ADDITIONAL RESOURCES

The following resources are available to help review the material in this chapter.

- Chapter Review Games and Activities (*Chapter 4 Resource Book*, p. 80)
- *Instant Replay: Video Review Games*
- *Personal Student Tutor*
- Cumulative Review, Chs. 1–4 (*Chapter 4 Resource Book*, p. 92)

2. Not possible; the two matrices do not have the same dimensions.

4. Not possible; the two matrices do not have the same dimensions.

5. $\begin{bmatrix} 8 & 12 & -2 \\ 20 & -10 & 4 \\ 0 & 22 & 2 \end{bmatrix}$

CHAPTER 4

Chapter Review

VOCABULARY

- matrix, p. 199
- dimensions of a matrix, p. 199
- entries of a matrix, p. 199
- row matrix, p. 199
- column matrix, p. 199
- square matrix, p. 199
- zero matrix, p. 199
- equal matrices, p. 199
- scalar, p. 200
- determinant, p. 214
- Cramer's rule, p. 216
- coefficient matrix, p. 216
- identity matrix, p. 223
- inverse matrix, p. 223
- matrix of variables, p. 230
- matrix of constants, p. 230

4.1 MATRIX OPERATIONS

Examples on pp. 199–202

EXAMPLES You can add or subtract matrices that have the same dimensions by adding or subtracting corresponding entries.

$$\begin{bmatrix} 5 & -2 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 5+9 & -2+1 \\ 0+(-4) & 6+4 \end{bmatrix} = \begin{bmatrix} 14 & -1 \\ -4 & 10 \end{bmatrix}$$

You cannot subtract these matrices because they have different dimensions.

$$\begin{bmatrix} -2 & 1 \\ 0 & -3 \end{bmatrix} - \begin{bmatrix} 1 & -5 & -4 \\ 2 & 7 & 1 \end{bmatrix}$$

To do scalar multiplication, multiply each entry in the matrix by the scalar.

$$-3\begin{bmatrix} -12 & -6 \\ 3 & 1 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} (-3)(-12) & (-3)(-6) \\ (-3)(3) & (-3)(1) \\ (-3)(2) & (-3)(8) \end{bmatrix} = \begin{bmatrix} 36 & 18 \\ -9 & -3 \\ -6 & -24 \end{bmatrix}$$

To solve this matrix equation, equate corresponding entries and solve for x and y.

$$\begin{bmatrix} x+2 & 2 \\ -1 & 9 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ -1 & 3y \end{bmatrix} \qquad \begin{aligned} x+2 &= -6 \\ x &= -8 \end{aligned} \qquad \begin{aligned} 3y &= 9 \\ y &= 3 \end{aligned}$$

Perform the indicated operation if possible. If not possible, state the reason.

1. $\begin{bmatrix} 15 & 4 \\ 3 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 9 \\ 2 & 7 \end{bmatrix}$ $\begin{bmatrix} 15 & -5 \\ 1 & 5 \end{bmatrix}$

2. $\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ **See margin.**

3. $\begin{bmatrix} 6 & 10 \\ 9 & 6 \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 7 \\ 4 & 7 \end{bmatrix}$ $\begin{bmatrix} 8 & 11 \\ 9 & 13 \\ 8 & 6 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 1 & 5 \\ -2 & 3 & 1 \\ 1 & 2 & -4 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 4 & 1 \\ 2 & -3 \end{bmatrix}$ **4 and 5. See margin.**

5. $2\begin{bmatrix} 4 & 6 & -1 \\ 10 & -5 & 2 \\ 0 & 11 & 1 \end{bmatrix}$

6. $\frac{1}{2}\begin{bmatrix} -2 & 0 \\ 4 & 8 \\ -6 & -2 \end{bmatrix}$ $\begin{bmatrix} -1 & 0 \\ 2 & 4 \\ -3 & -1 \end{bmatrix}$

Solve the matrix equation for x and y.

7. $\begin{bmatrix} 1 & 14 \\ -5x & 10 \end{bmatrix} = \begin{bmatrix} y-9 & 14 \\ 5 & 10 \end{bmatrix}$ $x = -1, y = 10$

8. $\begin{bmatrix} 3 & 4y \\ -1 & 13 \end{bmatrix} + \begin{bmatrix} -6 & 5 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -3 & -7 \\ x & 13 \end{bmatrix}$ $x = 7, y = -3$

9. $\begin{bmatrix} 2 & 3y \\ 4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & -4 \\ x & -2 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 3 & -3 \end{bmatrix}$ $x = -1, y = 5$

10. $\begin{bmatrix} 7y & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ x & -3 \end{bmatrix} = \begin{bmatrix} 6 & -7 \\ -2 & 8 \end{bmatrix}$ $x = -1, y = 1$

4.2 MULTIPLYING MATRICES

Examples on pp. 208–210

EXAMPLE You can multiply a matrix with n columns by a matrix with n rows.

$$\begin{bmatrix} -6 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (-6)(6) + (1)(0) & (-6)(3) + (1)(1) \\ (5)(6) + (-2)(0) & (5)(3) + (-2)(1) \end{bmatrix} = \begin{bmatrix} -36 & -17 \\ 30 & 13 \end{bmatrix}$$

Write the product. If it is not defined, state the reason.

11. $\begin{bmatrix} 12 \\ -4 \end{bmatrix}\begin{bmatrix} -10 & -7 \end{bmatrix}$ $\begin{bmatrix} -120 & -84 \\ 40 & 28 \end{bmatrix}$

12. $\begin{bmatrix} 2 & 15 \\ -3 & 10 \end{bmatrix}\begin{bmatrix} -5 & 12 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 5 & 24 \\ 25 & -36 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 7 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 3 & -1 & 8 \\ 2 & -4 & 8 \end{bmatrix}$ $\begin{bmatrix} 17 & -29 & 64 \\ 18 & -36 & 72 \end{bmatrix}$

4.3 DETERMINANTS AND CRAMER'S RULE

Examples on pp. 214–217

EXAMPLES You can evaluate the determinant of a 2 × 2 or a 3 × 3 matrix. Find products of the entries on the diagonals and subtract.

$$\det\begin{bmatrix} -2 & -6 \\ 1 & 4 \end{bmatrix} = \begin{vmatrix} -2 & -6 \\ 1 & 4 \end{vmatrix} = -2(4) - 1(-6) = -8 + 6 = -2$$

$$\det\begin{bmatrix} 2 & 1 & 5 \\ -1 & 6 & 3 \\ 2 & -4 & 2 \end{bmatrix} = \begin{vmatrix} 2 & 1 & 5 \\ -1 & 6 & 3 \\ 2 & -4 & 2 \end{vmatrix}\begin{matrix} 2 & 1 \\ -1 & 6 \\ 2 & -4 \end{matrix} = (24 + 6 + 20) - [60 + (-24) + (-2)] = 16$$

You can find the area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) using

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where ± indicates you should choose the sign that yields a positive value.

You can use Cramer's rule to solve a system of linear equations. First find the determinant of the coefficient matrix and then use Cramer's rule to solve for x and y.

$3x - 4y = 12$
$x + 2y = 14$

$$\det\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix} = 3(2) - 1(-4) = 6 + 4 = 10$$

$$x = \frac{\begin{vmatrix} 12 & -4 \\ 14 & 2 \end{vmatrix}}{10} = \frac{12(2) - 14(-4)}{10} = \frac{80}{10} = 8 \qquad y = \frac{\begin{vmatrix} 3 & 12 \\ 1 & 14 \end{vmatrix}}{10} = \frac{3(14) - 1(12)}{10} = \frac{30}{10} = 3$$

Evaluate the determinant of the matrix.

14. $\begin{bmatrix} -9 & 1 \\ 3 & 2 \end{bmatrix}$ **−21**

15. $\begin{bmatrix} 6 & -3 \\ 2 & 1 \end{bmatrix}$ **12**

16. $\begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 1 \\ 0 & 3 & 4 \end{bmatrix}$ **−5**

17. $\begin{bmatrix} 2 & -3 & 4 \\ 0 & 1 & -2 \\ 1 & 2 & -3 \end{bmatrix}$ **4**

18. Find the area of a triangle with vertices $A(0, 1)$, $B(2, 4)$, and $C(1, 8)$. $\frac{11}{2}$ **square units**

Use Cramer's rule to solve the linear system.

19. $7x - 4y = -3$
$2x + 5y = -7$ **(−1, −1)**

20. $2x + y = -2$
$x - 2y = 19$ **(3, −8)**

21. $5x - 4y + 4z = 18$
$-x + 3y - 2z = 0$ **(6, 0, −3)**
$4x - 2y + 7z = 3$

4.4 IDENTITY AND INVERSE MATRICES

Examples on pp. 222–226

EXAMPLES You can find the inverse of an $n \times n$ matrix provided its determinant does not equal zero.

The inverse of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

If $A = \begin{bmatrix} 7 & 3 \\ 5 & 2 \end{bmatrix}$, then $A^{-1} = \dfrac{1}{7(2) - 5(3)}\begin{bmatrix} 2 & -3 \\ -5 & 7 \end{bmatrix} = -1\begin{bmatrix} 2 & -3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 5 & -7 \end{bmatrix}$.

You can use the inverse of a matrix A to solve a matrix equation $AX = B$: $X = A^{-1}B$.

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} X = \begin{bmatrix} 3 & 0 \\ 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{7 - 6}\begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -1 & 2 \end{bmatrix}$$

Find the inverse of the matrix.

22. $\begin{bmatrix} 2 & 3 \\ 7 & 11 \end{bmatrix}$ $\begin{bmatrix} 11 & -3 \\ -7 & 2 \end{bmatrix}$ **23.** $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ $\begin{bmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$ **24.** $\begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$ **no inverse** **25.** $\begin{bmatrix} 6 & -1 \\ -5 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$

Solve the matrix equation.

26. $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} X = \begin{bmatrix} 0 & 9 \\ -1 & 4 \end{bmatrix}$ $\begin{bmatrix} 3 & 6 \\ -5 & -7 \end{bmatrix}$

27. $\begin{bmatrix} -7 & -5 \\ 4 & 3 \end{bmatrix} X + \begin{bmatrix} 8 & -2 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ 6 & 2 \end{bmatrix}$ $\begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}$

4.5 SOLVING SYSTEMS USING INVERSE MATRICES

Examples on pp. 230–232

EXAMPLE You can use inverse matrices to solve a system of linear equations.

$x + 3y = 10$
$2x + 5y = -2$

Write in matrix form. $\underset{A}{\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{B}{\begin{bmatrix} 10 \\ -2 \end{bmatrix}}$

Then $X = A^{-1}B = \dfrac{1}{1(5) - 2(3)}\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 10 \\ -2 \end{bmatrix} = -1\begin{bmatrix} 56 \\ -22 \end{bmatrix} = \begin{bmatrix} -56 \\ 22 \end{bmatrix}$.

The solution is $(-56, 22)$.

Use an inverse matrix to solve the linear system.

28. $9x + 8y = -6$
$-x - y = 1$
(2, −3)

29. $x - 3y = -2$
$5x + 3y = 17$
$\left(\frac{5}{2}, \frac{3}{2}\right)$

30. $4x - 14y = -15$
$18x - 12y = 9$
$\left(\frac{3}{2}, \frac{3}{2}\right)$

Use an inverse matrix and a graphing calculator to solve the linear system.

31. $x - y - 4z = 3$
$-x + 3y - z = -1$
$x - y + 5z = 3$
(4, 1, 0)

32. $4x + 10y - z = -3$
$11x + 28y - 4z = 1$
$-6x - 15y + 2z = -1$
(19, −9, −11)

33. $5x - 3y + 5z = -1$
$3x + 2y + 4z = 11$
$2x - y + 3z = 4$
(−3, 2, 4)