 0$

The center and radius of the given sphere is

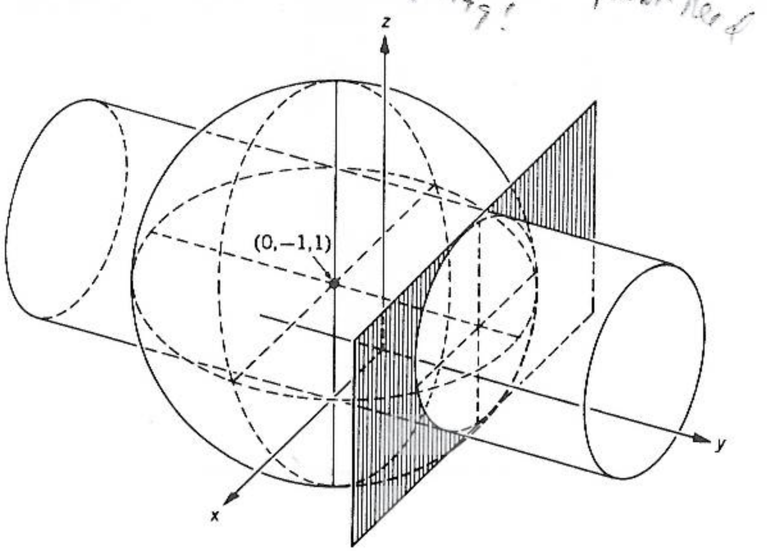
7. $(1/2, 1/2, 1/2), \sqrt{3}$
8. $(0, 0, 9/4), \sqrt{89}/4$
9. $(10, 10, 0), 1$
10. $(0, a, 0), a$

Set B

11. Empty
12. Sphere with center $(1/2, -4, 1)$ and radius $(1/2)\sqrt{93}$
13. A point
14. Sphere with center $(7, 4, 5)$ and radius 1

15. The radius of the sphere is 4 and the plane $y = 2$ is three units distant from the center of the sphere. By the Pythagorean Theorem, the radius of the cylinder is $\sqrt{7}$ units. Since the generators of the sylinder are parallel to the y-axis and the axis of the cylinder contains the point $(0, -1, 1)$, an equation of the cylinder is $x^2 + (z-1)^2 = 7$.

Sub in y = 2, don't need Pythag!



Set C

16. The point $(-2, -3, 2)$ is on the sphere $x^2 + y^2 + z^2 = 17$ because $(-2)^2 + (-3)^2 + (2)^2 = 4 + 9 + 4 = 17$. $c_1 = -2/\sqrt{17}, c_2 = -3/\sqrt{17}$, $c_3 = 2/\sqrt{17}$; $x = -2d/\sqrt{17}, y = -3d/\sqrt{17}, z = 2d/\sqrt{17}$.

17. $(1 + \sqrt{2}, -1 - \sqrt{2}, -2)$

18. The equation of the sphere is, equivalently, $(x+2)^2 + (y-1)^2 + (z-2)^2 = 10$. So the center of the sphere is $(-2, 1, 2)$ and the radius is $\sqrt{10}$. The distance from $(-2, 1, 2)$ to the plane $3x - y - 3 = 0$ is $|3(-2) - 1 - 3|/\sqrt{10}$ $= |-10|/\sqrt{10} = \sqrt{10}$. The distance from $(-2, 1, 2)$ to the plane is the same as the radius of the sphere, so the plane and sphere must be tangent.

19. Parametric equations of the ray from the center of the sphere through the point of tangency are given by: $x = -2 + 3d/\sqrt{10}, y = 1 - d/\sqrt{10}$, and $z = 2$. Choosing $d = r = \sqrt{10}$, we get $x = 1, y = 0$, and $z = 2$.

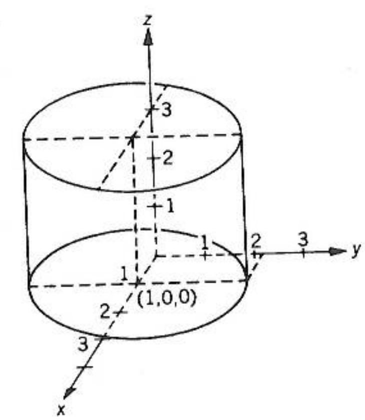
20. The sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ has its center at $(-1, 3, 0)$ and has radius 3. If O is the center of the sphere and $P = P(2, 0, 3)$, then $\overrightarrow{OP}$ is normal to the plane tangent to the sphere at the point where the ray intersects the sphere. $\overrightarrow{OP} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ and a unit vector having the same direction as $\overrightarrow{OP}$ is $\mathbf{n} = \mathbf{i}/\sqrt{3} - \mathbf{j}/\sqrt{3} + \mathbf{k}/\sqrt{3}$. Parametric equations of the ray from the center of the sphere are $x = -1 + d/\sqrt{3}, y = 3 + -d/\sqrt{3}, z = 0 + d/\sqrt{3}$. Choosing $d = r = 3$, we find the coordinates where the ray intersects the sphere to be $(-1 + \sqrt{3}, 3 - \sqrt{3}, \sqrt{3})$.

Calculator Problem

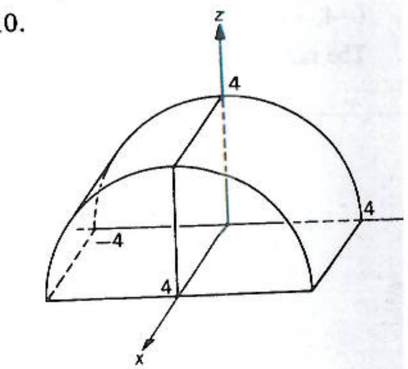
Centers of the spheres are $(4, 4, 3)$ and $(4, 12, 9)$ with $r = 4$ and $r = 6$, respectively. Parametric equations of the line of center with $(4, 4, 3)$ as (x_0, y_0, z_0) are $x = 4, y = 4 + (8/10)d, z = 3 + (6/10)d$. Choosing $d = 4$ since the radius of the smaller circle is 4, we have $x = 4, y = 36/5, z = 27/5$.

Test, page 436

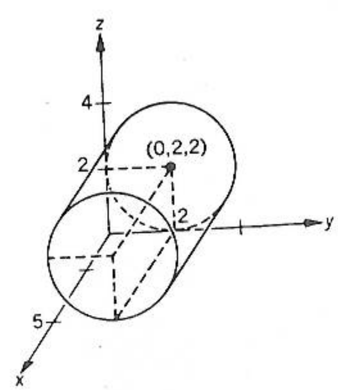
1. $a^2/4 + b^2/4 - c > 0$
2. $a^2/4 + b^2/4 - c = 0$
3. $a^2/4 + b^2/4 - c < 0$
4. $(7/2, -4)$; $r = \sqrt{131}/2$
5. $(-1/2, 1/2)$; $r = \sqrt{5}/2$
6. $(3, 0)$; $r = 3$
7. $x^2 + y^2 + 2x - 16 = 0$
8. $(x-5)^2 + (y-5)^2 = 25$
9.



10.



11.



Section 18-9, page 470

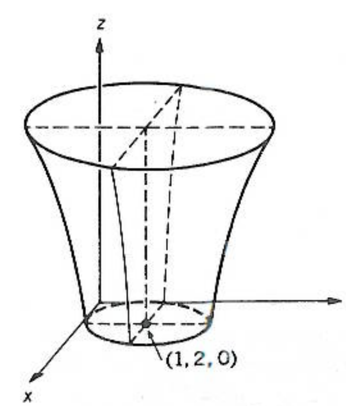
Set A

1-9. The sketches referred to are those given in Section 18-9 of the text.

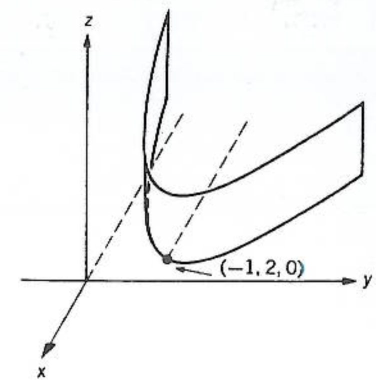
1. Ellipsoid with center (0, 0, 0) and intercepts $(\pm\sqrt{6}, 0, 0)$, $(0, \pm\sqrt{3}, 0)$, and $(0, 0, \pm\sqrt{2})$.
2. Hyperboloid of one sheet. The sketch is similar to sketch (b).
3. Hyperboloid of two sheets. The sketch is similar to sketch (c) rotated about the x-axis through an angle of 90°.
4. Quadric cone (circular sections). The sketch is similar to sketch (f) rotated about the x-axis through an angle of 90°.
5. Paraboloid. The sketch is similar to sketch (d) rotated about the x-axis through an angle of $-90°$.
6. Hyperbolic paraboloid. The sketch is similar to sketch (e).
7. The two intersecting planes $x + z = 1$ and $x = y$.
8. Elliptical cylinder with the z-axis as its axis. Its trace in the xy-plane is an ellipse with center (0, 0, 0) and intercepts $(\pm\sqrt{6}, 0, 0)$ and $(0, \pm\sqrt{3}, 0)$.
9. Parabolic cylinder. The sketch is similar to sketch (g).

Set B

10. The equation $x^2 + y^2 + z^2 - 4z = 0$ is equivalent to $x^2 + y^2 + (z - 2)^2 = 4$, which is an equation of the sphere with center at (0, 0, 2) and radius 2.
11. The equation $x^2 + 2y^2 + 3z^2 - 2x - 8y + 3 = 0$ is equivalent to $(x - 1)^2 + 2(y - 2)^2 + 3z^2 = 6$. This is an equation of an ellipsoid with center at (1, 2, 0).
12. The equation $x^2 + y^2 - z^2 - 2x - 4y + 1 = 0$ is equivalent to $(x - 1)^2 + (y - 2)^2 - z^2 = 4$ or $$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{4} + \frac{z^2}{4} = 1.$$ This is an equation of a hyperboloid of one sheet with a circular cross-section and axis parallel to the z-axis.



13. The equation $y^2 + 4x - 4y + 8 = 0$ is equivalent to $(y-2)^2 = -4(x+1)$. This is an equation of a parabolic cylinder.

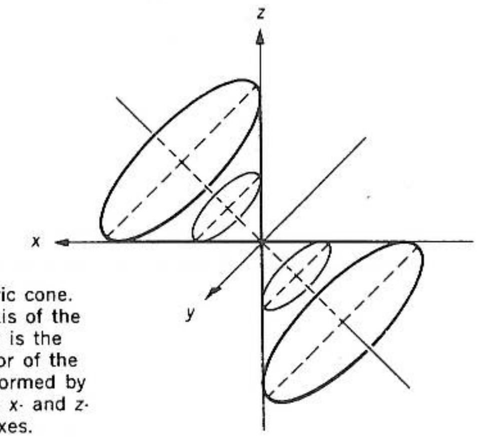


14. The given equation is equivalent to $(x-1)^2 + (y-1)^2 = 3(z-1)^2$. This is an equation of a cone.

15. The given equation is equivalent to $(x+1)^2/2 + (y+1)^2/3 = (z+1)$. This is an equation of a paraboloid.

Set C

16.



Quadric cone. The axis of the cone is the bisector of the angle formed by positive x- and z-axes.

17. The general quadric surface, represented by the equation

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0, \quad (1)$$

has a trace in the xy-plane ($z = 0$) given by

$$Ax^2 + By^2 + Dxy + Gx + Hy + J = 0. \quad (2)$$

By Theorem 18-2, equation (2), if not degenerate, represents one of the conic sections.

Setting $x = 0$ in (1), we get the trace of the general quadric surface in the yz-plane. It is

$$By^2 + Cz^2 + Eyz + Hy + Iz + J = 0, \quad (3)$$

by Theorem 18-2 and, if not degenerate, is a conic.

Setting $y = 0$, we get the trace in the xz-plane, which is

$$Ax^2 + Cz^2 + Fxz + Gx + Iz + J = 0, \quad (4)$$

and we see that (4) is also an equation of a conic.

In planes parallel to the coordinate planes, we will change only the constant term in each of the equations above, so that these equations will also represent conics.