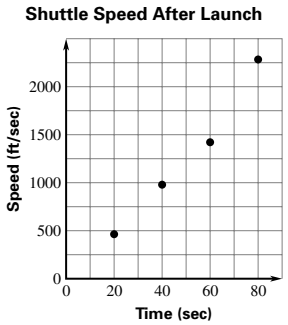
CHAPTER 6

Think & Discuss *(p. 321)*

1. 

about 41 seconds

2. A quadratic function would be a good model because the data lies on a curve.

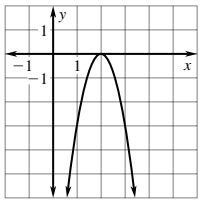
Skill Review *(p. 322)*

1. $4x^2 - 2x + x - x^2 = 3x^2 - x = x(3x - 1)$

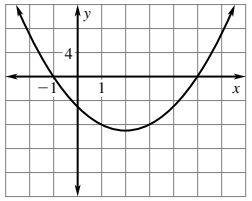
2. $2(8x + 5) - 19x = 16x + 10 - 19x = -3x + 10$

3. $-x^3 - 5x^4 - 3x^3 + 7x^2 = -5x^4 - 4x^3 + 7x^2$
$= x^2(-5x^2 - 4x + 7)$

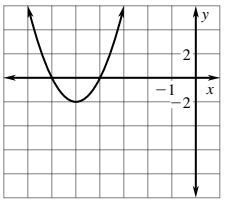
4. $y = -3(x - 2)^2$



5. $y = (x + 1)(x - 5)$



6. $y = 2(x + 6)(x + 4)$



7. $y = (x - 1)^2 - 7$
$= x^2 - 2x + 1 - 7$
$= x^2 - 2x - 6$

8. $y = 2(x + 4)^2$
$= 2(x^2 + 8x + 16)$
$= 2x^2 + 16x + 32$

9. $y = -(x - 2)(x + 8)$
$= (x^2 + 6x - 16)$
$= -x^2 - 6x + 16$

10. $x^2 + 6x - 27 = 0$
$x^2 + 6x + 9 = 27 + 9$
$(x + 3)^2 = 36$
$x + 3 = \pm 6$
$x = -3 \pm 6$
$x = -9$ or
$x = 3$

11. $x^2 + 20x + 100 = 0$
$(x + 10)^2 = 0$
$x + 10 = 0$
$x = -10$

12. $2x^2 + 5x = 12$
$2\left(x^2 + \frac{5}{2}x + \frac{25}{16}\right) = 12 + \frac{25}{8}$
$2\left(x + \frac{5}{4}\right)^2 = \frac{121}{8}$
$\left(x + \frac{5}{4}\right)^2 = \frac{121}{16}$
$x + \frac{5}{4} = \pm\frac{11}{4}$
$x = -\frac{5}{4} \pm \frac{11}{4}$
$x = -4$ or $x = \frac{3}{2}$

Lesson 6.1

Activity (p. 323)

1. 7; 2^7 **2. a.** 2^7 **b.** 2^7 **c.** 2^9 **d.** 2^8

3. 2^{m+n} **4. a.** 2^2 **b.** 2^3 **c.** 2^4 **d.** 2^4 **5.** 2^{m-n}

6.1 Guided Practice (p. 326)

1. a. product of powers **b.** power of a power

c. power of a product

2. a. The bases were multiplied; $(-2)^5$

b. The exponents were divided when they should have been subtracted; x^6

c. The exponents were mulitplied when they should have been added; x^7

3. 216; product of powers

4. 1; power of a power and product of powers

5. 64; power of a power

6. 3; negative exponent, power of power, and product of powers

7. $\frac{25}{9}$; negative exponent and power of a quotient

8. $\frac{1}{49}$; negative exponent quotient of power

9. 1; product of powers

10. $\frac{x^6y^4}{z}$; negative exponent, power of a power, and product of powers

11. $\frac{1}{16x^6}$; negative exponent and power of a power

12. $64x^{18}$; power of a quotient, negative exponent, and power of a product

13. $3y^3$; quotient of powers

14. x^3y^5; negative exponent, power of a product, quotient of powers, and product of powers

15. $\frac{\frac{4}{3}\pi(6.96 \times 10^5)^3}{\frac{4}{3}\pi(6.38 \times 10^3)^3} = \frac{\frac{4}{3}\pi(3.37 \times 10^{17})}{\frac{4}{3}\pi(2.60 \times 10^{11})} = \frac{1.41 \times 10^{18}}{1.09 \times 10^{12}}$

$= 1.29 \times 10^6$

sun's volume: 1.41×10^{18} km^3

Earth's volume: 1.09×10^{12} km^3

Ratio is about 1,290,000. Yes, the results match.

6.1 Practice and Applications (pp. 326–328)

16. $4^{2+4} = 4^6 = 4096$ **17.** $5^{-6} = \frac{1}{5^6} = \frac{1}{15{,}625}$

18. $(-9)^{3+1} = (-9)^4 = 6561$ **19.** $8^6 = 262{,}144$

20. $5^{2-5} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ **21.** $\left(\frac{3}{7}\right)^3 = \frac{3^3}{7^3} = \frac{27}{343}$

22. $\left(\frac{5}{9}\right)^{-3} = \frac{5^{-3}}{9^{-3}} = \frac{9^3}{5^3} = \frac{729}{125}$ **23.** $11^{-2+0} = \frac{1}{11^2} = \frac{1}{121}$

24. $4^{-2-(-3)} = 4^1 = 4$ **25.** $\left(\frac{1}{8}\right)^{-4} = \frac{1^{-4}}{8^{-4}} = 8^4 = 4096$

26. $2^8 = 256$ **27.** $\frac{2^2}{2^{-9}} = 2^2 \cdot 2^9 = 2^{9+2} = 2^{11} = 2048$

28. $\frac{6^2}{6^4 \cdot 5^{-2}} = 6^{2-4} \cdot 5^2 = 6^{-2} \cdot 5^2 = \frac{5^2}{6^2} = \frac{25}{36}$

29. $6^{0+3} \cdot \frac{1}{6^4} = 6^{3-4} = 6^{-1} = \frac{1}{6}$ **30.** $\frac{1}{10^3} \cdot 10^3 = 1$

31. $\left(\frac{2}{5}\right)^{-6} = \frac{2^{-6}}{5^{-6}} = \frac{5^6}{2^6} = \frac{15{,}625}{64}$ **32.** $x^{8-3} = x^5$

33. $2^{15}x^{10} = 32{,}768x^{10}$ **34.** $\frac{1}{x^2y^2}$ **35.** $x^5 \cdot x^2 = x^{5+2} = x^7$

36. $x^{5-4}y^{2-0} = xy^2$ **37.** $\frac{1}{(x^4y^7)^3} = \frac{1}{x^{12}y^{21}}$

38. $x^{11} \cdot x^3 \cdot y^{10} \cdot y^1 = x^{11+3}y^{10+1} = x^{14}y^{11}$

39. $-3\frac{y^0}{x^4} = \frac{-3}{x^4}$ **40.** $\frac{1}{(10x^3y^5)^3} = \frac{1}{1000x^9y^{15}}$

41. $\frac{y \cdot y^2}{x \cdot x} = \frac{y^{1+2}}{x^{1+1}}$ $\frac{y^3}{x^2}$ **42.** $\frac{1}{(4x^2y^5)^2} = \frac{1}{16x^4y^{10}}$

43. $\frac{2x^2y \cdot y}{6x} = \frac{1}{3}x^{2-1}y^{1+1} = \frac{1}{3}xy^2$

44. $\frac{5}{20}x^{3-2}y^{9+2} = \frac{1}{4}xy^{11}$

45. $\frac{xy^9}{3y^{-2}} \cdot \frac{-7y}{21x^5} = \frac{-1}{9}\frac{xy^{9+1}}{x^5y^{-2}} = \frac{-1}{9}x^{1-5}y^{10+2}$

$= \frac{-1}{9}x^{-4}y^{12} = -\frac{y^{12}}{9x^4}$

46. $\frac{y^{10}}{2x^3} \cdot \frac{20x^{14}}{xy^6} = \frac{10x^{14}y^{10}}{x^{3+1}y^6} = \frac{10x^{14}y^{10}}{x^4y^6}$

$= 10x^{14-4}y^{10-6} = 10x^{10}y^4$

47. $\frac{12xy}{7x^4} \cdot \frac{7x^5y^2}{4y} = \frac{3x^{5+1}y^{1+2}}{x^4 \cdot y} = 3x^{6-4}y^{3-1} = 3x^2y^2$

48. $A = \frac{\sqrt{3}}{4}\left(\frac{x}{2}\right)^2 = \frac{\sqrt{3} \cdot x^2}{4 \cdot 4} = \frac{\sqrt{3}}{16}x^2$

49. $A = (4x)^2\pi = 16x^2\pi$ **50.** $V = \pi(2x)^2x = 4x^3\pi$

51. $V = \frac{4}{3}\pi\left(\frac{x}{3}\right)^3 = \frac{4}{3}\pi\frac{x^3}{27} = \frac{4}{81}\pi x^3$

52. $\frac{\text{National debt}}{\text{population}} = \frac{\$5.608 \times 10^{12}}{2.73 \times 10^8} = \frac{\$5.608 \times 10^4}{2.73}$

$= \$2.054 \times 10^4$

53. France $= \frac{\$1.2496 \times 10^{12}}{5.8607 \times 10^7} = \$2.13 \times 10^4$

Germany $= \frac{\$1.8393 \times 10^{12}}{8.2061 \times 10^7} = \$2.24 \times 10^4$

Ireland $= \frac{\$7.13 \times 10^{10}}{3.661 \times 10^6} = \$1.95 \times 10^4$

Luxembourg $= \frac{\$1.36 \times 10^{10}}{4.2 \times 10^5} = \$3.24 \times 10^4$

The Netherlands $= \frac{\$3.334 \times 10^{11}}{1.56 \times 10^7} = \$2.14 \times 10^4$

Sweden $= \frac{\$1.773 \times 10^{11}}{8.849 \times 10^6} = 2.00 \times 10^4$

54. $\frac{4.56 \times 10^{-2}\text{ cm}}{7.5 \times 10^{-4}\text{ cm}} = 6.08 \times 10^1$ cm

55. $\frac{1.04 \times 10^{10}\text{ km}}{1.39 \times 10^6\text{ km/day}} = 7.48 \times 10^3$ days

56. $(1.2 \times 10^7\text{ birds/species})(8.6 \times 10^3\text{ species})$

$= 1.03 \times 10^{11}$ birds

57. a.; b.

State	*Total Area (acres)*	*Amount of park space*	*Park space/ Total area*
Alaska	3.937472×10^8	3.25×10^6	8.25×10^{-3}
California	1.01676×10^8	1.345×10^6	1.32×10^{-2}
Connecticut	3.548×10^6	1.76×10^5	4.96×10^{-2}
Kansas	5.26×10^7	2.9×10^4	5.51×10^{-4}
Ohio	2.869×10^7	2.04×10^5	7.11×10^{-3}
Pennsylvania	2.9477×10^7	2.83×10^5	9.60×10^{-3}

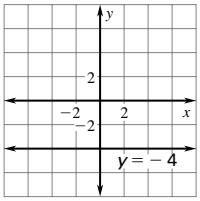
c. A good answer should include the percent of area in the state that is now park land, it should also include comparisons with the percents in other states.

58. $\frac{a^0}{a^m} = a^{0-m} = a^{-m}$

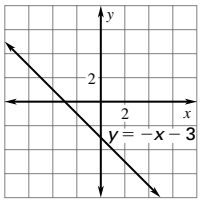
59. $a^m \cdot a^{-n} = a^{m+(-n)} = a^{m-n} = \frac{a^m}{a^n}$

6.1 Mixed Review (p. 328)

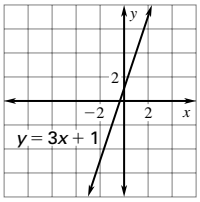
60. $y = -4$



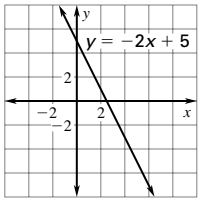
61. $y = -x - 3$



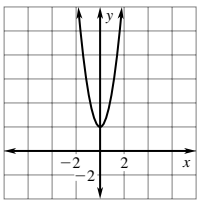
62. $y = 3x + 1$



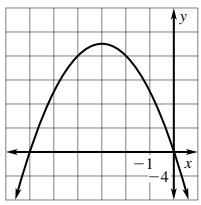
63. $y = -2x + 5$



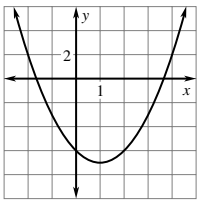
64. $y = 3x^2 + 2$



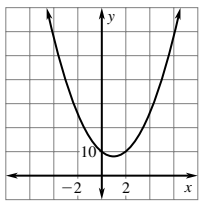
65. $y = -2x(x + 6)$



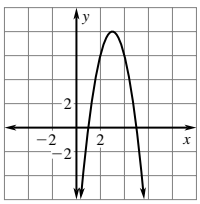
66. $y = x^2 - 2x - 6$



67. $y = 2x^2 - 4x + 10$



68. $y = -2(x - 3)^2 + 8$



69. $2x^2 = 32$

$x^2 = 16$

$x = \pm 4$

70. $-3x^2 = -24$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

71. $25x^2 = 16$

$x^2 = \frac{16}{25}$

$x = \pm\frac{4}{5}$

72. $3x^2 = 108$

$x^2 = 36$

$x = \pm 6$

73. $5x^2 = 5$

$x^2 = 1$

$x = \pm 1$

74. $4x^2 = 14$

$x^2 = \frac{14}{4}$

$x = \pm\frac{\sqrt{14}}{2}$

75. $3x^2 = 15$

$x^2 = 5$

$x = \pm\sqrt{5}$

76. $3x^2 = 20$

$x^2 = \frac{20}{3}$

$x = \pm\frac{2\sqrt{15}}{3}$

77. $3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

78. $18 + 3i$ **79.** $-3 + 4i$ **80.** $6 - 8i$

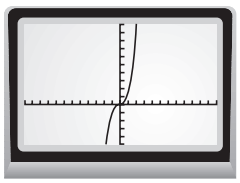
81. $-7i + 2 = 2 - 7i$ **82.** $11 - 55i$

83. $(27 - 1) + (9i + 3i) = 26 + 12i$

Lesson 6.2

Activity (p. 331)

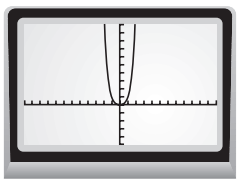
1. a. $y = x^3$



$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

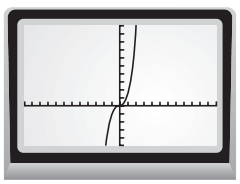
b. $y = x^4$



$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

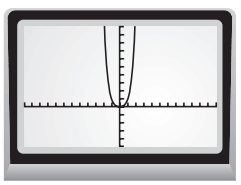
c. $y = x^5$



$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

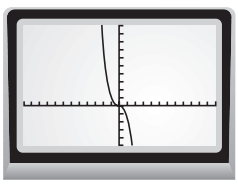
d. $y = x^6$



$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

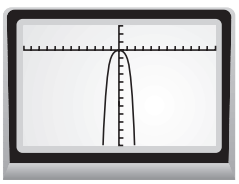
e. $y = -x^3$



$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

f. $y = -x^4$



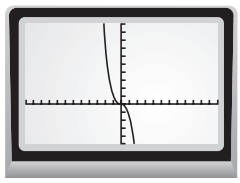
$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

—CONTINUED—

1. —CONTINUED—

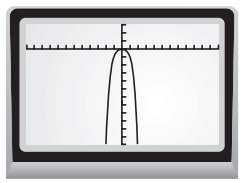
g. $y = -x^5$



$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

h. $y = -x^6$



$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

2. If the leading coefficient is positive, the values of the function approach $+\infty$; if the leading coefficient is negative, the values of the function approach $-\infty$.

3. When the function's degree is odd, the ends will go in opposite directions. When the function's degree is even, the ends will go in the same direction.

6.2 Guided Practice (p. 333)

1. 3, cubic, $-2, 0$ **2.** -2 | 3 1 -9 2 / -6 10 -2 / 3 -5 1 0

3. horizontal line

4. -1 | $\sqrt{5}$ 0 0 0 0 / $-\sqrt{5}$ $\sqrt{5}$ $-\sqrt{5}$ $\sqrt{5}$ / $\sqrt{5}$ $-\sqrt{5}$ $\sqrt{5}$ $-\sqrt{5}$ $\sqrt{5}$

5. no **6.** no

yes; $\sqrt{5} + 1$

7. -1 | 5 0 -21 0 14 / -5 5 16 -16 / 5 -5 -16 16 -2

yes; -2

8. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

9. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

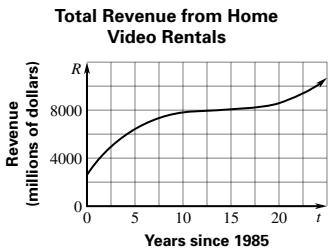
10. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

11. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

12. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) - \infty$ as $x \to +\infty$

13. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

14.



6.2 Practice and Applications (pp. 333–336)

15. yes; $f(x) = -5x + 12$; 1; linear; -5

16. yes; $f(x) = \frac{3}{5}x^4 + 2x + 9$; 4; quartic; $\frac{3}{5}$

17. yes; $f(x) = x + \pi$; 1; linear; 1

18. yes; $f(x) = x^2\sqrt{2} + x - 5$; 2; quadratic; $\sqrt{2}$

19. no

20. yes; $f(x) = -2$; 0; constant; -2

21. yes; $f(x) = x^2 - x + 1$; 2; quadratic; 1

22. no

23. yes; $f(x) = x^4 - x^3 + 36x^2$; 4; quartic; 1

24. no

25. yes; $f(x) = 3x^3$; 3; cubic; 3

26. no

27. $f(-2) = 2(-2)^3 + 5(-2)^2 + 4(-2) + 8$
$= -16 + 20 - 8 + 8$
$= 4$

28. $f(3) = 2(3)^3 - (3)^4 + 5(3)^2 - 3$
$= 54 - 81 + 45 - 3$
$= 15$

29. $f(4) = 4 + \frac{1}{2}(4)^3$
$= 4 + 32$
$= 36$

30. $f(-1) = (-1)^2 - (-1)^5 + 1$
$= 1 + 1 + 1$
$= 3$

31. $f(1) = 5(1)^4 - 8(1)^3 + 7(1)^2$
$= 5 - 8 + 7$
$= 4$

32. $f(-3) = (-3)^3 + 3(-3)^2 - 2(-3) + 5$
$= -27 + 27 + 6 + 5$
$= 11$

33. $f(0) = 11(0)^3 - 6(0)^2 + 2$
$= 0 + 0 + 2$
$= 2$

34. $f(2) = (2)^4 - 2(2) + 7$
$= 16 - 4 + 7$
$= 19$

35. $f(10) = 7(10)^3 + 9(10)^2 + 3(10)$
$= 7000 + 900 + 30$
$= 7930$

36. $f(-2) = -(-2)^5 - 4(-2)^3 + 6(-2)^2 - (-2)$
$= 32 + 32 + 24 + 2$
$= 90$

37. $2 \mid 5 \quad 4 \quad 8 \quad 1$
$\quad\quad\quad\ \ 10 \quad 28 \quad 72$
$\quad\quad 5 \quad 14 \quad 36 \quad 73$

38. $3 \mid -3 \quad 7 \quad -4 \quad 8$
$\quad\quad\quad\ \ -9 \quad -6 \quad -30$
$\quad\quad -3 \quad -2 \quad -10 \quad -22$

39. $-5 \mid 1 \quad 3 \quad 6 \quad -11$
$\quad\quad\quad\ \ -5 \quad 10 \quad -80$
$\quad\quad 1 \quad -2 \quad 16 \quad -91$

40. $-1 \mid 1 \quad -1 \quad 12 \quad 15$
$\quad\quad\quad\ \ -1 \quad 2 \quad -14$
$\quad\quad 1 \quad -2 \quad 14 \quad 1$

41. $2 \mid -4 \quad 0 \quad 3 \quad -5$
$\quad\quad\quad\ \ -8 \quad -16 \quad -26$
$\quad\quad -4 \quad -8 \quad -13 \quad -31$

42. $-3 \mid -1 \quad 1 \quad 0 \quad -1 \quad 1$
$\quad\quad\quad\ \ 3 \quad -12 \quad 36 \quad -105$
$\quad\quad -1 \quad 4 \quad -12 \quad 35 \quad -104$

43. $-1 \mid 2 \quad 1 \quad -3 \quad 5 \quad 0$
$\quad\quad\quad\ \ -2 \quad 1 \quad 2 \quad -7$
$\quad\quad 2 \quad -1 \quad -2 \quad 7 \quad -7$

44. $2 \mid 3 \quad 0 \quad 0 \quad -2 \quad 1 \quad 0$
$\quad\quad\quad\ \ 6 \quad 12 \quad 24 \quad 44 \quad 90$
$\quad\quad 3 \quad 6 \quad 12 \quad 22 \quad 45 \quad 90$

45. $5 \mid 2 \quad -1 \quad 6 \quad 0$
$\quad\quad\quad\ \ 10 \quad 45 \quad 255$
$\quad\quad 2 \quad 9 \quad 51 \quad 255$

46. $-2 \mid -1 \quad 8 \quad 0 \quad 13 \quad -4$
$\quad\quad\quad\ \ 2 \quad -20 \quad 40 \quad -106$
$\quad\quad -1 \quad 10 \quad -20 \quad 53 \quad -110$

47.

Function	as $x \to -\infty$	as $x \to +\infty$
$f(x) = -5x^3$	$f(x) \to +\infty$	$f(x) \to -\infty$
$f(x) = -x^3 + 1$	$f(x) \to +\infty$	$f(x) \to -\infty$
$f(x) = 2x - 3x^3$	$f(x) \to +\infty$	$f(x) \to -\infty$
$f(x) = 2x^2 - x^3$	$f(x) \to +\infty$	$f(x) \to -\infty$

48.

Function	as $x \to -\infty$	as $x \to +\infty$
$f(x) = x^4 + 3x^3$	$f(x) \to +\infty$	$f(x) \to +\infty$
$f(x) = x^4 + 2$	$f(x) \to +\infty$	$f(x) \to +\infty$
$f(x) = x^4 - 2x - 1$	$f(x) \to +\infty$	$f(x) \to +\infty$
$f(x) = 3x^4 - 5x^2$	$f(x) \to +\infty$	$f(x) \to +\infty$

49. C **50.** D **51.** B **52.** A

53. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

54. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

55. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

56. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

57. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

58. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

59. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

60. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

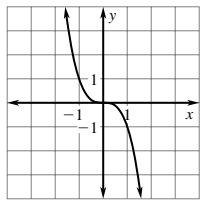
61. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

62. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

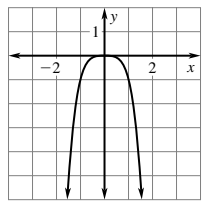
63. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$

64. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$

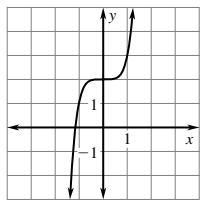
65. $f(x) = -x^3$



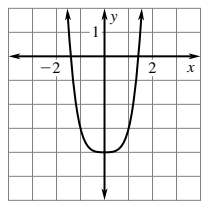
66. $f(x) = -x^4$



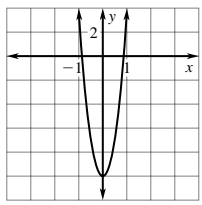
67. $f(x) = x^5 + 2$



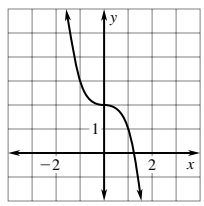
68. $f(x) = x^4 - 4$



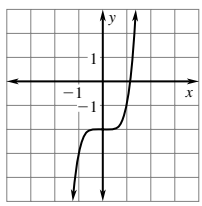
69. $f(x) = x^4 + 6x^2 - 5$



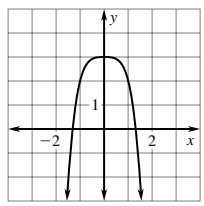
70. $f(x) = 2 - x^3$



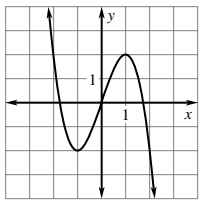
71. $f(x) = x^5 - 2$



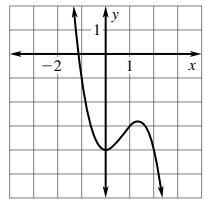
72. $f(x) = -x^4 + 3$



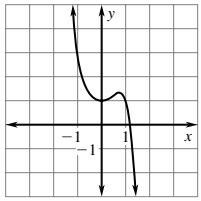
73. $f(x) = -x^3 + 3x$



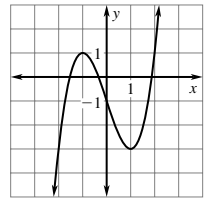
74. $f(x) = -x^3 + 2x^2 - 4$



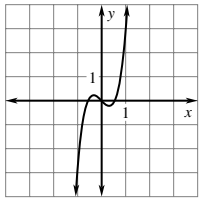
75. $f(x) = -x^5 + x^2 + 1$



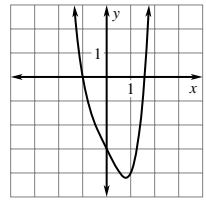
76. $f(x) = x^3 - 3x - 1$



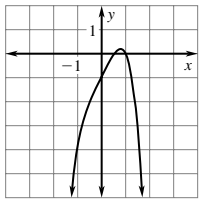
77. $f(x) = x^5 + 3x^3 - x$



78. $f(x) = x^4 - 2x - 3$



79. $f(x) = -x^4 + 2x - 1$



80. *Sample answer:* Any polynomial function of odd degree that has a positive leading coefficient will work; $f(x) = 4x^3$

81. $S = -0.0068(18)^3 - 0.27(18)^2 + 150(18) + 1700$

$S = 4272.9$

about 4272.9 million ft^2

82. $R = -0.0036(3)^3 + 0.13(3)^2 - 0.073(3) + 7.7$

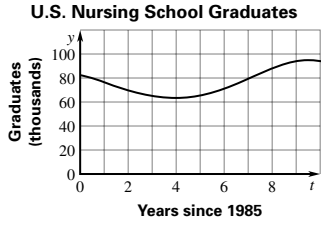
$R = 8.55$

about \$8.55

83. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to -\infty$ as $x \to +\infty$; less; the graph will go down over time

84. $y = -0.036t^4 + 0.605t^3 - 1.87t^2 - 4.67t + 82.5$

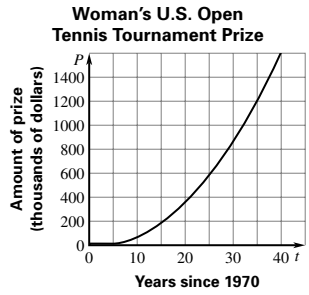
1992



85. $f(x) \to +\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$; more; the graph will go up over time

86. $P = 1.141t^2 - 5.837t + 14.31$

about \$1,208,000



87. a. $L = 0.0007(18)^3 - 0.061(18)^2 + 2.02(18) + 30$

$L = 50.7$

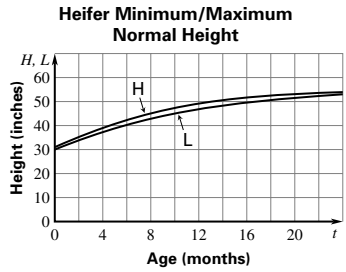
$H = 0.001(18)^3 - 0.08(18)^2 + 2.3(18) + 31$

$H = 52.3$

Normal range would be 50.7 in. to 52.3 in.

b. $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to +\infty$ as $x \to +\infty$; more; the graph will go up over time

c.



d. *Sample answer:* The calf is probably around 7 months old. I got this by using the graph in part c. I found the height and looked down to find how old the calf was.

88.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
50	125,000	120,205	1.03989
100	1,000,000	980,405	1.01999
500	1.25×10^8	124,502,005	1.004
1000	10^9	998,004,005	1.002
5000	1.25×10^{11}	1.2495×10^{11}	1.0004

89. 1; Eventually the combined values of the terms after the leading term will be negligible compared to the value of the leading terms.

6.2 Mixed Review (p. 336)

90. $-2x + 5$ **91.** $7x$ **92.** $-4x^2 - 1$ **93.** $x^2 + 4x - 11$

94. $-3x^2 + 4x - 1$ **95.** $-x^2 - x + 2$

96. $y = -4x^2 + 16x - 11$ **97.** $y = -2x^2 - 2x + 60$

98. $y = 2x^2 - 6x - 56$ **99.** $y = 4x^2 - 24x + 12$

100. $y = -x^2 - 10x - 13$ **101.** $y = -3x^2 + 30x - 72$

102. $x = \pm 3i$ **103.** $x = \pm i\sqrt{5}$ **104.** $x = \pm i\sqrt{2}$

105. $x = \pm i\sqrt{3}$ **106.** $x = \pm i$ **107.** $x = \pm i\frac{\sqrt{6}}{6}$

108. $x = \pm i\sqrt{5}$ **109.** $x = \pm i\frac{\sqrt{10}}{2}$ **110.** $x = \pm i\frac{\sqrt{7}}{7}$

Developing Concepts Activity 6.2 (p. 337)

1. $-10 \le x \le 10, -10 \le y \le 100$

2. $-10 \le x \le 30, 0 \le y \le 3000$

3. $-5 \le x \le 5, -5 \le y \le 10$

4. $-5 \le x \le 5, -5 \le y \le 30$

5. $-5 \le x \le 5, 0 \le y \le 20$

6. $0 \le x \le 5, -5 \le y \le 5$

7. $0 \le x \le 15, 0 \le y \le 300{,}000$

Lesson 6.3

6.3 Guided Practice (p. 341)

1. like terms

2. The negative sign was not distributed over all of the second polynomial.

3. 6 **4.** $7x^2 + 11$ **5.** $2x^3 - 5x^2 - 3x + 6$

6. $(x^2 + 7x - 5) - (3x^2 + 1) = -2x^2 + 7x - 6$

7. $(x^2 + 1) - (3x^2 - 4x + 3) = x^2 + 1 - 3x^2 + 4x - 3$
$= -2x^2 + 4x - 2$

8. $(x + 2)(2x^2 + 3) = 2x^3 + 3x + 4x^2 + 6$
$= 2x^3 + 4x^2 + 3x + 6$

9. $(x^2 + 3x + 10)(4x^2 - 2x - 7)$
$= x^2(4x^2 - 2x - 7) + 3x(4x^2 - 2x - 7)$
$+ 10(4x^2 - 2x - 7)$
$= 4x^4 - 2x^3 - 7x^2 + 12x^3 - 6x^2 - 21x$
$+ 40x^2 - 20x - 70$
$= 4x^4 + 10x^3 + 27x^2 - 41x - 70$

10. $(x - 1)(2x + 1)(x + 5)$
$= (2x^2 - x - 1)(x + 5)$
$= x(2x^2 - x - 1) + 5(2x^2 - x - 1)$
$= 2x^3 - x^2 - x + 10x^2 - 5x - 5$
$= 2x^3 + 9x^2 - 6x - 5$

11. $(-3x + 1)(-3x + 1)(-3x + 1)$
$= (9x^2 - 6x + 1)(-3x + 1)$
$= -3x(9x^2 - 6x + 1) + (9x^2 - 6x + 1)$
$= -27x^3 + 18x^2 - 3x + 9x^2 - 6x + 1$
$= -27x^3 + 27x^2 - 9x + 1$

12. $V = (x + 3)(x - 3)(x - 2)$
$= (x^2 - 9)(x - 2)$
$= x(x^2 - 9) - 2(x^2 - 9)$
$= x^3 - 9x - 2x^2 + 18$
$= x^3 - 2x^2 - 9x + 18$

6.3 Practice and Applications (pp. 341–343)

13. $(8x^2 + 1) + (3x^2 - 2) = 11x^2 - 1$

14. $3x^3 + 10x + 5 - x^3 + 4x - 6 = 2x^3 + 14x - 1$

15. $x^2 - 6x + 5 - x^2 - x + 2 = -7x + 7$

16. $16 - 13x + 10x - 11 = -3x + 5$

17. $7x^3 - 1 - 15x^3 - 4x^2 + x - 3 = -8x^3 - 4x^2 + x - 4$

18. $8x + 14x + 3 - 41x^2 + x^3 = x^3 - 41x^2 + 22x + 3$

19. $4x^2 - 11x + 10 + 5x - 31 = 4x^2 - 6x - 21$

20. $9x^3 - 4 + x^2 + 8x - 7x^3 + 3x - 7$
$= 2x^3 + x^2 + 11x - 11$

21. $-3x^3 + x - 11 - 4x^3 - x^2 + x$
$= -7x^3 - x^2 + 2x - 11$

22. $6x^2 - 19x + 5 - 19x^2 + 4x - 9 = -13x^2 - 15x - 4$

23. $10x^3 - 4x^2 + 3x - x^3 + x^2 - 1 = 9x^3 - 3x^2 + 3x - 1$

24. $50x - 3 + 8x^3 + 7x^2 + x + 4 = 8x^3 + 7x^2 + 51x + 1$

25. $10x - 3 + 7x^2 + x^3 - 2x + 17 = x^3 + 7x^2 + 8x + 14$

26. $3x^3 - 5x^4 - 10x + 1 + 17x^4 - x^3 = 12x^4 + 2x^3 - 10x + 1$

27. $x(x^2 + 6x - 7) = x^3 + 6x^2 - 7x$

28. $10x^2(x - 5) = 10x^3 - 50x^2$

29. $-4x(x^2 - 8x + 3) = -4x^3 + 32x^2 - 12x$

30. $5x(3x^2 - x + 3) = 15x^3 - 5x^2 + 15x$

31. $(x - 4)(x - 7) = x^2 - 11x + 28$

32. $(x + 9)(x - 2) = x^2 + 7x - 18$

33. $(x + 3)(x^2 - 4x + 9)$
$= x(x^2 - 4x + 9) + 3(x^2 - 4x + 9)$
$= x^3 - 4x^2 + 9x + 3x^2 - 12x + 27$
$= x^3 - x^2 - 3x + 27$

34. $(x + 8)(x^2 - 7x - 3)$
$= x(x^2 - 7x - 3) + 8(x^2 - 7x - 3)$
$= x^3 - 7x^2 - 3x + 8x^2 - 56x - 24$
$= x^3 + x^2 - 59x - 24$

35. $(2x + 5)(3x^3 - x^2 + x)$

$= 2x(3x^3 - x^2 + x) + 5(3x^3 - x^2 + x)$

$= 6x^4 - 2x^3 + 2x^2 + 15x^3 - 5x^2 + 5x$

$= 6x^4 + 13x^3 - 3x^2 + 5x$

36. $(6x + 2)(2x^2 - 6x + 1)$

$= 6x(2x^2 - 6x + 1) + 2(2x^2 - 6x + 1)$

$= 12x^3 - 36x^2 + 6x + 4x^2 - 12x + 2$

$= 12x^3 - 32x^2 - 6x + 2$

37. $(x + 11)(x^2 - 5x + 9)$

$= x(x^2 - 5x + 9) + 11(x^2 - 5x + 9)$

$= x^3 - 5x^2 + 9x + 11x^2 - 55x + 99$

$= x^3 + 6x^2 - 46x + 99$

38. $(4x^2 - 1)(x^2 - 6x + 9)$

$= 4x^2(x^2 - 6x + 9) - 1(x^2 - 6x + 9)$

$= 4x^4 - 24x^3 + 36x^2 - x^2 + 6x - 9$

$= 4x^4 - 24x^3 + 35x^2 + 6x - 9$

39. $(x - 1)(x^3 + 2x^2 + 2)$

$= x(x^3 + 2x^2 + 2) - (x^3 + 2x^2 + 2)$

$= x^4 + 2x^3 + 2x - x^3 - 2x^2 - 2$

$= x^4 + x^3 - 2x^2 + 2x - 2$

40. $(x + 1)(5x^3 - x^2 + x - 4)$

$= x(5x^3 - x^2 + x - 4) + (5x^3 - x^2 + x - 4)$

$= 5x^4 - x^3 + x^2 - 4x + 5x^3 - x^2 + x - 4$

$= 5x^4 + 4x^3 - 3x - 4$

41. $(3x^2 - 2)(x^2 + 4x + 3)$

$= 3x^2(x^2 + 4x + 3) - 2(x^2 + 4x + 3)$

$= 3x^4 + 12x^3 + 9x^2 - 2x^2 - 8x - 6$

$= 3x^4 + 12x^3 + 7x^2 - 8x - 6$

42. $(-x^3 - 2)(x^2 + 3x - 3)$

$= -x^3(x^2 + 3x - 3) - 2(x^2 + 3x - 3)$

$= -x^5 - 3x^4 + 3x^3 - 2x^2 - 6x + 6$

43. $(x^2 + x + 4)(2x^2 - x + 1)$

$= x^2(2x^2 - x + 1) + x(2x^2 - x + 1) + 4(2x^2 - x + 1)$

$= 2x^4 - x^3 + x^2 + 2x^3 - x^2 + x + 8x^2 - 4x + 4$

$= 2x^4 + x^3 + 8x^2 - 3x + 4$

44. $(x^2 - x - 3)(x^2 + 4x + 2)$

$= x^2(x^2 + 4x + 2) - x(x^2 + 4x + 2) - 3(x^2 + 4x + 2)$

$= x^4 + 4x^3 + 2x^2 - x^3 - 4x^2 - 2x - 3x^2 - 12x - 6$

$= x^4 + 3x^3 - 5x^2 - 14x - 6$

45. $(x + 9)(x - 2)(x - 7)$

$= (x^2 + 7x - 18)(x - 7)$

$= x(x^2 + 7x - 18) - 7(x^2 + 7x - 18)$

$= x^3 + 7x^2 - 18x - 7x^2 - 49x + 126$

$= x^3 - 67x + 126$

46. $(x + 3)(x - 4)(x - 5)$

$= (x^2 - x - 12)(x - 5)$

$= x(x^2 - x - 12) - 5(x^2 - x - 12)$

$= x^3 - x^2 - 12x - 5x^2 + 5x + 60$

$= x^3 - 6x^2 - 7x + 60$

47. $(x + 5)(x + 7)(-x + 1)$

$= (x^2 + 12x + 35)(-x + 1)$

$= -x(x^2 + 12x + 35) + (x^2 + 12x + 35)$

$= -x^3 - 12x^2 - 35x + x^2 + 12x + 35$

$= -x^3 - 11x^2 - 23x + 35$

48. $(2x - 3)(x^2 + 13x + 42)$

$= 2x(x^2 + 13x + 42) - 3(x^2 + 13x + 42)$

$= 2x^3 + 26x^2 + 84x - 3x^2 - 39x - 126$

$= 2x^3 + 23x^2 + 45x - 126$

49. $(x - 9)(x - 2)(3x + 2)$

$= (x^2 - 11x + 18)(3x + 2)$

$= 3x(x^2 - 11x + 18) + 2(x^2 - 11x + 18)$

$= 3x^3 - 33x^2 + 54x + 2x^2 - 22x + 36$

$= 3x^3 - 31x^2 + 32x + 36$

50. $(x - 1)(x - 8)(-2x - 5)$

$= (x^2 - 9x + 8)(-2x - 5)$

$= -2x(x^2 - 9x + 8) - 5(x^2 - 9x + 8)$

$= -2x^3 + 18x^2 - 16x - 5x^2 + 45x - 40$

$= -2x^3 + 13x^2 + 29x - 40$

51. $(2x + 1)(3x + 1)(x + 4)$

$= (2x + 1)(3x^2 + 13x + 4)$

$= 2x(3x^2 + 13x + 4) + (3x^2 + 13x + 4)$

$= 6x^3 + 26x^2 + 8x + 3x^2 + 13x + 4$

$= 6x^3 + 29x^2 + 21x + 4$

52. $(4x - 1)(2x - 1)(3x - 2)$

$= (8x^2 - 6x + 1)(3x - 2)$

$= 3x(8x^2 - 6x + 1) - 2(8x^2 - 6x + 1)$

$= 24x^3 - 18x^2 + 3x - 16x^2 + 12x - 2$

$= 24x^3 - 34x^2 + 15x - 2$

53. $(x + 7)(x - 7) = x^2 + 7x - 7x - 49 = x^2 - 49$

54. $(x + 4)(x + 4) = x^2 + 4x + 4x + 16 = x^2 + 8x + 16$

55. $(4x - 3)(4x - 3)(4x - 3)$

$= (16x^2 - 24 + 9)(4x - 3)$

$= 4x(16x^2 - 24x + 9) - 3(16x^2 - 24x + 9)$

$= 64x^3 - 96x^2 + 36x - 48x^2 + 72x - 27$

$= 64x^3 - 144x^2 + 108x - 27$

56. $(10x + 3)(10x - 3) = 100x^2 - 30x + 30x - 9$

$= 100x^2 - 9$

57. $(6 - x^2)(6 - x^2) = 36 - 6x^2 - 6x^2 + x^4$

$= 36 - 12x^2 + x^4$

58. $(2y + 5x)(2y + 5x) = 4y^2 + 10xy + 10xy + 25x^2$

$= 4y^2 + 20xy + 25x^2$

59. $(3x + 7)(3x + 7)(3x + 7)$

$= (3x + 7)(9x^2 + 42x + 49)$

$= 3x(9x^2 + 42x + 49) + 7(9x^2 + 42x + 49)$

$= 27x^3 + 126x^2 + 147x + 63x^2 + 294x + 343$

$= 27x^3 + 189x^2 + 441x + 343$

60. $(7y - x)(7y - x) = 49y^2 - 7xy - 7xy + x^2$

$= 49y^2 - 14xy + x^2$

61. $(2x + 3y)(2x + 3y)(2x + 3y)$

$= (2x + 3y)(4x^2 + 12xy + 9y^2)$

$= 2x(4x^2 + 12xy + 9y^2) + 3y(4x^2 + 12xy + 9y^2)$

$= 8x^3 + 24x^2y + 18xy^2 + 12x^2y + 36xy^2 + 27y^3$

$= 8x^3 + 36x^2y + 54xy^2 + 27y^3$

62. $V = \pi(x - 2)^2(x + 3)$

$= \pi[(x^2 - 4x + 4)(x + 3)]$

$= \pi[x(x^2 - 4x + 4) + 3(x^2 - 4x + 4)]$

$= \pi[x^3 - 4x^2 + 4x + 3x^2 - 12x + 12]$

$= \pi[x^3 - x^2 - 8x + 12]$

$= \pi x^3 - \pi x^2 - 8\pi x + 12\pi$

63. $V = (2x + 3)(x)(x + 1)$

$= (2x^2 + 3x)(x + 1)$

$= x(2x^2 + 3x) + (2x^2 + 3x)$

$= 2x^3 + 3x^2 + 2x^2 + 3x$

$= 2x^3 + 5x^2 + 3x$

64. $C = -1.63t^4 + 49.5t^3 - 476t^2 + 1370t + 6705$

$T = -1.052t^4 + 31.6t^3 - 296t^2 + 1097t + 2290$

$V = -2.682t^4 + 81.1t^3 - 772t^2 + 2467t + 8995$

$= -2.682(7)^4 + 81.1(7)^3 - 772(7)^2 + 2467(7) + 8995$

≈ 9814

about 9,813,818 total vehicles

65. $P - S = Y$

$P = -0.804t^4 + 26.9t^3 - 262t^2 + 3010t + 227{,}000$

$S = 0.0206t^4 - 0.67t^3 + 6.42t^2 + 213t + 7740$

$Y = -0.8246t^4 + 27.57t^3 - 268.42t^2 + 2797t + 219{,}260$

$= -0.8246(15)^4 + 27.57(15)^3 - 268.42(15)^2$

$+ 2797(15) + 219.260$

≈ 252

about 252 million people

66. $P = 0.00267s(0.0116s^2 + 0.789)$

$= 0.000030972s^3 + .00210663s$

$= 0.000030972(10)^3 + .00210663(10)$

$= 0.030972 + 0.0210663$

$= 0.0520383$

about 0.05 horsepower

67. $P \times D = W \times 100$

$= (0.43t + 49)(-0.096t^4 + 3t^3 - 27t^2 + 91t + 1700)$

$= 0.04128t^5 + 1.29t^4 - 11.61t^3 + 39.13t^2 + 731$

$- 4.704t^4 + 147t^3 - 1323t^2 + 4459t + 83300$

$= -0.04128t^5 - 3.414t^4 + 135.39t^3 - 1283.87t^2$

$+ 5190t + 83300$

$W = -0.0004128t^5 - 0.03414t^4 + 1.3539t^3 - 12.8387t^2$

$+ 51.9t + 833$

$t = 11$

$W = -0.0004128(11)^5 - 0.03414(11)^4 + 1.3539(11)^3$

$- 12.8387(11)^2 + 51.9(11) + 833$

$\approx 1{,}086$

about 1,086,000 degrees

68. $R = N \times P$

$R = 0.67t(-0.27t^3 + 3.9t^2 + 7.9t + 650)$

$+ 9.4(-0.27t^3 + 3.9t^2 + 7.9t + 650)$

$R = -0.1809t^4 + 2.613t^3 + 5.293t^2 + 435.5t$

$- 2.538t^3 + 36.66t^2 + 74.26t + 6110$

$R = -0.1809t^4 + 0.075t^3 + 41.953t^2 + 509.76t + 6110$

$t = 6$

$R = -0.1809(6)^4 + 0.075(6)^3 + 41.953(6)^2$

$+ 509.76(6) + 6110$

$R \approx 10{,}461$

about $10,461 million

69. $I = 4000(1 + r)^3 + 5000(1 + r)^2 + 7000(1 + r)$
$= 4000(1 + 3r + 3r^2 + r^3) + 5000(1 + 2r + r^2) + 7000(1 + r)$
$= 1(16{,}000) + r(29{,}000) + r^2(17{,}000) + r^3(4000)$
$m = 6000(1 + 3r + 3r^2 + r^3) + 800(1 + 2r + r^2) + 9000(1 + r)$
$= (23{,}000) + r(43{,}000) + r^2(26{,}000) + r^3(6{,}000)$
$T + M = 39{,}000 + 72{,}000r + 43{,}000r^2 + 10{,}000r^3$

70. $2x^4 + 5x^3 - 8x^2 - x + 10 + 8x^4 - 4x^3 + x^2 - x + 2$
$= 10x^4 + x^3 - 7x^2 - 2x + 12$
C

71. $(3x - 8)(3x - 8)(3x - 8)$
$= (9x^2 - 48x + 64)(3x - 8)$
$= 3x(9x^2 - 48 + 64) - 8(9x^2 - 48x + 64)$
$= 27x^3 - 144x^2 + 192x - 72x^2 + 384 - 512$
$= 27x^3 - 216x^2 + 576x - 512$
A

72. a. $(x^5 - 1) = (x - 1)(x^4 + x^3 + x^2 + x + 1)$
$(x^6 - 1) = (x - 1)(x^5 + x^4 + x^3 + x^2 + x + 1)$

b. $x^n - 1 = (x - 1)(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + x + 1)$

Multiply:
$x(x^{n-1}) - (x^{n-1}) + (x)(x^{n-2}) - (x^{n-2}) + \cdots - 1$

Pairs of middle terms will cancel out.

6.3 Mixed Review (p. 344)

73. $4x^2 = 36$
$x^2 = 9$
$x = \pm 3$

74. $(x + 8)(x - 5) = 0$
$x = -8$ or $x = 5$

75. $x^2 + 16x + 64 = 0$
$(x + 8)(x + 8) = 0$
$x = -8$

76. $x^2 - x - 56 = 0$
$(x - 8)(x + 7) = 0$
$x = 8$ or $x = -7$

77. $(2x + 3)(x - 5) = 0$
$x = -\frac{3}{2}$ or $x = 5$

78. $2(3x^2 + 5x - 2) = 0$
$2(3x - 1)(x + 2) = 0$
$x = \frac{1}{3}$ or $x = -2$

79. $16a - 4b + c = 0$
$4a + 2b + c = 0$
$a + b + c = 6$
$y = -\frac{6}{5}x^2 - \frac{12}{5}x + \frac{48}{5}$

80. $100a + 10b + c = 0$
$a + b + c = 0$
$16a + 4b + c = 3$
$y = -\frac{1}{6}x^2 + \frac{11}{6}x - \frac{5}{3}$

81. $36a - 6b + c = 0$
$36a + 6b + c = 0$
$9a - 3b + c = -9$
$y = \frac{1}{3}x^2 - 12$

82. $9a - 3b + c = 0$
$25a + 5b + c = 0$
$4a - 2b + c = 7$
$y = -x^2 + 2x + 15$

83. $x^5 \cdot \frac{1}{x^2} = x^{5-2} = x^3$

84. $\frac{x^4y^5}{xy^3} = x^{4-1}y^{5-3} = x^3y^2$

85. $-5^{-2}y^0 = \frac{1}{-5^2} = -\frac{1}{25}$

86. $(4x^{-3})^4 \cdot \left(\frac{x^6}{2}\right)^2 = \frac{4^4}{(x^3)^4} \cdot \frac{x^{12}}{2^2} = \frac{256}{x^{12}} \cdot \frac{x^{12}}{4} = 64$

87. $\frac{3x^5y^8}{6xy^{-3}} = \frac{1}{2}x^{5-1}y^{8+3} = \frac{1}{2}x^4y^{11}$

88. $\frac{6x^4y^2}{30x^2y^{-1}} = \frac{1}{5}x^{4-2}y^{2+1} = \frac{1}{5}x^2y^3$

6.3 Quiz 1 (p. 344)

1. $7^0 \cdot 5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

2. $\left(\frac{4}{9}\right)^{-2} = \frac{9^2}{4^2} = \frac{81}{16}$

3. $\left(\frac{5}{3^2}\right)^2 = \frac{25}{9^2} = \frac{25}{81}$

4. $3^2 \cdot (3^2 \cdot 2^4)^{-1} = 3^2 \cdot (3^{-2} \cdot 2^{-4}) = \frac{1}{2^4} = \frac{1}{16}$

5. $(8^2 \cdot 8^{-3})^2 \cdot 8^2 = (8^{-1})^2 \cdot 8^2 = \frac{8^2}{8^2} = 1$

6. $\frac{(2^5 \cdot 3^2)^{-1}}{2^{-2} \cdot 3^2} = \frac{2^{-5} \cdot 3^{-2}}{2^{-2} \cdot 3^2} = 2^{-5-(-2)} = 3^{-2-2}$
$= 2^{-3} \cdot 3^{-4} = \frac{1}{2^3 \cdot 3^4} = \frac{1}{648}$

7. $(-5)^{-2}y^0 = \frac{1}{25}$

8. $(3x^3y^6)^{-2} = \frac{1}{(3x^3y^6)^2} = \frac{1}{9x^6y^{12}}$

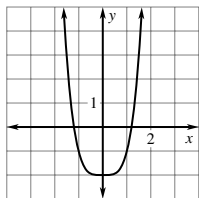
9. $x^3y^{-5}(x^2y)^2 = x^{3+4}y^{-5+2} = \frac{x^7}{y^3}$

10. $(x^2y^{-3})(xy^2) = \frac{x^3}{y}$

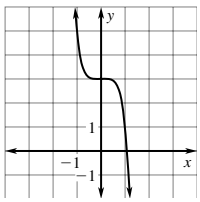
11. $\left(\frac{2x}{y^2}\right)^{-3} = \left(\frac{y^2}{2x}\right)^3 = \frac{y^6}{8x^3}$

12. $\frac{x^6y^{-2}}{x^{-1}y^5} = \frac{x^7}{y^7}$

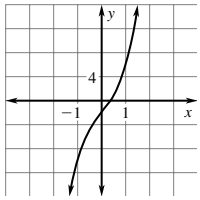
13. $f(x) = x^4 - 2$



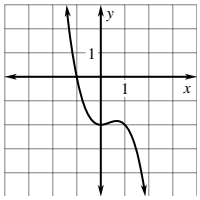
14. $f(x) = -2x^5 + 3$



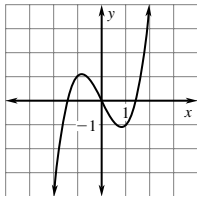
15. $f(x) = 3x^3 + 5x - 2$



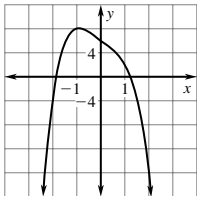
16. $f(x) = -x^3 + x^2 - 2$



17. $f(x) = x^3 - 2x$



18. $f(x) = -x^4 - 3x + 6$



19. $7x^3 + 3x^2 + 7x - 3$

20. $3x^2 + 3x - 11$

21. $2x^2 + 18x - 2$

22. $3x^2 + 4x - 1 + x^3 - 2x - 5 = x^3 + 3x^2 + 2x - 6$

23. $x(4x^2 - x - 1) + 5(4x^2 - x - 1)$

$= 4x^3 - x^2 - x + 20x^2 - 5x - 5$

$= 4x^3 + 19x^2 - 6x - 5$

24. $(x^2 - x - 6)(2x + 5)$

$= 2x^3 - 2x^2 - 12x + 5x^2 - 5x - 30$

$= 2x^3 + 3x^2 - 17x - 30$

25. $(x - 6)(x - 6)(x - 6)$

$= (x^2 - 12x + 36)(x - 6)$

$= x^3 - 12x^2 + 36x - 6x^2 + 72x - 216$

$= x^3 - 18x^2 + 108x - 216$

26. $(2x^2 + 3)(2x^2 + 3) = 4x^4 + 6x^2 + 6x^2 + 9$

$= 4x^4 + 12x^2 + 9$

27. $\dfrac{4.95 \times 10^8 \text{ mi}}{2.5 \times 10^4 \text{ mi/hr}} = 1.98 \times 10^4 \text{ hours}$

$\dfrac{1.98 \times 10^4 \text{ hr}}{2.4 \times 10^1 \text{ hr/day}} = .825 \times 10^3 \text{ day} = 825 \text{ days}$

Lesson 6.4

Activity (p. 345)

1. *Sample answer:* The cube a is missing a small part. The part is the cube b. But the total volume can be broken into three parts. By adding the three parts, we get the same volume as we would have if we had taken the total volume of the cube a and subtracted the cube b.

2. Solid I $= (a)(a)(a - b)$

Solid II $= (b)(b)(a - b)$

Solid III $= (a)(b)(a - b)$

3. $a^3 - b^3 = [a \cdot a \cdot (a - b)] + [b \cdot b \cdot (a - b)]$

$+ (a \cdot b \cdot a - b)$

$= (a - b)(a^2 + ab + b^2)$

6.4 Guided Practice (p. 348)

1. *Sample answer:* $9x^3 - 1$

2. **a.** grouping **b.** difference of cubes
c. factoring polynomial

3. You can't divide by $2x^2$, which contains a variable. Zero is also a solution.

4. **a.** $(x^3 + 1) = (x + 1)(x^2 - x + 1)$

b. *Sample answer:* The graph of $y = x^2 - x + 1$ does not intersect the x-axis, so $x^2 - x + 1$ is not factorable.

5. $x^6 + 125 = (x^2)^3 + (5)^3 = (x^2 + 5)(x^4 - 5x^2 + 25)$

6. $4x^3 + 16x^2 + x + 4 = 4x^2(x + 4) + (x + 4)$

$= (x + 4)(4x^2 + 1)$

7. $x^4 - 1 = (x^2)^2 - (1^2)^2$

$= (x^2 - 1)(x^2 + 1)$

$= (x + 1)(x - 1)(x^2 + 1)$

8. $2x^3 - 3x^2 - 10x + 15 = x^2(2x - 3) + 5(-2x + 3)$

$= x^2(2x - 3) - 5(2x - 3)$

$= (2x - 3)(x^2 - 5)$

9. $5x^3 - 320 = 5(x^3 - 64) = 5(x - 4)(x^2 + 4 + 16)$

10. $x^4 + 7x^2 + 10 = (x^2 + 5)(x^2 + 2)$

11. $x^3 - 27 = 0$

$x^3 = 27$

$x = 3$

12. $3x^3 + 7x^2 - 12x - 28 = 0$

$x^2(3x + 7) - 4(3x + 7) = 0$

$(3x + 7)(x^2 - 4) = 0$

$(3x + 7)(x + 2)(x - 2) = 0$

$x = -\frac{7}{3}$ or $x = \pm 2$

13. $x^3 + 2x^2 - 9x - 18 = 0$

$x^2(x + 2) - 9(x + 2) = 0$

$(x^2 - 9)(x + 2) = 0$

$(x - 3)(x + 3)(x + 2) = 0$

$x = \pm 3$ or $x = -2$

14. $54x^3 = -2$

$x^3 = -\frac{1}{27}$

$x = -\frac{1}{3}$

15. $(3x^2 - 2)^2 = 0$

$3x^2 = 2$

$x^2 = \frac{2}{3}$

$x = \pm\frac{\sqrt{6}}{3}$

16. $16x^8 - 81 = 0$

$(4x^4)^2 - (9)^2 = 0$

$(4x^4 - 9)(4x^4 + 9) = 0$

$(2x^2 - 3)(2x^2 + 3)(4x^4 + 9) = 0$

$x = \pm\frac{\sqrt{6}}{2}$

17. $R = t^3 - 8t^2 + t + 82$

$90 = t^3 - 8t^2 + t + 82$

$t^3 - 8t^2 + t - 8 = 0$

$t^2(t - 8) + (t - 8) = 0$

$t = 8$

The year was 1998.

6.4 Practice and Application (pp. 348–350)

18. $14x^2 + 8x + 72 = 2(7x^2 + 4x + 36)$

19. $3x^4 - 12x^3 = 3x^3(x - 4)$

20. $7x + 28x^2 - 35x^3 = 7x(1 + 4x - 7x^2)$

21. $24x^4 - 6x = 6x(4x^3 - 1)$

22. $39x^5 + 13x^3 - 78x^2 = 13x^2(3x^3 + x - 6)$

23. $145x^9 - 17 = 1(145x^9 - 17)$

24. $6x^6 - 3x^4 - 9x^2 = 3x^2(2x^4 - x^2 - 3)$

25. $72x^9 + 15x^6 + 9x^3 = 3x^3(24x^6 + 5x^3 + 3)$

26. $6x^4 - 18x^3 + 15x^2 = 3x^2(2x^2 - 6x + 5)$

27. $3x^2 + 11x + 6 = (3x + 2)(x + 3)$ C

28. $x^3 - 4x^2 + 4x - 16 = (x^2 + 4)(x - 4)$ D

29. $125x^3 - 216 = (5x)^3 - (6)^3 = (5x - 6)(25x^2 + 30x + 36)$

F

30. $2x^7 - 32x^3 = 2x^3(x^4 - 16) = 2x^3(x + 2)(x - 2)(x^2 + 4)$

A

31. $2x^5 + 4x^4 - 4x^3 - 8x^2 = 2x^2(x^3 + 2x^2 - 2x - 4)$

$= 2x^2(x^2 - 2)(x + 2)$

E

32. $2x^3 - 32x = 2x(x^2 - 16) = 2x(x + 4)(x - 4)$

B

33. $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

34. $x^3 + 64 = (x + 4)(x^2 - 4x + 16)$

35. $216x^3 + 1 = (6x + 1)(36x^2 - 6x + 1)$

36. $125x^3 - 8 = (5x - 2)(25x^2 + 10x + 4)$

37. $1000x^3 + 27 = (10x + 3)(100x^2 - 30x + 9)$

38. $27x^3 + 216 = 27(x + 2)(x^2 - 2x + 4)$

39. $32x^3 - 4 = 4(8x^3 - 1) = 4(2x - 1)(4x^2 + 2x + 1)$

40. $2x^3 + 54 = 2(x^3 + 27) = 2(x + 3)(x^2 - 3x + 9)$

41. $x^3 + x^2 + x + 1 = x^2(x + 1) + (x + 1)$

$= (x + 1)(x^2 + 1)$

42. $10x^3 + 20x^2 + x + 2 = 10x^2(x + 2) + (x + 2)$

$= (x + 2)(10x^2 + 1)$

43. $x^3 + 3x^2 + 10x + 30 = x^2(x + 3) + 10(x + 3)$

$= (x + 3)(x^2 + 10)$

44. $x^3 - 2x^2 + 4x - 8 = x^2(x - 2) + 4(x - 2)$

$= (x - 2)(x^2 + 4)$

45. $2x^3 - 5x^2 + 18x - 45 = x^2(2x - 5) + 9(2x - 5)$

$= (2x - 5)(x^2 + 9)$

46. $-2x^3 - 4x^2 - 3x - 6 = -2x^2(x + 2) - 3(x + 2)$

$= (x + 2)(-2x^2 - 3)$

47. $3x^3 - 6x^2 + x - 2 = 3x^2(x - 2) + (x - 2)$

$= (x - 2)(3x^2 + 1)$

48. $2x^3 - x^2 + 2x - 1 = x^2(2x - 1) + (2x - 1)$

$= (2x - 1)(x^2 + 1)$

49. $3x^3 - 2x^2 - 9x + 6 = x^2(3x - 2) - 3(3x - 2)$

$= (3x - 2)(x^2 - 3)$

50. $16x^4 - 1 = (4x^2 - 1)(4x^2 + 1) = (2x - 1)(2x + 1)(4x^2 + 1)$

51. $x^4 + 3x^2 + 2 = (x^2 + 1)(x^2 + 2)$

52. $x^4 - 81 = (x^2 - 9)(x^2 + 9) = (x - 3)(x + 3)(x^2 + 9)$

53. $81x^4 - 256 = (9x^2 - 16)(9x^2 + 16)$

$= (3x - 4)(3x + 4)(9x^2 + 16)$

54. $4x^4 - 5x^2 - 9 = (4x^2 - 9)(x^2 + 1)$

$= (2x - 3)(2x + 3)(x^2 + 1)$

55. $x^4 + 10x^2 + 16 = (x^2 + 2)(x^2 + 8)$

56. $81 - 16x^4 = (9 - 4x^2)(9 + 4x^2)$

$= (3 - 2x)(3 + 2x)(9 + 4x^2)$

57. $32x^6 - 2x^2 = 2x^2(16x^4 - 1)$

$= 2x^2(4x^2 + 1)(4x^2 - 1)$

$= 2x^2(4x^2 + 1)(2x - 1)(2x + 1)$

58. $6x^5 - 51x^3 - 27x = 3x(2x^4 - 17x^2 - 9)$

$= 3x(2x^2 + 1)(2x^2 - 9)$

$= 3x(2x^2 + 1)(x + 3)(x - 3)$

59. $18x^3 - 2x^2 + 27x - 3 = 2x^2(9x - 1) + 3(9x - 1)$
$= (9x - 1)(2x^2 + 3)$

60. $6x^3 + 21x^2 + 15x = 3x(2x^3 + 7x + 5)$
$= 3x(2x + 5)(x + 1)$

61. $4x^4 + 39x^2 - 10 = (4x^2 - 1)(x^2 + 10)$
$= (2x + 1)(2x - 1)(x^2 + 10)$

62. $8x^3 - 12x^2 - 2x + 3 = 4x^2(2x - 3) - (2x - 3)$
$= (2x - 3)(4x^2 - 1)$
$= (2x - 3)(2x - 1)(2x + 1)$

63. $8x^3 - 64 = (2x - 4)(4x^2 + 8x + 16)$
$= 8(x - 2)(x^2 + 2x + 4)$

64. $3x^4 - 300x^2 = 3x^2(x^2 - 100) = 3x^2(x + 10)(x - 10)$

65. $3x^4 - 24x = 3x(x^3 - 8) = 3x(x - 2)(x^2 + 2x + 4)$

66. $5x^4 + 31x^2 + 6 = (5x^2 + 1)(x^2 + 6)$

67. $3x^4 + 9x^3 + x^2 + 3x = 3x^3(x + 3) + x(x + 3)$
$= x(x + 3)(3x^2 + 1)$

68. $x^3 - 3x^2 = 0$
$x^2(x - 3) = 0$
$0, 3$

69. $2x^3 - 6x^2 = 0$
$2x^2(x - 3) = 0$
$0, 3$

70. $3x^4 + 15x^2 - 72 = 0$
$(3x^2 - 9)(x^2 + 8) = 0$
$x = \pm\sqrt{3}$

71. $x^3 + 27 = 0$
$x^3 = -27$
$x = -3$

72. $x^3 + 2x^2 - x - 2 = 0$
$x^2(x + 2) - 1(x + 2) = 0$
$(x + 2)(x^2 - 1) = 0$
$(x + 2)(x + 1)(x - 1) = 0$
$-2, -1, 1$

73. $x^4 + 7x^3 - 8x - 56 = 0$
$x^3(x + 7) - 8(x + 7) = 0$
$(x + 7)(x^3 - 8) = 0$
$-7, 2$

74. $2x^4 - 26x^2 + 72 = 0$
$(2x^2 - 8)(x^2 - 9) = 0$
$2(x^2 - 4)(x^2 - 9) = 0$
$\pm 2, \pm 3$

75. $3x^7 - 243x^3 = 0$
$3x^3(x^4 - 81) = 0$
$3x^3(x^2 - 9)(x^2 + 9) = 0$
$0, \pm 3$

76. $x^3 + 3x^2 - 2x - 6 = 0$
$x^2(x + 3) - 2(x + 3) = 0$
$(x + 3)(x^2 - 2) = 0$
$-3, \pm\sqrt{2}$

77. $8x^3 - 1 = 0$
$x^3 = \frac{1}{8}$
$x = \frac{1}{2}$

78. $x^3 + 8x^2 + 16x = 0$
$x(x^2 + 8x + 16) = 0$
$x(x + 4)^2 = 0$
$0, -4$

79. $x^3 - 5x^2 + 5x - 25 = 0$
$x^2(x - 5) + 5(x - 5) = 0$
$(x - 5)(x^2 + 5) = 0$
5

80. $3x^4 + 3x^3 - 6x^2 - 6x = 0$
$3x^3(x + 1) - 6x(x + 1) = 0$
$3x(x^2 - 2)(x + 1) = 0$
$0, -1, \pm\sqrt{2}$

81. $x^4 + x^3 - x - 1 = 0$
$x^3(x + 1) - (x + 1) = 0$
$(x + 1)(x^3 - 1) = 0$
$-1, 1$

82. $4x^4 + 20x^2 = -25$
none; left side will always be a positive number

83. $-2x^6 = 16$
$x^6 = -8$
none; left side is always positive

84. $3x^7 - 81x^4 = 0$
$3x^4(x^3 - 27) = 0$
$0, 3$

85. $2x^5 - 12x^3 + 16x = 0$
$2x(x^4 - 6x^2 + 8) = 0$
$2x(x^2 - 4)(x^2 - 2) = 0$
$0, \pm 2, \pm\sqrt{2}$

86. Sample answer should include:

1. For two terms, finding a common factor and using the sum/difference of cubes.
2. For 3 terms, look for a quadratic pattern.
3. For 4 or more terms, grouping and looking for a common factor.

87. $30 = x(x - 2)(x + 5)$
$30 = x(x^2 + 3x - 10)$
$0 = x^3 + 3x^2 - 10x - 30$
$0 = x^2(x + 3) - 10(x + 3)$
$0 = (x + 3)(x^2 - 10)$
$x = \sqrt{10} \approx 3.16$
3.16 in. $\times$ 1.16 in. $\times$ 8.16 in.

88. $72 = x(x - 1)(x + 9)$
$72 = x^3 + 8x^2 - 9x$
$0 = x^3 + 8x^2 - 9x - 72$
$0 = x^2(x + 8) - 9(x + 8)$
$0 = (x + 8)(x^2 - 9)$
$0 = (x + 8)(x - 3)(x + 3)$
$x = 3$
width: 2 ft length: 3 ft height: 12 ft

89. $4 = x(6x - 2)(3x - 2)$
$4 = 18x^3 - 18x^2 + 4x$
$0 = 18x^2(x - 1) + 4(x - 1)$
$0 = (18x^2 + 4)(x - 1)$
$x = 1$
6 ft $\times$ 3 ft $\times$ 1 ft

90. $25 = x(5x - 10)(2x - 1)$

$25 = x(10x^2 - 25x + 10)$

$25 = 10x^3 - 25x^2 + 10x$

$0 = 5x^2(2x - 5) + 5(2x - 5)$

$0 = (2x - 5)(5x^2 + 5)$

$x = 2\frac{1}{2}$

$2\frac{1}{2}$ ft $\times$ $2\frac{1}{2}$ ft $\times$ 4 ft

91. $250 = \frac{1}{3}(x - 5)^2\, 3x$

$250 = x(x^2 - 10x + 25)$

$0 = x^3 - 10x^2 + 25x - 250$

$0 = x^2(x - 10) + 25(x - 10)$

$0 = (x - 10)(x^2 + 25)$

$x = 10$

5 ft $\times$ 5 ft $\times$ 30 ft

92. $250 = (x - 5)^2x$

$250 = (x^2 - 10x + 25)x$

$250 = x^3 - 10x^2 + 25x$

$x^3 - 10x^2 + 25x - 250 = 0$

$x^2(x - 10) + 25(x - 10) = 0$

$(x^2 + 25)(x - 10) = 0$

$x = 10$

5 in. by 5 in. by 10 in.

93. $(3x - 4)(9x^2 + 12x + 16)$

$27x^2 - 64$

C

94. $x^3 - 8$

$(x - 2)(x^2 + 2x + 4)$

D

95. $x^5 = 81x$

$x(x^4 - 81) = 0$

$x(x^2 - 9)(x^2 + 9) = 0$

$x(x - 3)(x + 3)(x^2 + 9) = 0$

$0, \pm 3$

D

96. *Sample answer:* If we think of the total volume equal to the 3 prisms: $a^2(a - b)$, $ab(a - b)$, and $b^2(a + b)$ Then $a(a - b)(a + b) + b^2(a + b)(a + b)(a^2 - ab + b^2) = a^3 + b^3$

97. $30x^2y + 36x^2 - 20xy - 24x$

$= 2x(15xy + 18x - 10y - 12)$

$= 2x(3x - 2)(5y + 6)$

98. $2x^7 - 127x = x(2x^6 - 127)$

6.4 Mixed Review (p. 351)

99. $\frac{6x^3y^9}{36x^3y^{-2}} = \frac{1}{6}y^{11}$

100. $\frac{5^{-2}x^2y^{-1}}{5^2xy^3} = \frac{x}{625y^4}$

101. $\frac{7^2x^{-3}y^2}{49x^{-3}y^{-2}} = y^4$

102. $f(3) = 3(3)^4 + 2(3)^3 - (3)^2 - 12(3) + 1$

$= 243 + 54 - 9 - 36 + 1$

$= 253$

103. $f(3) = 2(3)^5 - (3)^3 + 7(3) + 1$

$= 486 - 27 + 21 + 1$

$= 481$

104. $T = 4s + 6p + 8$

Math and History (p. 351)

1.

y	1	2	3	4	5
$f(y)$	2	12	36	80	150

y	6	7	8	9	10
$f(y)$	252	392	576	810	1100

2. $x^3 + x^2 = 252$

$x = 6$

3. $x^3 + 2x^2 = 288$

$\left(\frac{x}{2}\right)^3 + \left(\frac{x}{2}\right)^2 = 36$

$y = 3$

$x = \frac{(2)(3)}{1} = 6$

4. $3x^3 + x^2 = 90$

$27x^3 + x^2 = 810$

$(3x)^3 + (x)^2 = 810$

$y = 9$

$x = \frac{(1)(9)}{3} = 3$

5. $2x^3 + 5x^2 = 2500$

$\left(\frac{2x}{5}\right)^3 + \left(\frac{2x}{5}\right)^2 = 80$

$y = 4$

$x = \frac{(5)(4)}{2} = 10$

6. $7x^3 + 6x^2 = 1728$

$\left(\frac{7x}{6}\right)^3 + \left(\frac{7x}{6}\right)^2 = 392$

$y = 7$

$x = \frac{(6)(7)}{(7)} = 6$

7. $10x^3 + 3x^2 = 297$

$\left(\frac{10x}{3}\right)^3 + \left(\frac{10x}{3}\right)^2 = 1100$

$y = 10$

$x = \frac{(3)(10)}{10} = 3$

Lesson 6.5

6.5 Activity (p. 353)

1.

$$x - 2\overline{)\,3x^3 - 2x^2 + 2x - 5}$$

$3x^2 + 4x + 10$; $3x^2 + 4x + 10 + \frac{15}{x - 2}$; $3x^2 + 4x + 10$; 15

$-3x^2 + 6x^2$

$4x^2 + 2x$

$-4x^2 + 8x$

$10x - 5$

$-10x + 20$

15

2.
$$\begin{array}{r|rrrr} 2 & 3 & -2 & 2 & -5 \\ & & 6 & 8 & 20 \\ \hline & 3 & 4 & 10 & 15 \end{array}$$
They are equal; they match the coefficients of the quotient.

6.5 Guided Practice (p. 356)

1. For any number k, the remainder obtained when a polynomial $f(x)$ is divided by $x - k$ is the value of $f(x)$ when $x = k$.

2. *Sample answer:* $\dfrac{3x^3 + x^2 + 5x + 8}{3x - 4}, \dfrac{3x^3 + x^2 + 5x - 8}{x - 2}$

3. $x + 3; x^3 - 2x^2 - 9x + 18; x^2 - 5x + 6$

4.
$$\begin{array}{r} x^2 - 5x - 1 \\ 2x + 3 \overline{)\,2x^3 - 7x^2 - 17x - 3} \\ \underline{-2x^3 - 3x^2} \\ -10x^2 - 17x \\ \underline{10x^2 + 15x} \\ -2x - 3 \\ \underline{2x + 3} \\ 0 \end{array}$$

5.
$$\begin{array}{r} x^2 + x - 4 + \dfrac{14}{x + 4} \\ x + 4 \overline{)\,x^3 + 5x^2 + 0x - 2} \\ \underline{-x^3 - 4x^2} \\ x^2 + 0x \\ \underline{-x^2 - 4x} \\ -4x - 2 \\ \underline{4x + 16} \\ 14 \end{array}$$

6.
$$\begin{array}{r} -3x^2 - 3x + 1 \\ x - 1 \overline{)\,-3x^3 + 0x^2 + 4x - 1} \\ \underline{3x^3 - 3x^2} \\ -3x^2 + 4x \\ \underline{3x^2 - 3x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

7.
$$\begin{array}{r} -x + 2 + \dfrac{-3x + 5}{x^2 - 1} \\ x^2 - 1 \overline{)\,-x^3 + 2x^2 - 2x + 3} \\ \underline{x^3 \quad - x} \\ 2x^2 - 3x \\ \underline{-2x^2 \quad + 2} \\ -3x + 5 \end{array}$$

8.
$$\begin{array}{r|rrrr} -3 & 1 & 0 & -8 & 3 \\ & & -3 & 9 & -3 \\ \hline & 1 & -3 & 1 & 0 \end{array}$$
$x^2 - 3x + 1$

9.
$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -16 & 1 & 4 \\ & & -4 & 16 & 0 & -4 \\ \hline & 1 & -4 & 0 & 1 & 0 \end{array}$$
$x^3 - 4x^2 + 1$

10.
$$\begin{array}{r|rrr} 3 & 1 & 2 & 15 \\ & & 3 & 15 \\ \hline & 1 & 5 & 30 \end{array}$$
$x + 5 + \dfrac{30}{x - 3}$

11.
$$\begin{array}{r|rrr} 2 & 1 & 7 & -2 \\ & & 2 & 18 \\ \hline & 1 & 9 & 16 \end{array}$$
$x + 9 + \dfrac{16}{x - 2}$

12.
$$\begin{array}{r|rrrr} 4 & 1 & -8 & 4 & 48 \\ & & 4 & -16 & -48 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$
$f(x) = (x - 4)(x^2 - 4x - 12)$

$f(x) = (x - 4)(x + 2)(x - 6)$

$4, -2, 6$

13.
$$\begin{array}{r|rrrr} 10 & 2 & -14 & -56 & -40 \\ & & 20 & 60 & 40 \\ \hline & 2 & 6 & 4 & 0 \end{array}$$
$f(x) = (x - 10)(2x^2 + 6x + 4)$

$= 2(x - 10)(x^2 + 3x + 2)$

$= 2(x - 10)(x + 1)(x + 2)$

$10, -1, -2$

14. $21 = -4x^3 + 25x$

$4x^3 - 25x + 21 = 0$

$$\begin{array}{r} 4x^2 + 4x - 21 \\ x - 1 \overline{)\,4x^3 + 0x^2 - 25x + 21} \\ \underline{-4x^3 + 4x^2} \\ 4x^2 - 25x \\ \underline{-4x^2 + 4x} \\ -21x + 21 \\ \underline{21x - 21} \\ 0 \end{array}$$

$(x - 1)(4x^2 + 4x - 21) = 0$

$x = 1$ and $x = 1.85$

about 1.85 million radios

6.5 Practice and Applications (pp. 356-358)

15.
$$\begin{array}{r} x + 9 \\ x-2\overline{)\,x^2+7x-5} \\ \underline{-x^2+2x} \\ 9x-5 \\ \underline{-9x+18} \\ 13 \end{array} \quad + \frac{13}{x-2}$$

16.
$$\begin{array}{r} 3x+20 \\ x-3\overline{)\,3x^2+11x+1} \\ \underline{-3x^2+9x} \\ 20x+1 \\ \underline{-20x+60} \\ 61 \end{array} \quad + \frac{61}{x-3}$$

17.
$$\begin{array}{r} 2x-5 \\ x+4\overline{)\,2x^2+3x-1} \\ \underline{-2x^2-8x} \\ -5x-1 \\ \underline{5x+20} \\ 19 \end{array} \quad + \frac{19}{x+4}$$

18.
$$\begin{array}{r} x-7 \\ x+1\overline{)\,x^2-6x+4} \\ \underline{-x^2-x} \\ -7x+4 \\ \underline{7x+7} \\ 11 \end{array} \quad + \frac{11}{x+1}$$

19.
$$\begin{array}{r} x+15 \\ x-10\overline{)\,x^2+5x-3} \\ \underline{-x^2+10x} \\ 15x-3 \\ \underline{-15x+150} \\ 147 \end{array} \quad + \frac{147}{x-10}$$

20.
$$\begin{array}{r} x^2-2x-1 \\ x-1\overline{)\,x^3-3x^2+x-8} \\ \underline{-x^3+x^2} \\ -2x^2+x \\ \underline{2x^2-2x} \\ -x-8 \\ \underline{-x+1} \\ -9 \end{array} \quad - \frac{9}{x-1}$$

21.
$$\begin{array}{r} 2x^2+2 \\ x^2-1\overline{)\,2x^4+0x^3+0x^2+0x+7} \\ \underline{-2x^4+2x^2} \\ 2x^2+7 \\ \underline{-2x^2+2} \\ 9 \end{array} \quad + \frac{9}{x^2-1}$$

22.
$$\begin{array}{r} x+8 \\ x^2+5\overline{)\,x^3+8x^2-3x+16} \\ \underline{-x^3-5x} \\ 8x^2-8x+16 \\ \underline{-8x^2-40} \\ -8x-24 \end{array} \quad - \frac{8x+24}{x^2+5}$$

23.
$$\begin{array}{r} 3x-4 \\ 2x+3\overline{)\,6x^2+x-7} \\ \underline{-6x^2-9x} \\ -8x-7 \\ \underline{8x+12} \\ 5 \end{array} \quad + \frac{5}{2x+3}$$

24.
$$\begin{array}{r} 10x+7 \\ x^2+2x\overline{)\,10x^3+27x^2+14x+5} \\ \underline{-10x^3-20x^2} \\ 7x^2+14x \\ \underline{-7x^2-14x} \\ 0+5 \end{array} \quad + \frac{5}{x^2+2x}$$

25.
$$\begin{array}{r} 5x^2-x+3 \\ x^2+3x\overline{)\,5x^4+14x^3+0x^2+9x} \\ \underline{-5x^4-15x^3} \\ -x^3+0x^2 \\ \underline{x^3+3x^2} \\ 3x^2+9x \\ \underline{-3x^2-9x} \\ 0 \end{array}$$

26.
$$\begin{array}{r} 2x \\ x^3+x^2-5\overline{)\,2x^4+2x^3+0x^2-10x-9} \\ \underline{-2x^4-2x^3+10x} \\ 0-9 \end{array} \quad - \frac{9}{x^3+x^2-5}$$

27.
$$\begin{array}{c|cccc} 2 & 1 & 0 & -7 & -6 \\ & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & -12 \end{array}$$

$$x^2+2x-3-\frac{12}{x-2}$$

28. $-4 \mid 1 \quad 0 \quad -14 \quad 8$ / $-4 \quad 16 \quad -8$ / $1 \quad -4 \quad 2 \quad 0$

$x^2 - 4x + 2$

29. $-1 \mid 4 \quad 5 \quad -4$ / $-4 \quad -1$ / $4 \quad 1 \quad -5$

$4x + 1 - \frac{5}{x+1}$

30. $2 \mid 1 \quad -4 \quad 3$ / $2 \quad -4$ / $1 \quad -2 \quad -1$

$x - 2 - \frac{1}{x-2}$

31. $2 \mid 2 \quad 7 \quad 8$ / $4 \quad 22$ / $2 \quad 11 \quad 30$

$2x + 11 + \frac{30}{x-2}$

32. $6 \mid 3 \quad -10 \quad 0$ / $18 \quad 48$ / $3 \quad 8 \quad 48$

$3x + 8 + \frac{48}{x-6}$

33. $-4 \mid 1 \quad 0 \quad 10$ / $-4 \quad 16$ / $1 \quad -4 \quad 26$

$x - 4 + \frac{26}{x+4}$

34. $-3 \mid 1 \quad 0 \quad 3$ / $-3 \quad 9$ / $1 \quad -3 \quad 12$

$x - 3 + \frac{12}{x+3}$

35. $-1 \mid 10 \quad 5 \quad 4 \quad 0 \quad -9$ / $-10 \quad 5 \quad -9 \quad 9$ / $10 \quad -5 \quad 9 \quad -9 \quad 0$

$10x^3 - 5x^2 + 9x - 9$

36. $7 \mid 1 \quad -6 \quad 0 \quad -40 \quad 33$ / $7 \quad 7 \quad 49 \quad 63$ / $1 \quad 1 \quad 7 \quad 9 \quad 96$

$x^3 + x^2 + 7x + 9 + \frac{96}{x-7}$

37. $3 \mid 2 \quad -6 \quad 1 \quad -3 \quad -3$ / $6 \quad 0 \quad 3 \quad 0$ / $2 \quad 0 \quad 1 \quad 0 \quad -3$

$2x^3 + x - \frac{3}{x-3}$

38. $-1 \mid 4 \quad 5 \quad 2 \quad 0 \quad -1$ / $-4 \quad -1 \quad -1 \quad 1$ / $4 \quad 1 \quad 1 \quad -1 \quad 0$

$4x^3 + x^2 + x - 1$

39. $-2 \mid 1 \quad -5 \quad -2 \quad 24$ / $-2 \quad 14 \quad -24$ / $1 \quad -7 \quad 12 \quad 0$

$f(x) = (x + 2)(x^2 - 7x + 12)$

$= (x + 2)(x - 3)(x - 4)$

40. $6 \mid 1 \quad -3 \quad -16 \quad -12$ / $6 \quad 18 \quad 12$ / $1 \quad 3 \quad 2 \quad 0$

$f(x) = (x - 6)(x^2 + 3x + 2)$

$= (x - 6)(x + 1)(x + 2)$

41. $10 \mid 1 \quad -12 \quad 12 \quad 80$ / $10 \quad -20 \quad -80$ / $1 \quad -2 \quad -8 \quad 0$

$f(x) = (x^2 - 2x - 8)(x - 10)$

$= (x - 4)(x + 2)(x - 10)$

42. $9 \mid 1 \quad -18 \quad 95 \quad -126$ / $9 \quad -81 \quad 126$ / $1 \quad -9 \quad 14 \quad 0$

$f(x) = (x^2 - 9x + 14)(x - 9)$

$= (x - 7)(x - 2)(x - 9)$

43. $-5 \mid 1 \quad -1 \quad -21 \quad 45$ / $-5 \quad 30 \quad -45$ / $1 \quad -6 \quad 9 \quad 0$

$f(x) = (x + 5)(x^2 - 6x + 9)$

$= (x + 5)(x - 3)(x - 3)$

44. $8 \mid 1 \quad -11 \quad 14 \quad 80$ / $8 \quad -24 \quad -80$ / $1 \quad -3 \quad -10 \quad 0$

$f(x) = (x - 8)(x^2 - 3x - 10)$

$= (x - 8)(x - 5)(x + 2)$

45. $1 \mid 4 \quad -4 \quad -9 \quad 9$ / $4 \quad 0 \quad -9$ / $4 \quad 0 \quad -9 \quad 0$

$f(x) = (x - 1)(4x^2 - 9)$

$= (x - 1)(2x - 3)(2x + 3)$

46. $-6 \mid 2 \quad 7 \quad -33 \quad -18$ / $72 \quad 30 \quad 18$ / $2 \quad -5 \quad -3 \quad 0$

$f(x) = (x + 6)(2x^2 - 5x - 3)$

$= (x + 6)(2x + 1)(x - 3)$

47. $-2 \mid 9 \quad 10 \quad -17 \quad -2$ / $-18 \quad 16 \quad 2$ / $9 \quad -8 \quad -1 \quad 0$

$f(x) = (x + 2)(9x^2 - 8x - 1)$

$= (x + 2)(9x + 1)(x - 1)$

$-2, -\frac{1}{9}, 1$

48. $-14 \mid 1 \quad 11 \quad -150 \quad -1512$ / $-14 \quad 42 \quad 1512$ / $1 \quad -3 \quad -108 \quad 0$

$f(x) = (x + 14)(x^2 - 3x - 108)$

$= (x + 14)(x - 12)(x + 9)$

$-14, 12, -9$

49. $4 \mid 2 \quad 3 \quad -39 \quad -20$
$\quad\quad\quad\quad\;\; 8 \quad 44 \quad 20$
$\quad\quad\quad\quad 2 \quad 11 \quad 5 \quad 0$

$f(x) = (x - 4)(2x^2 + 11x + 5)$

$f(x) = (x - 4)(2x + 1)(x + 5)$

$4, -\frac{1}{2}, -5$

50. $8 \mid 15 \quad -119 \quad -10 \quad 16$
$\quad\quad\quad\quad\;\; 120 \quad 8 \quad -16$
$\quad\quad\quad\quad 15 \quad 1 \quad -2 \quad 0$

$f(x) = (x - 8)(15x^2 + x - 2)$

$f(x) = (x - 8)(5x + 2)(3x - 1)$

$8, -\frac{2}{5}, \frac{1}{3}$

51. $9 \mid 1 \quad -14 \quad 47 \quad -18$
$\quad\quad\quad\quad\;\; 9 \quad -45 \quad 18$
$\quad\quad\quad\quad 1 \quad -5 \quad 2 \quad 0$

$f(x) = (x - 9)(x^2 - 5x + 2)$

$9, \frac{5 \pm \sqrt{17}}{2}$

52. $-5 \mid 4 \quad 9 \quad -52 \quad 15$
$\quad\quad\quad\quad\;\; -20 \quad 55 \quad -15$
$\quad\quad\quad\quad 4 \quad -11 \quad 3 \quad 0$

$f(x) = (x + 5)(x^2 - 11x + 3)$

$-5, \frac{11 \pm \sqrt{73}}{8}$

53. $-3 \mid 1 \quad 1 \quad 2 \quad 24$
$\quad\quad\quad\quad\;\; -3 \quad 6 \quad -24$
$\quad\quad\quad\quad 1 \quad -2 \quad 8 \quad 0$

$f(x) = (x + 3)(x^2 - 2x + 8)$

$-3, 1 \pm \sqrt{7}i$

54. $6 \mid 5 \quad -27 \quad -17 \quad -6$
$\quad\quad\quad\quad\;\; 30 \quad 18 \quad 6$
$\quad\quad\quad\quad 5 \quad 3 \quad 1 \quad 0$

$f(x) = (x - 6)(5x^2 + 3x + 1)$

$6, \frac{-3 \pm i\sqrt{11}}{10}$

55.
$$\begin{array}{r} 3x^2 + 5x - 50 \\ x + 1 \overline{)\, 3x^3 + 8x^2 - 45x - 50} \\ \underline{-3x^3 - 3x^2} \\ 5x^2 - 45x \\ \underline{-5x - 5x} \\ -50x - 50 \\ \underline{50x + 50} \\ 0 \end{array}$$

$$\begin{array}{r} 3x - 10 \\ x + 5 \overline{)\, 3x^2 + 5x - 50} \\ \underline{-3x^2 - 15x} \\ -10x - 50 \\ \underline{10x + 50} \\ 0 \end{array}$$

$f(x) = (x + 1)(x + 5)(3x - 10)$

56.
$$\begin{array}{r} 2x^2 + 15x + 25 \\ x + 1 \overline{)\, 2x^3 + 17x^2 + 40x + 25} \\ \underline{-2x^3 - 2x^2} \\ 15x^2 + 40x \\ \underline{-15x^2 - 15x} \\ 25x + 25 \\ \underline{-25x - 25} \\ 0 \end{array}$$

$$\begin{array}{r} 2x + 5 \\ x + 5 \overline{)\, 2x^2 + 15x + 25} \\ \underline{-2x^2 - 10x} \\ 5x + 25 \\ \underline{-5x - 25} \\ 0 \end{array}$$

$f(x) = (x + 1)(2x + 5)(x + 5)$

57. $x^3 + x^2 - 5x = -x^2 - 4x + 2$

$x^3 + 2x^2 - x - 2 = 0$

$1 \mid 1 \quad 2 \quad -1 \quad -2$
$\quad\quad\quad\;\; 1 \quad 3 \quad 2$
$\quad\quad 1 \quad 3 \quad 2 \quad 0$

$(x - 1)(x^2 + 3x + 2) = 0$

$(x - 1)(x + 2)(x + 1) = 0$

x at $1, -2, -1$

$(1, -3), (-1, 5), (-2, 6)$

58. $x^3 - 6x^2 + 6x + 3 = -x^2 + 7x - 2$

$x^3 - 5x^2 - x + 5 = 0$

$$\begin{array}{c|cccc} 1 & 1 & -5 & -1 & 5 \\ & & 1 & -4 & -5 \\ \hline & 1 & -4 & -5 & 0 \end{array}$$

$(x - 1)(x^2 - 4x - 5) = 0$

$(x - 1)(x - 5)(x + 1) = 0$

x at 1, 5, -1

$(1, 4), (5, 8), (-1, -10)$

59. $5x^3 - 3x^2 + 21x - 8$; I multiplied $5x^2 - 13x + 47$ by $x + 2$ and added -102.

60. $62.5 = (100 - 8x^2)x - 25x$

$62.5 = 100x - 8x^3 - 25x$

$8x^3 - 75x + 62.5 = 0$

$$\begin{array}{c|cccc} 2.5 & 8 & 0 & -75 & 62.5 \\ & & 20 & 50 & -62.5 \\ \hline & 8 & 20 & -25 & 0 \end{array}$$

$(x - 2.5)(8x^2 + 20x - 25) = 0$

$x = 2.5$ or $x = 0.92$

about 0.92 million cameras

61.
$$\begin{array}{r} 0.0031x^2 + 0.158x + 11.155 \\ -8.4x + 580 \overline{)\ -0.026x^3 + 0.47x^2 - \ \ 2.2x + 72} \\ \underline{0.026x^3 - 1.80x^2} \\ -1.33x^2 - \ \ 2.2x \\ \underline{+1.33x^2 - \ \ 91.5x} \\ -93.7x + 72 \\ \underline{93.7x - 6469.76} \\ -6398 \end{array}$$

$C = 0.0031x^2 + 0.158x + 11.155 - \dfrac{6398}{-8.4x + 580}$

$C = 0.0031(10)^2 + 1.58 + 11.155 - \dfrac{6398}{496}$

$C = 0.31 + 1.58 + 11.155 - 12.90$

$C = 0.145$

about 145 million cars

62. *Sample answer*: $-1.1686x^2 + 137.4713x - 13{,}097.384 + \dfrac{3{,}240{,}124}{2.61x + 247}$

$$\begin{array}{r} 2.61x + 247 \overline{)\ -3.05x^3 + 70.2x^2 - \ \ 225x + \ \ 5070} \\ \underline{3.05x^3 + 288.6x^2} \\ 358.8x^2 - 225x \\ \underline{-358.8x^2 - 33959x} \\ -34184x + 5070 \\ \underline{34184x + 3235054} \\ 3{,}240{,}124 \end{array}$$

about \$20.50

63.
$$\begin{array}{c|cccc} 3 & 1 & 0 & -9 & 5 \\ & & 3 & 9 & 0 \\ \hline & 1 & 3 & 0 & 5 \end{array}$$

$x^2 + 3x + \dfrac{5}{x - 3}$

C

64. $2x^3 - 19x^2 - 20x + 100$

$$\begin{array}{r} x^2 - 12x + \ \ 20 \\ 2x + 5 \overline{)\ 2x^3 - 19x^2 - 20x + 100} \\ \underline{-2x^3 - \ \ 5x^2} \\ -24x^2 - 20x \\ \underline{24x^2 + 60x} \\ 40x + 100 \\ \underline{-40x - 100} \\ 0 \end{array}$$

E

65.
$$\begin{array}{r} 6x^2 - 7x + 6 - \dfrac{4}{2x + 1} \\ 2x + 1 \overline{)\ 12x^3 - 8x^2 + 5x + 2} \\ \underline{-12x^3 - 6x^2} \\ -14x^2 + 5x \\ \underline{14x^2 + 7x} \\ 12x + 2 \\ \underline{-12x - 6} \\ -4 \end{array}$$

$$\begin{array}{r} 4x^2 - 4x + 3 - \dfrac{1}{3x + 1} \\ 3x + 1 \overline{)\ 12x^3 - 8x^2 + 5x + 2} \\ \underline{-12x^3 - 4x^2} \\ -12x^2 + 5x \\ \underline{12x^2 + 4x} \\ 9x + 2 \\ \underline{-9x - 3} \\ -1 \end{array}$$

—CONTINUED—

65. —CONTINUED—

$$3x^2 - \frac{11}{4}x + \frac{31}{16} + \frac{1}{16(4x+1)}$$
$$4x+1 \overline{)\,12x^3 - 8x^2 + 5x + 2}$$
$$\underline{-12x^3 - 3x^2}$$
$$-11x^2 + 5x$$
$$\underline{11x^2 + \tfrac{11}{4}x}$$
$$\tfrac{31}{4}x + 2$$
$$\underline{-\tfrac{31}{4}x - \tfrac{31}{16}}$$
$$\tfrac{1}{16}$$

$$\begin{array}{r|cccc} -\frac{1}{2} & 12 & -8 & 5 & 2 \\ & & -6 & 7 & -6 \\ \hline & 12 & -14 & 12 & -4 \end{array} \qquad \begin{array}{r|cccc} -\frac{1}{3} & 12 & -8 & 5 & 2 \\ & & -4 & 4 & -3 \\ \hline & 12 & -12 & 9 & -1 \end{array}$$

$$\begin{array}{r|cccc} -\frac{1}{4} & 12 & -8 & 5 & 2 \\ & & -3 & \frac{11}{4} & -\frac{31}{16} \\ \hline & 12 & -11 & \frac{31}{4} & \frac{1}{16} \end{array}$$

The remainders are all the same, but the coefficients are 2, 3, and 4 times larger with synthetic division.

6.5 Mixed Review (p.358)

66. $6 + 7(-2) \le -8$
$-8 \le -8$ yes
$-2 + 7(-3) \le -8$
$-2 - 21 \le -8$
$-23 \le -8$ yes

67. $2(-2) + 5(4) \ge 1$
$-4 + 20 \ge 1$
$16 \ge 1$ yes
$2(8) + 5(-3) \ge 1$
$16 - 15 \ge 1$
$1 \ge 1$ yes

68. $9(-1) - 4(-4) > 7$
$-9 + 16 > 7$
$7 > 7$ no
$9(2) - 4(2) > 7$
$18 - 8 > 7$
$10 > 7$ yes

69. $-3(2) - 2(0) < -6$
$-6 < -6$ no
$-3(1) - 2(4) < -6$
$-3 - 8 < -6$
$-11 < -6$ yes

70. $x = \dfrac{5 \pm \sqrt{25 - 12}}{2}$
$x = \dfrac{5 \pm \sqrt{13}}{2}$

71. $x = \dfrac{8 \pm \sqrt{64 - 12}}{2}$
$x = 4 \pm \sqrt{13}$

72. $x = \dfrac{10 \pm \sqrt{100 - 60}}{2}$
$x = 5 \pm \sqrt{10}$

73. $x = \dfrac{7 \pm \sqrt{49 - 16}}{8}$
$x = \dfrac{7 \pm \sqrt{33}}{8}$

74. $x = \dfrac{9 \pm \sqrt{81 + 48}}{-12}$
$x = -\dfrac{9 \pm \sqrt{129}}{12}$ or
$x = \dfrac{-3}{4} \pm \dfrac{\sqrt{129}}{12}$

75. $x = \dfrac{-1 \pm \sqrt{1 + 40}}{10}$
$x = \dfrac{-1 \pm \sqrt{41}}{10}$

76. $x = \dfrac{-3 \pm \sqrt{9 - 40}}{4}$
$x = \dfrac{-3 \pm i\sqrt{31}}{4}$

77. $x = \dfrac{1 \pm \sqrt{1 - 160}}{-10}$
$x = \dfrac{1 \pm i\sqrt{159}}{-10}$

78. $x = \dfrac{-3 \pm \sqrt{9 - 12}}{6} = -\dfrac{3 \pm i\sqrt{3}}{6}$ or
$x = \dfrac{-1}{2} \pm \dfrac{i\sqrt{3}}{6}$

79. $(x^2 - 3x + 8) - (x^2 + x - 1) = -4x + 9$

80. $(14x^2 - 15x + 3) + (11x - 7) = 14x^2 - 4x - 4$

81. $(8x^3 - 1) - (22x^3 + 2x^2 - x - 5) = -14x^3 - 2x^2 + x + 4$

82. $(x + 5)(x^2 - x + 5) = x^3 + 4x^2 + 25$

83. $c + v = 120$
$24c + 21v = 2766$
$24c + 21(120 - c) = 2766$
$24c - 21c = 246$
$3c = 246$
$c = 82$

Lesson 6.6

6.6 Guided Practice (p. 362)

1. constant term, leading coefficient

2. a. yes; coefficients are all integers

b. no; coefficients are not all integers

c. no; coefficients are not all integers

3. Make a graph **4.** $\pm1, \pm2, \pm4, \pm7, \pm8, \pm14, \pm28, \pm56$

5. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm9, \pm12, \pm18, \pm24, \pm36, \pm72$

6. $\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}, \pm1, \pm2, \pm3, \pm5, \pm6, \pm10, \pm15, \pm30$

7. $\pm\frac{1}{5}, \pm\frac{2}{5}, \pm1, \pm2, \pm5, \pm10$

8.
$$\begin{array}{r|cccc} 1 & 1 & -3 & -6 & 8 \\ & & 1 & -2 & -8 \\ \hline & 1 & -2 & -8 & 0 \end{array}$$
$f(x) = (x^2 - 2x - 8)(x - 1)$
$= (x - 4)(x + 2)(x - 1)$
$x = 4, -2, 1$

9.
$$\begin{array}{r|cccc} 1 & 1 & 4 & -1 & -4 \\ & & 1 & 5 & 4 \\ \hline & 1 & 5 & 4 & 0 \end{array}$$
$f(x) = (x - 1)(x^2 + 5x + 4)$
$= (x - 1)(x + 1)(x + 4)$
$x = 1, -1, -4$

10. $1 \mid 2 \quad -5 \quad -2 \quad 5$
$\qquad\quad\ \ 2 \quad -3 \quad -5$
$\qquad 2 \quad -3 \quad -5 \quad 0$

$f(x) = (x - 1)(2x^2 - 3x - 5)$

$= (x - 1)(2x - 5)(x + 1)$

$x = 1, \frac{5}{2}, -1$

11. $2 \mid 2 \quad -1 \quad -15 \quad 18$
$\qquad\quad\ \ 4 \quad 6 \quad -18$
$\qquad 2 \quad 3 \quad -9 \quad 0$

$f(x) = (x - 2)(2x^2 + 3x - 9)$

$= (x - 2)(x + 3)(2x - 3)$

$x = 2, -3, \frac{3}{2}$

12. $1 \mid 1 \quad 4 \quad 1 \quad -6$
$\qquad\quad\ \ 1 \quad 5 \quad 6$
$\qquad 1 \quad 5 \quad 6 \quad 0$

$f(x) = (x - 1)(x^2 + 5x + 6)$

$= (x - 1)(x + 3)(x + 2)$

$x = 1, -2, -3$

13. $1 \mid 1 \quad 5 \quad -1 \quad -5$
$\qquad\quad\ \ 1 \quad 6 \quad 5$
$\qquad 1 \quad 6 \quad 5 \quad 0$

$f(x) = (x - 1)(x^2 + 6x + 5)$

$= (x - 1)(x + 5)(x + 1)$

$x = 1, -5, -1$

14. $18 = \frac{1}{3}x^2(x + 3)$

$54 = x^3 + 3x^2$

$0 = x^3 + 3x^2 - 54$

$3 \mid 1 \quad 3 \quad 0 \quad -54$
$\qquad\quad\ \ 3 \quad 18 \quad 54$
$\qquad 1 \quad 6 \quad 18 \quad 0$

$0 = (x - 3)(x^2 + 6x + 18)$

$x = 3$

3 in. by 3 in. by 6 in.

6.6 Practice and Applications (pp. 362–364)

15. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24$ **16.** $\pm 1, \pm\frac{1}{2}$

17. $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm\frac{1}{2}$

18. $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60$
$\pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{15}{2}$

19. $\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3}, \pm\frac{1}{6}, \pm\frac{5}{6}$

20. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$

21. $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}, \pm\frac{1}{8}, \pm\frac{3}{8}$

22. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{3}, \pm\frac{5}{3}$

23. $-2 \mid 1 \quad 7 \quad -4 \quad -28$
$\qquad\quad\ \ -2 \quad -10 \quad 28$
$\qquad 1 \quad 5 \quad -14 \quad 0$

$f(x) = (x + 2)(x^2 + 5x - 14)$

$= (x + 2)(x - 2)(x + 7)$

$x = -2, 2$

24. $1 \mid 1 \quad 5 \quad 2 \quad -8$
$\qquad\quad\ \ 1 \quad 6 \quad 8$
$\qquad 1 \quad 6 \quad 8 \quad 0$

$f(x) = (x - 1)(x^2 + 6x + 8)$

$= (x - 1)(x + 2)(x + 4)$

$x = 1, -2$

25. $-1 \mid 1 \quad 3 \quad -7 \quad -27 \quad -18$
$\qquad\quad\ \ -1 \quad -2 \quad 9 \quad 18$
$\qquad 1 \quad 2 \quad -9 \quad -18 \quad 0$

$-2 \mid 1 \quad 2 \quad -9 \quad -18$
$\qquad\quad\ \ -2 \quad 0 \quad 18$
$\qquad 1 \quad 0 \quad -9 \quad 0$

$f(x) = (x + 1)(x - 2)(x^2 - 9)$

$= (x + 1)(x + 2)(x - 3)(x + 3)$

$x = -1, -2$

26. $1 \mid 2 \quad -9 \quad 8 \quad 9 \quad -10$
$\qquad\quad\ \ 2 \quad -7 \quad 1 \quad 10$
$\qquad 2 \quad -7 \quad 1 \quad 10 \quad 0$

$1 \mid 2 \quad -7 \quad 1 \quad 10$
$\qquad\quad\ \ -2 \quad 9 \quad -10$
$\qquad 2 \quad -9 \quad 10 \quad 0$

$2 \mid 2 \quad -9 \quad 10$
$\qquad\quad\ \ 4 \quad -10$
$\qquad 2 \quad -5 \quad 0$

$f(x) = (x - 1)(x + 1)(x - 2)(2x - 5)$

$x = 1, -1, 2$

27. $1 \mid 1 \quad 3 \quad 3 \quad -3 \quad -4$
$\qquad\quad\ \ 1 \quad 4 \quad 7 \quad 4$
$\qquad 1 \quad 4 \quad 7 \quad 4 \quad 0$

$-1 \mid 1 \quad 4 \quad 7 \quad 4$
$\qquad\quad\ \ -1 \quad -3 \quad -4$
$\qquad 1 \quad 3 \quad 4 \quad 0$

$f(x) = (x - 1)(x + 1)(x^2 + 3x + 4)$

$x = 1, -1$

28. none **29.** none

30. $2 \mid 1 \quad 1 \quad -11 \quad 10$
$\qquad\quad\ \ 2 \quad 6 \quad -10$
$\qquad 1 \quad 3 \quad -5 \quad 0$

$f(x) = (x - 2)(x^2 + 3x - 5)$

$x = 2$

31. $-2 \mid 1 \quad 0 \quad -2 \quad 0 \quad -11 \quad 0 \quad 12$
$\quad\quad\quad\quad -2 \quad 4 \quad -4 \quad 8 \quad 6 \quad -12$
$\quad\quad 1 \quad -2 \quad 2 \quad -4 \quad -3 \quad 6 \quad 0$

$2 \mid 1 \quad -2 \quad 2 \quad -4 \quad -3 \quad 6$
$\quad\quad\quad 2 \quad 0 \quad 4 \quad 0 \quad -6$
$\quad\quad 1 \quad 0 \quad 2 \quad 0 \quad -3 \quad 0$

$1 \mid 1 \quad 0 \quad 2 \quad 0 \quad -3$
$\quad\quad\quad 1 \quad 1 \quad 3 \quad 3$
$\quad\quad 1 \quad 1 \quad 3 \quad 3 \quad 0$

$-1 \mid 1 \quad 1 \quad 3 \quad 3$
$\quad\quad\quad -1 \quad 0 \quad -3$
$\quad\quad 1 \quad 0 \quad 3 \quad 0$

$x = \pm 2, \pm 1$

32. $2 \mid 1 \quad -1 \quad -2 \quad -1 \quad 1 \quad 2$
$\quad\quad\quad 2 \quad 2 \quad 0 \quad -2 \quad -2$
$\quad\quad 1 \quad 1 \quad 0 \quad -1 \quad -1 \quad 0$

$-1 \mid 1 \quad 1 \quad 0 \quad -1 \quad -1$
$\quad\quad\quad -1 \quad 0 \quad 0 \quad 1$
$\quad\quad 1 \quad 0 \quad 0 \quad -1 \quad 0$

$1 \mid 1 \quad 0 \quad 0 \quad -1$
$\quad\quad\quad 1 \quad 1 \quad 1$
$\quad\quad 1 \quad 1 \quad 1 \quad 0$

$f(x) = (x - 1)(x + 1)(x - 2)(x^2 + x + 1)$

$x = \pm 1, 2$

33. $10 \mid 1 \quad -8 \quad -23 \quad 30$
$\quad\quad\quad 10 \quad 20 \quad -30$
$\quad\quad 1 \quad 2 \quad -3 \quad 0$

$f(x) = (x - 10)(x^2 + 2x - 3)$
$\quad\quad = (x - 10)(x + 3)(x - 1)$

$x = 10, -3, 1$

34. $-1 \mid 1 \quad 2 \quad -11 \quad -12$
$\quad\quad\quad -1 \quad -1 \quad 12$
$\quad\quad 1 \quad 1 \quad -12 \quad 0$

$f(x) = (x + 1)(x^2 + x - 12)$
$\quad\quad = (x + 1)(x + 4)(x - 3)$

$x = -1, -4, 3$

35. $-2 \mid 1 \quad -7 \quad 2 \quad 40$
$\quad\quad\quad -2 \quad 18 \quad -40$
$\quad\quad 1 \quad -9 \quad 20 \quad 0$

$f(x) = (x + 2)(x^2 - 9x + 20)$
$\quad\quad = (x + 2)(x - 5)(x - 4)$

$x = -2, 5, 4$

36. $-1 \mid 1 \quad 1 \quad -2 \quad -2$
$\quad\quad\quad -1 \quad 0 \quad 2$
$\quad\quad 1 \quad 0 \quad -2 \quad 0$

$f(x) = (x + 1)(x^2 - 2)$

$x = -1, \pm\sqrt{2}$

37. $3 \mid 1 \quad -5 \quad -18 \quad 72$
$\quad\quad\quad 3 \quad -6 \quad -72$
$\quad\quad 1 \quad -2 \quad -24 \quad 0$

$f(x) = (x - 3)(x^2 - 2x - 24)$
$\quad\quad = (x - 3)(x - 6)(x + 4)$

$x = 3, 6, -4$

38. $2 \mid 1 \quad 9 \quad -4 \quad -36$
$\quad\quad\quad 2 \quad 22 \quad 36$
$\quad\quad 1 \quad 11 \quad 18 \quad 0$

$f(x) = (x - 2)(x^2 + 11x + 18)$
$\quad\quad = (x - 2)(x + 9)(x + 2)$

$x = 2, -2, -9$

39. $2 \mid 1 \quad -5 \quad 7 \quad 3 \quad -10$
$\quad\quad\quad 2 \quad -6 \quad 2 \quad 10$
$\quad\quad 1 \quad -3 \quad 1 \quad 5 \quad 0$

$-1 \mid 1 \quad -3 \quad 1 \quad 5$
$\quad\quad\quad -1 \quad 4 \quad -5$
$\quad\quad 1 \quad -4 \quad 5 \quad 0$

$f(x) = (x - 2)(x + 1)(x^2 - 4x + 5)$

$x = 2, -1$

40. $2 \mid 1 \quad 1 \quad 1 \quad -9 \quad -10$
$\quad\quad\quad 2 \quad 6 \quad 14 \quad 10$
$\quad\quad 1 \quad 3 \quad 7 \quad 5 \quad 0$

$-1 \mid 1 \quad 3 \quad 7 \quad 5$
$\quad\quad\quad -1 \quad -2 \quad -5$
$\quad\quad 1 \quad 2 \quad 5 \quad 0$

$f(x) = (x - 2)(x + 1)(x^2 + 2x + 5)$

$x = 2, -1$

41. $1 \mid 1 \quad 1 \quad -11 \quad -9 \quad 18$
$\quad\quad\quad 1 \quad 2 \quad -9 \quad -18$
$\quad\quad 1 \quad 2 \quad -9 \quad -18 \quad 0$

$-2 \mid 1 \quad 2 \quad -9 \quad -18$
$\quad\quad\quad -2 \quad 0 \quad 18$
$\quad\quad 1 \quad 0 \quad -9 \quad 0$

$f(x) = (x - 1)(x + 2)(x^2 - 9)$
$\quad\quad = (x - 1)(x + 2)(x - 3)(x + 3)$

$x = 1, -2, 3, -3$

42.
$$\begin{array}{r|rrrrr} 2 & 1 & -3 & 6 & -2 & -12 \\ & & 2 & -2 & 8 & 12 \\ \hline & 1 & -1 & 4 & 6 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -1 & 1 & -1 & 4 & 6 \\ & & -1 & 2 & -6 \\ \hline & 1 & -2 & 6 & 0 \end{array}$$
$f(x) = (x - 2)(x + 1)(x^2 - 2x + 6)$

$x = 2, -1$

43.
$$\begin{array}{r|rrrrrr} -2 & 1 & 1 & -9 & -5 & 0 & -36 \\ & & -2 & 2 & 14 & -18 & 36 \\ \hline & 1 & -1 & -7 & 9 & -18 & 0 \end{array}$$
$$\begin{array}{r|rrrrr} 3 & 1 & -1 & -7 & 9 & -18 \\ & & 3 & 6 & -3 & 18 \\ \hline & 1 & 2 & -1 & 6 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -3 & 1 & 2 & -1 & 6 \\ & & -3 & 3 & -6 \\ \hline & 1 & -1 & 2 & 0 \end{array}$$
$f(x) = (x + 2)(x + 3)(x - 3)(x^2 - x + 2)$

$x = -2, 3, -3$

44.
$$\begin{array}{r|rrrrrr} -3 & 1 & -1 & -7 & 11 & -8 & 12 \\ & & -3 & 12 & -15 & 12 & -12 \\ \hline & 1 & -4 & 5 & -4 & 4 & 0 \end{array}$$
$$\begin{array}{r|rrrrr} 2 & 1 & -4 & 5 & -4 & 4 \\ & & 2 & -4 & 2 & -4 \\ \hline & 1 & -2 & 1 & -2 & 0 \end{array}$$
$f(x) = (x + 3)(x - 2)(x^3 - 2x^2 + x - 2)$

$x = -3, 2$

45.
$$\begin{array}{r|rrrr} \frac{3}{2} & 4 & -12 & -1 & 15 \\ & & 6 & -9 & -15 \\ \hline & 4 & -6 & -10 & 0 \end{array}$$
$$\begin{array}{r|rrr} \frac{5}{2} & 4 & -6 & -10 \\ & & 10 & 10 \\ \hline & 4 & 4 & 0 \end{array}$$
$x = \frac{3}{2}, \frac{5}{2}, -1$

46.
$$\begin{array}{r|rrrr} 4 & -3 & 20 & -36 & 16 \\ & & -12 & 32 & -16 \\ \hline & -3 & 8 & -4 & 0 \end{array}$$
$$\begin{array}{r|rrr} 2 & -3 & 8 & -4 \\ & & -6 & 4 \\ \hline & -3 & 2 & 0 \end{array}$$
$x = 4, 2, \frac{2}{3}$

47.
$$\begin{array}{r|rrrr} -2 & 2 & 4 & -2 & -4 \\ & & -4 & 0 & 4 \\ \hline & 2 & 0 & -2 & 0 \end{array}$$
$f(x) = (x + 2)(2x^2 - 2)$

$x = -2, \pm 1$

48.
$$\begin{array}{r|rrrr} 4 & 2 & -5 & -14 & 8 \\ & & 8 & 12 & -8 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$
$f(x) = (x - 4)(2x^2 + 3x - 2)$

$= (x - 4)(2x - 1)(x + 2)$

$x = 4, \frac{1}{2}, -2$

49.
$$\begin{array}{r|rrrr} 2 & 2 & -5 & -1 & 6 \\ & & 4 & -2 & -6 \\ \hline & 2 & -1 & -3 & 0 \end{array}$$
$f(x) = (x - 2)(2x^2 - x - 3)$

$= (x - 2)(2x - 3)(x + 1)$

$x = 2, \frac{3}{2}, -1$

50.
$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & -50 & -25 \\ & & -1 & 0 & 25 \\ \hline & 2 & 0 & -50 & 0 \end{array}$$
$f(x) = \left(x + \frac{1}{2}\right)(2x^2 - 50)$

$x = -\frac{1}{2}, +5, -5$

51.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -1 & -32 & 16 \\ & & 1 & 0 & -16 \\ \hline & 2 & 0 & -32 & 0 \end{array}$$
$f(x) = \left(x - \frac{1}{2}\right)(2x^2 - 32)$

$x = \frac{1}{2}, 4, -4$

52.
$$\begin{array}{r|rrrr} -3 & 3 & 12 & 3 & -18 \\ & & -9 & -9 & 18 \\ \hline & 3 & 3 & -6 & 0 \end{array}$$
$f(x) = (x + 3)(3x^2 + 3x - 6)$

$= (x + 3)(x + 2)(3x - 3)$

$x = -3, -2, 1$

53.
$$\begin{array}{r|rrrrr} 1 & 2 & 3 & -3 & 3 & -5 \\ & & 2 & 5 & 2 & 5 \\ \hline & 2 & 5 & 2 & 5 & 0 \end{array}$$
$$\begin{array}{r|rrrr} -\frac{5}{2} & 2 & 5 & 2 & 5 \\ & & -5 & 0 & -5 \\ \hline & 2 & 0 & 2 & 0 \end{array}$$
$f(x) = (x - 1)\left(x + \frac{5}{2}\right)(2x^2 + 2)$

$x = 1, -\frac{5}{2}$

54. $x = \dfrac{-1 \pm \sqrt{61}}{6}$

$x = \dfrac{3 \pm \sqrt{5}}{2}$

55. $-1 \mid 2 \quad 1 \quad -1 \quad -1 \quad -1$

$\quad\quad\quad\quad -2 \quad 1 \quad 0 \quad 1$

$\quad\quad 2 \quad -1 \quad 0 \quad -1 \quad 0$

$1 \mid 2 \quad -1 \quad 0 \quad -1$

$\quad\quad\quad 2 \quad 1 \quad 1$

$\quad 2 \quad 1 \quad 1 \quad 0$

$f(x) = (x - 1)(x + 1)(2x^2 + x + 1)$

$x = 1, -1$

56. $-2 \mid 3 \quad 11 \quad 11 \quad 1 \quad -2$

$\quad\quad\quad -6 \quad -10 \quad -2 \quad 2$

$\quad 3 \quad 5 \quad 1 \quad -1 \quad 0$

$-1 \mid 3 \quad 5 \quad 1 \quad -1$

$\quad\quad\quad -3 \quad -2 \quad 1$

$\quad 3 \quad 2 \quad -1 \quad 0$

$f(x) = (x + 2)(x + 1)(3x^2 + 2x - 1)$

$= (x + 2)(x + 1)(3x - 1)(x + 1)$

$x = -2, -1, \frac{1}{3}$

57. $-\frac{1}{2} \mid 2 \quad 1 \quad 0 \quad 0 \quad -32 \quad -16$

$\quad\quad\quad -1 \quad 0 \quad 0 \quad 0 \quad 16$

$\quad 2 \quad 0 \quad 0 \quad 0 \quad -32 \quad 0$

$2 \mid 2 \quad 0 \quad 0 \quad 0 \quad -32$

$\quad\quad\quad 4 \quad 8 \quad 16 \quad 32$

$\quad 2 \quad 4 \quad 8 \quad 16 \quad 0$

$-2 \mid 2 \quad 4 \quad 8 \quad 16$

$\quad\quad\quad -4 \quad 0 \quad -16$

$\quad 2 \quad 0 \quad 8 \quad 0$

$f(x) = \left(x + \frac{1}{2}\right)(x - 2)(x + 2)(2x^2 + 8)$

$x = -\frac{1}{2}, 2, -2$

58. $-3 \mid 3 \quad 1 \quad 0 \quad 0 \quad -243 \quad -81$

$\quad\quad\quad -9 \quad 24 \quad -72 \quad 216 \quad 81$

$\quad 3 \quad -8 \quad 24 \quad -72 \quad -27 \quad 0$

$3 \mid 3 \quad -8 \quad 24 \quad -72 \quad -27$

$\quad\quad\quad 9 \quad 3 \quad 81 \quad 27$

$\quad 3 \quad 1 \quad 27 \quad 9 \quad 0$

$-\frac{1}{3} \mid 3 \quad 1 \quad 27 \quad 9$

$\quad\quad\quad -1 \quad 0 \quad -9$

$\quad\quad 3 \quad 0 \quad 27$

$f(x) = (x - 3)(x + 3)\left(x + \frac{1}{3}\right)(3x^2 + 27)$

$x = 3, -3, -\frac{1}{3}$

59. $777 = 2t^3 + 23t^2 + 5t + 501$

$2t^3 + 23t^2 + 5t - 276 = 0$

$3 \mid 2 \quad 23 \quad 5 \quad -276$

$\quad\quad\quad 6 \quad 87 \quad 276$

$\quad 2 \quad 29 \quad 92 \quad 0$

$0 = (t - 3)(2t^2 + 29 + 92)$

$t = 3 \qquad 1993$

60. $90 = 3x^3 + x^2$

$3x^3 + x^2 - 90 = 0$

$3 \mid 3 \quad 1 \quad 0 \quad -90$

$\quad\quad\quad 9 \quad 30 \quad 90$

$\quad 3 \quad 10 \quad 30 \quad 0$

$0 = (x - 3)(3x^2 + 10x + 30)$

$x = 3$

3 ft

61. $20 = x^2(x + 3)$

$x^3 + 3x^2 - 20 = 0$

$2 \mid 1 \quad 3 \quad 0 \quad -20$

$\quad\quad\quad 2 \quad 10 \quad 20$

$\quad 1 \quad 5 \quad 10 \quad 0$

$f(x) = (x - 2)(x^2 + 5x + 10)$

$x = 2$

2 in. by 2 in. by 5 in

62. $48\pi = \frac{1}{3}\pi(x^2)(x + 5)$

$144 = x^3 + 5x^2$

$x^3 + 5x^2 - 144 = 0$

$4 \mid 1 \quad 5 \quad 0 \quad -144$

$\quad\quad\quad 4 \quad 36 \quad 144$

$\quad 1 \quad 9 \quad 36 \quad 0$

$(x - 4)(x^2 + 9x + 36) = 0$

$x = 4$

4 in. radius, 9 in. height

63. $2000 = (x^2 + 5x)(x + 35)$

$x^3 + 40x^2 + 175x - 2000 = 0$

$5 \mid 1 \quad 40 \quad 175 \quad -2000$

$\quad\quad\quad 5 \quad 225 \quad 2000$

$\quad 1 \quad 45 \quad 400 \quad 0$

$(x - 5)(x^2 + 45x + 400) = 0$

$x = 5$

5 ft deep, 10 ft wide, 40 ft long

64. $150 = \frac{1}{2}15x^2(2x + 1)$

$150 = 15x^3 + \frac{15x^2}{2}$

$15x^3 + \frac{15}{2}x^2 - 150 = 0$

$2 \mid 15 \quad \frac{15}{2} \quad 0 \quad -150$

$\quad\quad\quad 30 \quad 75 \quad 150$

$\quad 15 \quad \frac{75}{2} \quad 75 \quad 0$

$(x - 2)\left(15x^2 + \frac{75}{2}x + 75\right) = 0$

$x = 2$

2 ft by 6 ft by 25 ft

65. C **66.** A **67.** $-2, -1, 1$; B **68.** $-2, 1$; A

69. -1; C

70. no, no; If a cubic polynomial had 4 or more distinct real zeros, then there would be 4 or more binomials of the form $x - a$ that divide the polynomial to give a zero remainder. This would imply that the ploynomial has degree 4 or greater. However, this is impossible since the polynomial is a cubic polynomial. So a cubic polynomial has at most 3 real zeros. As $x \to -\infty$ and $x \to +\infty$, the values of a cubic polynomial approach $-\infty$ and $+\infty$, respectively, or else $+\infty$ and $-\infty$. At some value of x, therefore, the graph is below the x-axis, and at some other values of x, the graph is above the x-axis. This means that the graph crosses the x-axis somewhere between these two values, and the x-coordinate of the point where the graph crosses the x-axis is a zero.

6.6 Mixed Review (p. 365)

71. $x^2 - 6x + 9 = 0$
$(x - 3)^2 = 0$
$x - 3 = 0$
$x = 3$

72. $x^2 - 10x + 25 = 0$
$(x - 5)^2 = 0$
$x - 5 = 0$
$x = 5$

73. $x^2 - 2x + 1 = 0$
$(x - 1)^2 = 0$
$x - 1 = 0$
$x = 1$

74. $2x^2 - 12x - 18 = 0$
$2(x - 3)(x - 3) = 0$
$x - 3 = 0$
$x = 3$

75. $x^2 - 20x + 100 = 0$
$(x - 10)^2 = 0$
$x - 10 = 0$
$x = 10$

76. $x^2 - 18x + 81 = 0$
$(x - 9)^2 = 0$
$x - 9 = 0$
$x = 9$

77. $y = a(x + 3)(x - 3)$
$5 = a(-9)$
$y = -\frac{5}{9}(x + 3)(x - 3)$

78. $y = a(x + 5)(x - 1)$
$-6 = a(-2 + 5)(-2 - 1)$
$-6 = a - 9$
$\frac{2}{3} = a$
$y = \frac{2}{3}(x + 5)(x - 1)$

79. $y = a(x + 1)(x - 5)$
$10 = a(-5)$
$-2 = a$
$y = -2(x + 1)(x - 5)$

80. $y = a(x - 12)(x - 7)$
$7 = a(-23)(-18)$
$y = \frac{7}{414}(x - 12)(x - 7)$

81. $y = a(x + 12)(x + 6)$
$-5 = a(21)(15)$
$a = -\frac{1}{63}$
$y = -\frac{1}{63}(x + 12)(x + 6)$

82. $y = a(x - 2)(x - 8)$
$-4 = a(-5)$
$\frac{4}{5} = a$
$y = \frac{4}{5}(x - 2)(x - 8)$

83. $y = a(x - 4)(x - 10)$
$3 = a(3)(-3)$
$-\frac{1}{3} = a$
$y = -\frac{1}{3}(x - 4)(x - 10)$

84. $y = a(x + 6)(x + 0)$
$16 = a(16)$
$1 = a$
$y = (x + 6)(x + 0)$

85. $y = a(x + 9)(x + 1)$
$20 = a(20)$
$a = (1)$
$y = (x + 9)(x + 1)$

86. $(16 + 2x)(12 + 2x) = 204 + 192$
$(8 + x)(6 + x) = 99$
$48 + 14x + x^2 = 99$
$x^2 + 14x - 51 = 0$
$(x - 3)(x + 17) = 0$
$x = 3$ or $x = -17$
width of mat: 3 in.
overall: 18 in. by 22 in.

Quiz 2 (p. 365)

1. $5x^3 + 135 = 5(x^3 + 27) = 5(x + 3)(x^2 - 3x + 9)$

2. $6x^3 + 12x^2 + 12x + 24 = 6(x^3 + 2x^2 + 2x + 4)$
$= 6(x + 2)(x^2 + 2)$

3. $4x^5 - 16x = 4x(x^4 - 4) = 4x(x^2 + 2)(x^2 - 2)$

4. $3x^3 - x^2 - 15x + 5 = x^2(3x - 1) - 5(3x - 1)$
$= (3x - 1)(x^2 - 5)$

5. $7x^4 = 252x^2$
$x^2 = 36$
$x = \pm 6, 0$

6. $16x^6 = 54x^3$
$x^3 = \frac{27}{8}$
$x = \frac{3}{2}, 0$

7. $6x^5 - 18x^4 + 12x^3 - 36x^2 = 0$
$6x^4(x - 3) + 12x^2(x - 3) = 0$
$6x^2(x^2 + 2)(x - 3) = 0$
$x = 0, 3$

8. $2x^3 + 5x^2 - 8x - 20 = 0$
$x^2(2x + 5) - 4(2x + 5) = 0$
$(x + 2)(x - 2)(2x + 5) = 0$
$x = 2, -2, -\frac{5}{2}$

9.

4	1	7	-44
		4	44
	1	11	0

$(x - 4)(x + 11)$

10.
$$3x + 2 \overline{)\, 3x^2 - 8x + 20}$$
$$x - \frac{10}{3} + \frac{80}{3(3x + 2)}$$
$$-3x^2 - 2x$$
$$-10x + 20$$
$$10x + \frac{20}{3}$$
$$\frac{80}{3}$$

11.
$$\begin{array}{r} 4x - 7 + \dfrac{11x - 11}{x^2 - 3} \\ x^2 - 3 \overline{)\,4x^3 - 7x^2 - x + 10} \\ \underline{-4x^3 \qquad + 12x} \\ -7x^2 + 11x \\ \underline{7x^2 \qquad -21} \\ 11x - 11 \end{array}$$

12.
$$\begin{array}{r} 12x^3 - 7x^2 + 10x - 10 + \dfrac{5}{x + 1} \\ x + 1 \overline{)\,12x^4 + 5x^3 + 3x^2 + 0x - 5} \\ \underline{-12x^4 - 12x^3} \\ -7x^3 + 3x^2 \\ \underline{7x^3 + 7x^2} \\ 10x^2 + 0x \\ \underline{-10x^2 - 10x} \\ -10x - 5 \\ \underline{10x + 10} \\ 5 \end{array}$$

13.
$$\begin{array}{r} x + \dfrac{2x^2 + 6x + 6}{x^3 - 3} \\ x^3 - 3 \overline{)\,x^4 + 0x^3 + 2x^2 + 3x + 6} \\ \underline{-x^4 \qquad\qquad + 3x} \\ 2x^2 + 6x + 6 \end{array}$$

14.
$$\begin{array}{r} 5x^3 - 23x^2 + 115x - 576 + \dfrac{2875}{x + 5} \\ x + 5 \overline{)\,5x^4 + 2x^3 + 0x^2 - x - 5} \\ \underline{-5x^4 - 25x^3} \\ -23x^3 + 0x^2 \\ \underline{23x^2 + 115x^2} \\ 115x^2 - x \\ \underline{-115x^2 - 575x} \\ -576x - 5 \\ \underline{576x + 2880} \\ 2875 \end{array}$$

15. $f(x) = x^3 - 4x^2 - 7x + 28$

$= x^2(x - 4) - 7(x - 4)$

$= (x - 4)(x^2 - 7)$

$x = 4, \pm\sqrt{7}$

16.
$$\begin{array}{r|rrrr} 2 & 1 & -6 & 21 & -26 \\ & & 2 & -8 & 26 \\ \hline & 1 & -4 & 13 & 0 \end{array}$$

$f(x) = (x - 2)(x^2 - 4x + 13)$

$x = 2$

17.
$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 15 & 22 & -15 \\ & & 1 & 8 & 15 \\ \hline & 2 & 16 & 30 & 0 \end{array}$$

$f(x) = \left(x - \frac{1}{2}\right)2(x^2 + 8x + 15)$

$x = \frac{1}{2}, -3, -5$

18.
$$\begin{array}{r|rrrr} 2 & 2 & 7 & -28 & 12 \\ & & 4 & 22 & -12 \\ \hline & 2 & 11 & -6 & 0 \end{array}$$

$f(x) = (x - 2)(2x^2 + 11 - 6)$

$x = 2, \frac{1}{2}, -6$

19. $128 = x^2(x - 15.5)$

$x^3 - 15.5x^2 - 128 = 0$

$$\begin{array}{r|rrrr} 16 & 1 & -15.5 & 0 & -128 \\ & & 16 & 8 & 128 \\ \hline & 1 & 0.5 & 8 & 0 \end{array}$$

$0 = (x - 16)(x^2 + 0.5x + 8)$

$x = 16$

16 ft × 16 ft × 0.5 ft

Lesson 6.7

Activity (p. 366)

1. a. $2x - 1 = 0$

$x = \frac{1}{2}$;

1; rational

b. $x^2 - 2 = 0$

$x = \pm\sqrt{2}$;

2; irrational

c. $x = 1, \dfrac{-1 \pm i\sqrt{3}}{2}$;

3; 1 is rational, $\dfrac{-1 \pm i\sqrt{3}}{2}$ are imaginary

Sample answer: If $f(x)$ has a degree $n > 1$, then $f(x) = 0$ has n solutions.

2. $x^3 + x^2 - x - 1 = 0$

$x^2(x + 1) - (x + 1) = 0$

$(x + 1)(x^2 - 1) = 0$

$(x + 1)(x + 1)(x - 1) = 0$

$x = 1, -1$;

2 different solutions; -1 is a solution twice

6.7 Guided Practice (p. 369)

1. *Sample answer:* If $f(x)$ is a polynomial of positive degree, then $f(x) = 0$ has at least one root in the set of complex numbers.

2. *Sample answer:* The existence of an imaginary zero would imply that there are two distinct imaginary zeros which is not consistent with the fact that $f(x)$ is degree 3.

3. *Sample answer:* 2 real zeros; no imaginary zeros; the existence of an imaginary zero would imply the existence of two distinct imaginary zeros, which would not be consistent with the fact that $f(x)$ has degree 3. The real number 2 is a repeated zero.

4. $f(x) = x^3 - x^2 - 2x$

$= x(x^2 - x - 2)$

$= x(x - 2)(x + 1)$

$x = 0, 2, -1$

5. $f(x) = x^4 + x^2 - 12$

$= (x^2 - 3)(x^2 + 4)$

$= \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x^2 + 4)$

$x = \pm\sqrt{3}, \pm 2i$

6. $f(x) = x^3 + 5x^2 - 9x - 45$

$= (x + 5)(x^2 - 9)$

$= (x + 5)(x + 3)(x - 3)$

$x = -5, -3, 3$

7. $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$

$= (x + 1)(x^3 - 2x^2 + 4x - 8)$

$= (x + 1)(x - 2)(x^2 + 4)$

$x = -1, 2, \pm 2i$

8. $x = 3, 0, -2$

$f(x) = (x - 3)(x + 2)x$

$= (x^2 - x - 6)x$

$= x^3 - x^2 - 6x$

9. $x = 1, 1, i, -i$

$f(x) = (x - 1)(x - 1)(x - i)(x + i)$

$= (x^2 - 2x + 1)(x^2 + 1)$

$= x^4 - 2x^3 + 2x^2 - 2x + 1$

10. $x = 5, 2 + 3i, 2 - 3i$

$f(x) = (x - 5)[x - (2 + 3i)][x - (2 - 3i)]$

$= (x - 5)[(x - 2) - 3i][(x - 2) + 3i]$

$= (x - 5)[(x - 2)^2 + 9]$

$= x^3 - 9x^2 + 33x - 65$

11. $x = 1, -1, 2, -2, 3$

$f(x) = (x^2 - 1)(x^2 - 4)(x - 3)$

$= (x^4 - 5x^2 + 4)(x - 3)$

$= x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$

12. $x = 3, -2, -1 + i, -1 - i$

$f(x) = (x - 3)(x + 2)[x - (-1 + i)][x - (-1 - i)]$

$= (x^2 - x - 6)[(x + 1) - i][(x + 1) + i]$

$= (x^2 - x - 6)[(x + 1)^2 - i^2]$

$= (x^2 - x - 6)[x^2 + 2x + 2]$

$= x^4 + 2x^3 + 2x^2 - x^3 - 2x^2 - 2x - 6x^2 - 12x - 12$

$= x^4 + x^3 - 6x^2 - 14x - 12$

13. $x = 4i, 4i, -4i, -4i$

$f(x) = (x^2 + 16)(x^2 + 16)$

$= x^4 + 32x + 256$

14. $1.5 = \frac{1}{10{,}000}(-t^4 + 12t^3 - 77t^2 + 600t + 13{,}650)$

$t^4 - 12t^3 + 77t^2 - 600t + 1350 = 0$

3	1	−12	77	−600	1350
		3	−27	150	−1350
	1	−9	50	−450	0

9	1	−9	50	−450
		9	0	450
	1	0	50	0

$t = 3, t = 9$

6.7 Practice and Applications (pp. 369–371)

15. $f(x) = x^3 - x^2 + 4x - 4$

$f(1) = (1)^3 - (1)^2 + 4(1) - 4 = 0$

yes

16. $f(x) = x^3 + 3x^2 - 5x + 8$

$f(4) = (4)^3 + 3(4)^2 - 5(4) + 8 = 100$

no

17. $f(x) = x^4 - x^2 - 3x + 3$

$f(0) = 0^4 - 0^2 - 3(0) + 3 = 3$

no

18. $f(x) = x^3 + 5x^2 + x + 5$

$f(-5) = (-5)^3 + 5(-5)^2 - 5 + 5 = 0$

yes

19. $f(x) = x^3 - 4x^2 + 16x - 64$

$f(4i) = (4i)^3 - 4(4i)^2 + 16(4i) - 64 = 0$

yes

20. $f(x) = x^3 - 3x^2 + x - 3$

$f(-i) = (-i)^3 - 3(i)^2 - i - 3 = 0$

yes

21. $f(x) = x^4 + 5x^3 + 5x^2 - 5x - 6$

$= (x - 1)(x^3 + 6x^2 + 11x + 6)$

$= (x - 1)(x + 1)(x^2 + 5x + 6)$

$= (x - 1)(x + 1)(x + 3)(x + 2)$

$x = 1, -1, -3, -2$

22. $f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$

$= (x - 3)(x^3 + 7x^2 + 15x + 9)$

$= (x - 3)(x + 3)(x^2 + 4x + 3)$

$= (x - 3)(x + 3)(x + 3)(x + 1)$

$x = 3, -3, -3, -1$

23. $f(x) = x^3 - 4x^2 + 3x$
$= x(x^2 - 4x + 3)$
$= x(x - 3)(x - 1)$
$x = 0, 3, 1$

24. $f(x) = x^3 + 5x^2 - 4x - 20$
$= (x + 5)(x^2 - 4)$
$= (x + 5)(x - 2)(x + 2)$
$x = -5, 2, -2$

25. $f(x) = x^4 + 7x^3 - x^2 - 67x - 60$
$= (x + 5)(x^3 + 2x^2 - 11x - 12)$
$= (x + 5)(x + 4)(x^2 - 2x - 3)$
$x = -5, -4, 3, -1$

26. $f(x) = x^4 - 5x^2 - 36$
$= (x^2 + 4)(x^2 - 9)$
$= (x + 3)(x - 3)(x^2 + 4)$
$x = 3, -3, \pm 2i$

27. $f(x) = x^3 - x^2 + 49x - 49$
$= (x^2 + 49)(x - 1)$
$x = 1, \pm 7i$

28. $f(x) = x^3 - x^2 + 25x - 25$
$= (x^2 + 25)(x - 1)$
$x = 1, \pm 5i$

29. $f(x) = x^4 + 6x^3 + 14x^2 + 54x + 45$
$= (x + 5)(x^3 + x^2 + 9x + 9)$
$= (x + 5)(x + 1)(x^2 + 9)$
$x = -5, -1, \pm 3i$

30. $f(x) = x^3 + 3x^2 + 25x + 75$
$= (x^2 + 25)(x + 3)$
$x = \pm 5i, -3$

31. $f(x) = x^4 - x^3 - 5x^2 - x - 6$
$= (x + 2)(x^3 - 3x^2 + x - 3)$
$= (x + 2)(x - 3)(x^2 + 1)$
$x = -2, 3, \pm i$

32. $f(x) = x^4 + x^3 + 2x^2 + 4x - 8$
$= (x + 2)(x^3 - x^2 + 4x - 4)$
$= (x + 2)(x - 1)(x^2 + 4)$
$x = -2, 1, \pm 2i$

33. $f(x) = 2x^4 - 7x^3 - 27x^2 + 63x + 81$
$= (x^2 - 9)(2x^2 - 7x - 9)$
$= (x^2 - 9)(x + 1)(2x - 9)$
$x = 3, -3, -1, \frac{9}{2}$

34. $f(x) = 2x^4 - x^3 - 42x^2 + 16x + 160$
$= (x^2 - 16)(2x^2 - x - 10)$
$x = \pm 4, x \approx 2.5, x \approx -2$

35. $x = 2, 1, 4$
$f(x) = (x - 2)(x - 1)(x - 4)$
$= (x - 2)(x^2 - 5x + 4)$
$= x^3 - 7x^2 + 14x - 8$

36. $x = 1, -4, 5$
$f(x) = (x - 1)(x + 4)(x - 5)$
$= (x^2 + 3x - 4)(x - 5)$
$= x^3 - 2x^2 - 19x + 20$

37. $x = -6, 3, 5$
$f(x) = (x + 6)(x - 3)(x - 5)$
$= (x^2 + 3x - 18)(x - 5)$
$= x^3 - 2x^2 - 33x + 90$

38. $x = -5, 2, -2$
$f(x) = (x + 5)(x^2 - 4)$
$= x^3 + 5x^2 - 4x - 20$

39. $x = -2, -4, -7$
$f(x) = (x + 2)(x + 4)(x + 7)$
$= (x^2 + 6x + 8)(x + 7)$
$= x^3 + 13x^2 + 50x + 56$

40. $x = 8, -i, i$
$f(x) = (x - 8)(x^2 + 1)$
$= x^3 - 8x^2 + x - 8$

41. $x = 5, 3i, -3i$
$f(x) = (x - 5)(x - 3i)(x + 3i)$
$= (x - 5)(x^2 + 9)$
$= x^3 - 5x^2 + 9x - 45$

42. $x = 2, -2, -6i, 6i$
$f(x) = (x^2 - 4)(x^2 + 36)$
$= x^4 + 32x^2 - 144$

43. $x = i, -i, -3i, 3i$
$f(x) = (x^2 + 1)(x^2 + 9)$
$= x^4 + 10x^2 + 9$

44. $x = 3 - i, 3 + i, 5i, -5i$
$f(x) = [x - (3 - i)][x - (3 + i)](x - 5i)(x + 5i)$
$= [(x - 3) + i][(x - 3) - i](x^2 - 25i^2)$
$= [(x - 3)^2 - i^2](x^2 + 25)$
$= (x^2 - 6x + 10)(x^2 + 25)$
$= x^4 - 6x^3 + 35x^2 - 150x + 250$

45. $x = 4, 4, 2 + i, 2 - i$

$f(x) = (x - 4)(x - 4)[x - (2 + i)][x - (2 - i)]$

$= (x^2 - 8x + 16)[(x - 2) - i][(x - 2) + i]$

$= (x^2 - 8x + 16)[(x - 2)^2 - i^2]$

$= (x^2 - 8x + 16)(x^2 - 4x + 5)$

$f(x) = x^4 - 12x^3 + 53x^2 - 104x + 80$

46. $x = -2, -2, 3, -4i, 4i$

$f(x) = (x + 2)^2(x - 3)(x^2 + 16)$

$= (x^2 + 4x + 4)(x - 3)(x^2 + 16)$

$= x^5 + x^4 + 8x^3 + 4x^2 - 128x - 192$

47. $f(x) = x^3 - x^2 - 5x + 3$

$x \approx -2.09, 0.57, 2.51$

48. $f(x) = 2x^3 - x^2 - 3x - 1$

$x \approx -0.62, -0.5, 1.62$

49. $f(x) = x^3 - 2x^2 + x + 1$

$x \approx -0.47$

50. $f(x) = x^4 - 2x - 1$

$x \approx -0.47, 1.40$

51. $f(x) = x^4 - x^3 - 4x^2 - 3x - 2$

$x \approx -1.27, 2.86$

52. $f(x) = x^4 - x^3 - 3x^2 - x + 1$

$x \approx 0.42, 2.37$

53. $f(x) = x^4 + 3x^2 - 2$

$x \approx -0.75, 0.75$

54. $f(x) = x^4 - x^3 - 20x^2 + 10x + 27$

$x \approx -4.09, -0.98, 1.47, 4.60$

55. $E = -0.131t^3 + 5.033t^2 - 23.2t + 233$

$312.76 = -0.13t^3 + 5.033t^2 - 23.2t + 233$

$-0.131t^3 + 5.033t^2 - 23.2t - 79.76 = 0$

$t \approx 8.3$

1988

56. $D = 1.78t^3 - 6.02t^2 + 752t + 6701$

$14{,}300 = 1.78t^3 - 6.02t^2 + 752t + 6701$

$1.78t^3 - 6.02t^2 + 752t - 7599 = 0$

$t \approx 9.02$

1992

57. $S = -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2 - 318t + 1520$

$2000 = -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2 - 318t + 1520$

$= -0.982t^5 + 24.6t^4 - 211t^3 + 661t^2 - 318t - 480$

$= 0$

$t \approx 1.62, 6.3$

late 1988, 1993

58. $S = -0.213t^3 + 3.96t^2 + 10.2t + 366$

$455 = -0.213t^3 + 3.96t^2 + 10.2t + 366$

$-0.213t^3 + 3.96t^2 + 10.2t - 89 = 0$

$t \approx 3.95$

late 1993

59. $P = 0.00496t^3 - 0.432t^2 + 11.3t + 212$

$0.00496t^3 - 0.432t^2 + 11.3t - 510 = 0$

$t = 75$

1965

60. a. $1000g$, $1000g^2$, $1000g$

b. $1000g^3 + 1000g^2 + 1000g + 1000$

c. 1.05, 5%, *Sample answer:* I graphed $x^3 + x^2 + x + 1 = y$ and $y = 4.3$ and found the x–coordinate of the intersection point.

61. a.

Zeros	*Sum of zeros*	*Product of zeros*
2, 3	5	6
-3, 1, 2	0	-6
-3, 1, $\pm 2i$	-2	-12
-3, 2, 0, $2 \pm \sqrt{3}$	3	0

b. *Sample answer:* If $f(x)$ is a polynomial with leading coefficient 1 and degree n, where $n > 0$, then the sum of the roots is the opposite of the coefficient of the x^{n-1} term.

c. *Sample answer:* If $f(x)$ is a polynomial of degree n, where $n > 0$, then the product of the zeros is the constant term if n is even and the opposite of the constant term if n is odd.

62. $(a + bi) + (a - bi) = (a + a) + (bi - bi) = 2a$;

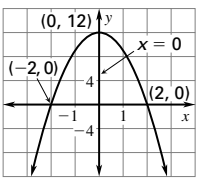
Since a is real, $2a$ must be real.

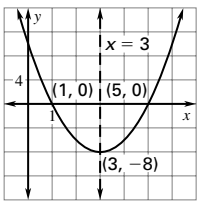
63. $(a + bi)(a - bi) = [a^2 + abi - abi + (bi)^2] = a^2 - b^2$;

Since a and b are real, $a^2 - b^2$ is real.

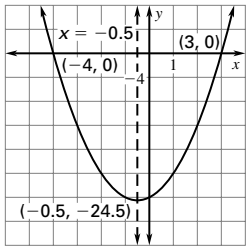
6.7 Mixed Review (p. 371)

64. $y = -3(x - 2)(x + 2)$ **65.** $y = 2(x - 1)(x - 5)$

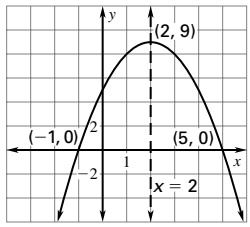




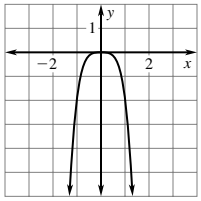
66. $y = 2(x + 4)(x - 3)$



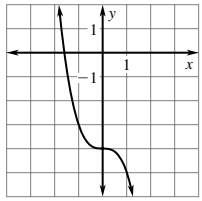
67. $y = -(x + 1)(x - 5)$



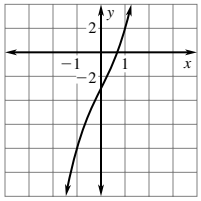
68. $f(x) = -2x^4$



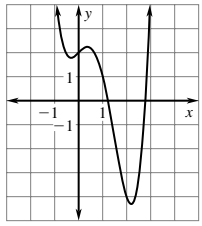
69. $y = -x^3 - 4$



70. $f(x) = x^3 + 4x - 3$



71. $f(x) = x^4 - 3x^3 + x + 2$



Developing Concepts Activity 6.8 (p. 372)

1. $-0.640, 1.135, 5.505$ **2.** -0.640 **3.** 5

4. $-0.219, 2.047, 14.839$

5. $-2.334, -0.742, 0.742, 2.334$

6. $-3.629, -0.629, 1.085, 18.173$

7. $-1.088, -0.668, 1.191$ **8.** $-0.735, 0.722, 1.326$

9. $-7.349, 16.429, 30.921$; yes

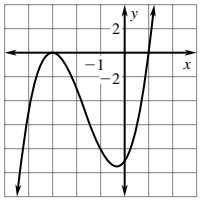
Lesson 6.8

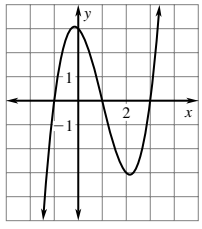
6.8 Guided Practice (p. 376)

1. The y-coordinate of a point of the graph that is higher than all nearby points.

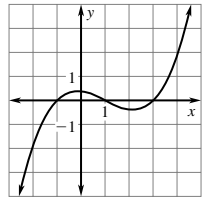
2. **a.** 4 **b.** 4 **c.** $-2, 6$ **3.** 4

4. $f(x) = (x - 1)(x + 3)^2$ **5.** $f(x) = (x - 1)(x + 1)(x - 3)$

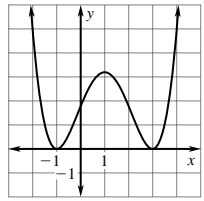




6. $f(x) = \frac{1}{8}(x + 1)(x - 1)(x - 3)$



7. $f(x) = \frac{1}{5}(x - 3)^2(x + 1)^2$



8. x-intercepts: $-1.40, -0.29$
local max: $(0.21, 1.21)$
local min: $(-1, -3), (0.79, 0.63)$

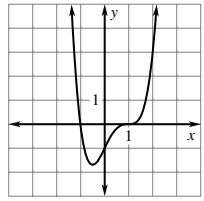
9. x-intercepts: $-0.41, 1, 2.41$
local max: $(0.18, 1.09)$
local min: $(1.82, -1.09)$

10. x-intercepts: $-1.19, 0, 1.69$
local max: $(1, 3)$
local min: $(-0.67, -1.63)$

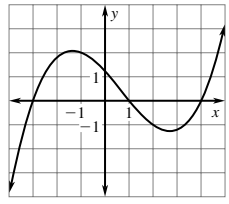
11. x-intercepts: $0, 1, 1.51$
local max: $(-1.59, -3.23), (0.49, 1.35)$
local min: $(-1, -4), (1.30, -0.79)$

12. **a.** $0 < x < 9$; the flaps can't be more than 9 in.
b. 3 in. **c.** 432 in.^{3}

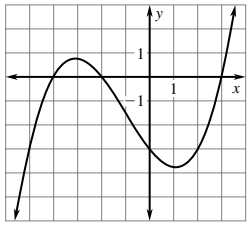
13. $f(x) = (x - 1)^3(x + 1)$



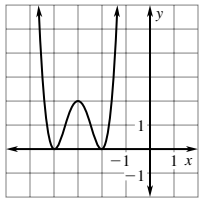
14. $f(x) = \frac{1}{10}(x + 3)(x - 1)(x - 4)$



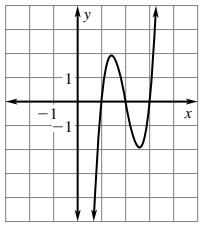
15. $f(x) = \frac{1}{8}(x + 4)(x + 2)(x - 3)$



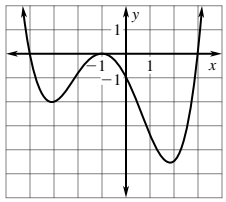
16. $f(x) = 2(x + 2)^2(x + 4)^2$



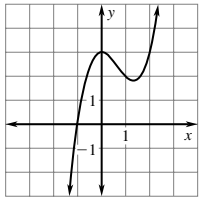
17. $f(x) = 5(x - 1)(x - 2)(x - 3)$



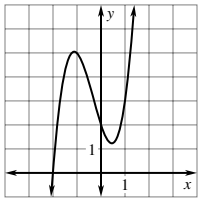
18. $f(x) = \frac{1}{12}(x + 4)(x - 3)(x + 1)^2$



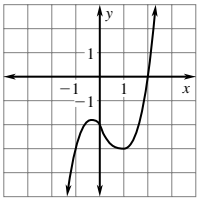
19. $f(x) = (x + 1)(x^2 - 3x + 3)$



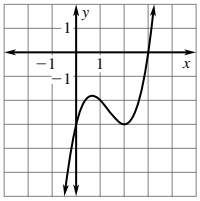
20. $f(x) = (x + 2)(2x^2 - 2x + 1)$



21. $f(x) = (x - 2)(x^2 + x + 1)$



22. $f(x) = (x - 3)(x^2 - x + 1)$



23. local max: $\left(-\frac{1}{2}, \frac{2}{3}\right)$
local min: $\left(\frac{1}{2}, -\frac{1}{3}\right)$
real zeros: $-1, 0, 1$
degree: 3

24. local max: $\left(-\frac{1}{2}, 5\right)$
local min: $(-2, 0), (1, 0)$
real zeros: $-2, 1$
degree: 4

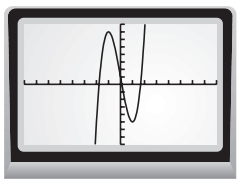
25. local max: $(0, 2)$
local min: $(-2, 1)$
real zeros: $\frac{4}{3}$
degree: 3

26. local max: $\left(-2, 2\frac{1}{2}\right), (1, -1)$
local min: $\left(0, -1\frac{1}{4}\right), \left(2\frac{1}{2}, -2\right)$
real zeros: $-2.5, -1, 3$
degree: 5

27. local max: $(-2, -1), (1, -2)$
local min: $(0, -2)$
real zeros: 0
degree: 4

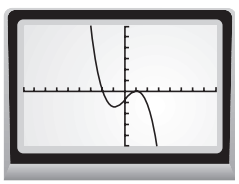
28. local max: $\left(-2\frac{1}{2}, 2\right), (2, 1)$
local min: $\left(0, \frac{1}{3}\right)$
real zeros: $-3\frac{2}{3}, 2\frac{2}{3}$
degree: 4

29. $f(x) = 3x^3 - 9x + 1$



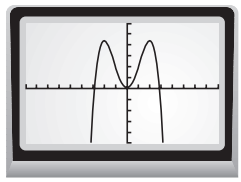
x-intercepts:
$-1.79, 0.11, 1.67$
local max: $(-1, 7)$
local min: $(1, -5)$

30. $f(x) = -\frac{1}{3}x^3 + x - \frac{2}{3}$



x-intercepts: $-2, 1$
local max: $(1, 0)$
local min: $\left(-1, -\frac{4}{3}\right)$

31. $f(x) = -\frac{1}{4}x^4 + 2x^2$

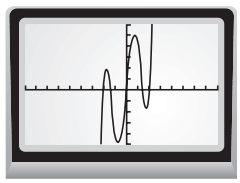


x-intercepts: $-2.83, 0, 2.83$

local max: $(-2, 4), (2, 4)$

local min: $(0, 0)$

32. $f(x) = x^5 - 6x^3 + 9x$

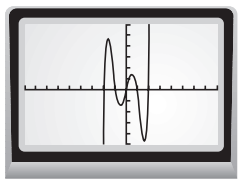


x-intercepts: $-1.73, 0, 1.73$

local max: $(-1.73, 0), (0.77, 4.46)$

local min: $(-0.77, -4.46), (1.73, 0)$

33. $f(x) = x^5 - 5x^3 + 4x$

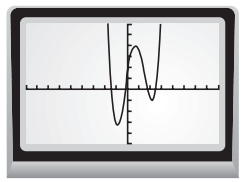


x-intercepts: $-2, -1, 0, 1, 2$

local max: $(-1.64, 3.63), (0.54, 1.42)$

local min: $(-0.54, -1.42), (1.64, -3.63)$

34. $f(x) = x^4 - 2x^3 - 3x^2 + 5x + 2$

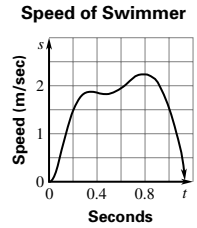


x-intercepts: $-1.53, -0.35, 1.88, 2$

local max: $(0.61, 3.62)$

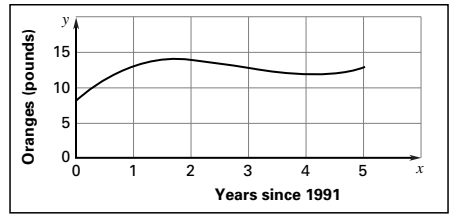
local min: $(-1.05, -3.03), (1.94, -0.03)$

35. $S = -241t^7 + 1062t^6 - 1871t^5 + 1647t^4 - 737t^3 + 144t^2 - 2.432t$



at about 0.8 seconds

36. $f(x) = 0.298x^3 - 2.73x^2 + 7.05x + 8.45$



The points are the average of oranges in pounds eaten in a given year since 1991.

37. $600 = \pi r^2 + \pi rl$

$\pi rl = 600 - \pi r^2$

$l = \dfrac{600 - \pi r^2}{\pi r}$

38. $V = \frac{1}{2}\pi r^2\left(\dfrac{600 - \pi r^2}{\pi r}\right)$

$V = \frac{1}{2}r(600 - \pi r^2)$

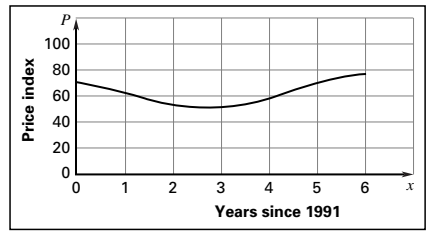
$V = 300r - \frac{1}{2}\pi r^3$

39. about 1600 ft^3

$r \approx 8$ ft

$l \approx 16$ ft

40. $P = -0.233x^4 + 2.64x^3 - 6.59x^2 - 3.93x + 69.1$

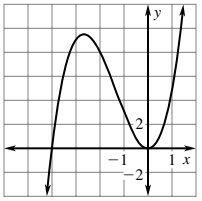


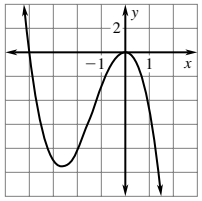
reaches a local min at (2.71, 50.03); the producer price index declined from 1991 to a low of about 50.03 around September 1993, after which it began to increase.

41. A polynomial with 3 turning points must be of degree four or higher.

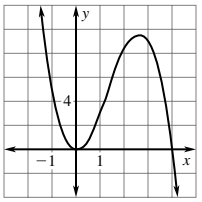
42. A **43.** B

44. $f(x) = x^3 + 4x^2$ $y = -f(x)$





$y = f(-x)$



6.8 Mixed Review (p. 378)

45. $y = 7x$ **46.** $y = -\frac{3}{2}x$ **47.** $y = \frac{1}{4}x$ **48.** $y = -\frac{5}{2}x$

49. $y = -\frac{3}{5}x$ **50.** $y = \frac{5}{2}x$ **51.** yes; 4×1 **52.** yes; 2×5

53. no **54.** yes; 6×5

55.
$y - 4 = a(x - 1)^2$
$-5 - 4 = a(4 - 1)^2$
$-9 = 9a$
$-1 = a$
$y = -(x - 1)^2 + 4$

56.
$y - 6 = a(x + 2)^2$
$-4 = 4a$
$-1 = a$
$y = -(x + 2)^2 + 6$

57.
$y = a(x + 5)(x - 5)$
$5 = a(7 + 5)(7 - 5)$
$\frac{5}{24} = a$
$y = \frac{5}{24}(x + 5)(x - 5)$

58.
$y = a(x + 2)(x - 4)$
$-4 = a(1 + 2)(1 - 4)$
$\frac{4}{9} = a$
$y = \frac{4}{9}(x + 2)(x - 4)$

59. $\frac{60 - 30}{3} = \frac{30}{3} = 10$ in./day

Lesson 6.9

Developing Concepts Activity (p. 379)
Drawing Conclusions

1. a. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

4 7 10 13 16 19

3 3 3 3 3

b. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

0 5 12 21 32 45

5 7 9 11 13

2 2 2 2

c. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

5 18 55 128 249 430

13 37 73 121 181

24 36 48 60

12 12 12

2. degrees: 1, 2, 3; 1, 2, 3; number of times differences were calculated before arriving at a row of constant, nonzero differences: 1, 2, 3; 1, 2, 3; the degree equals the number of times differences were calculated.

3. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

2 18 84 260 630 1302

16 66 176 370 672

50 110 194 302

60 84 108

24 24

6.9 Guided Practice (p. 383)

1. the differences between $f(n)$ and $f(n + 1)$; the differences of adjacent first-order differences

2. 5

3. because the points will not lie exactly on the curve generated by the model

4.
$2 = a(1 - 3)(1 + 1)(1 + 2)$
$2 = a(-12)$
$a = -\frac{1}{6}$
$f(x) = -\frac{1}{6}(x - 3)(x + 1)(x + 2)$

5. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

4 17 40 73 116 169

13 23 33 43 53

10 10 10 10

6. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

1 11 35 79 149 251

10 24 44 70 102

14 20 26 32

6 6 6

7. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

3 20 87 264 635 1308

17 67 177 371 673

50 110 194 302

60 84 108

24 24

8. $f(1)\ f(2)\ f(3)\ f(4)\ f(5)\ f(6)$

−12 −39 −66 −81 −72 −27

−27 −27 −15 9 45

0 12 24 36

12 12 12

9. −1 3 3 5 15 39

4 0 2 10 24

−4 2 8 14

6 6 6

3rd degree

10.

0	8	12	12	8	0	
	8	4	0	−4	−8	
		4	4	4	4	

2nd degree

11. $f(x) = -x^3 + 5x^2 + x + 1$ **12.** $f(x) = x^3 - 4x^2 + 2x$

13. $d = \frac{1}{2}n^2 - \frac{3}{2}n$

6.9 Practice and Applications (pp. 383–385)

14. $f(x) = a(x + 1)(x - 1)(x - 3)$

$3 = a(3)$

$1 = a$

$f(x) = (x + 1)(x - 1)(x - 3)$

15. $f(x) = a(x - 3)(x - 2)(x + 1)$

$-2 = a(-2)(-1)(2)$

$-\frac{1}{2} = a$

$f(x) = -\frac{1}{2}(x + 1)(x - 2)(x - 3)$

16. $f(x) = a(x + 3)(x + 1)(x - 3)$

$1 = a(1)(-1)(-5)$

$\frac{1}{5} = a$

$f(x) = \frac{1}{5}(x + 3)(x + 1)(x - 3)$

17. $f(x) = a(x + 1)(x + 2)(x + 0)$

$-3 = a(2)(3)(1)$

$-\frac{1}{2} = a$

$f(x) = -\frac{1}{2}(x + 1)(x + 2)(x + 0)$

18. $f(x) = a(x - 3)(x - 2)(x + 3)$

$-1 = a(-2)(-1)(4)$

$-\frac{1}{8} = a$

$f(x) = -\frac{1}{8}(x - 3)(x - 2)(x + 3)$

19. $f(x) = a(x - 1)(x - 3)(x + 2)$

$1 = a(1)(-1)(4)$

$-\frac{1}{4} = a$

$f(x) = -\frac{1}{4}(x - 1)(x - 3)(x + 2)$

20. $f(x) = a(x + 1)(x + 4)(x - 4)$

$3 = a(1)(4)(-4)$

$-\frac{3}{16} = a$

$f(x) = -\frac{3}{16}(x + 1)(x + 4)(x - 4)$

21. $f(x) = a(x - 3)(x - 2)(x + 1)$

$4 = a(-2)(-1)(2)$

$1 = a$

$f(x) = (x - 3)(x - 2)(x + 1)$

22. $f(x) = a(x + 0)(x + 3)(x - 5)$

$3 = a(-2)(1)(-7)$

$\frac{3}{14} = a$

$f(x) = \frac{3}{14}(x + 0)(x + 3)(x - 5)$

23.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
5	5	7	11	17	25
	0	2	4	6	8
		2	2	2	2

24.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
−4	−6	6	44	120	246
	−2	12	38	76	126
		14	26	38	50
			12	12	12

25.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
−3	−3	−9	−27	−63	−123
	0	−6	−18	−36	−60
		−6	−12	−18	−24
			−6	−6	−6

26.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
−2	−8	0	64	250	648
	−6	8	64	186	398
		14	56	122	212
			42	66	90
				24	24

27.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
−18	−8	102	432	1150	2472
	10	110	330	718	1322
		100	220	388	604
			120	168	216
				48	48

28.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
3	−8	−27	−54	−89	−132
	−11	−19	−27	−35	−43
		−8	−8	−8	−8

29.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
4	4	−36	−176	−500	−1116
	0	−40	−140	−324	−616
		−40	−100	−184	−292
			−60	−84	−108
				−24	−24

30.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
−4	2	34	110	248	466
	6	32	76	138	218
		26	44	62	80
			18	18	18

31.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
3	−2	−13	−30	−53	−82
	−5	−11	−17	−23	−29
		−6	−6	−6	−6

32. $f(x) = 3x^2 - 5x - 2$ **33.** $f(x) = -3x^2 + 20x$

34. $f(x) = x^3 - 3x^2 - 2$ **35.** $f(x) = x^3 - 4x^2 + x$

36. $f(x) = 0.5x^3 - 4x^2 + 3.5x - 3$

37. $f(x) = x^3 + 4x^2 - x - 2$

38. $f(x) = -x^3 + 8x^2 - 12x$

39. $f(x) = 2x^3 - 16x^2 + 37x - 25$

40. $f(x) = -2x^3 + 22x^2 - 74x + 74$

41. $f(x) = -x^3 + 10x^2 - 30x + 23$

42. $f(x) = x^4 - 15x^3 + 81x^2 - 183x + 142$

43. $f(x) = -x^4 + 13x^3 - 58x^2 + 104x - 58$

44.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
1	5	12	22	35	51
	4	7	10	13	16
		3	3	3	3

45.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
1	6	15	28	45	66
	5	9	13	17	21
		4	4	4	4

46. $f(n) = \frac{n}{6}(2n + 1)(n + 1)$

47. $f(t) = 0.641t^3 - 4.93t^2 + 25.8t + 232$ where t is the number of years since 1989.

$f(11) = 0.641(11)^3 - 4.93(11)^2 + 25.8(11) + 232$

$f(11) \approx 772.4$; about 772,000 Girl Scouts

48. $y = 0.242t^3 - 3.00t^2 + 13.5t + 140$ where t is the number of years since 1987.

$y = 0.242(13)^3 - 3.00(13)^2 + 13.5(13) + 140$

$y \approx 340.2$

about \$340,000

49. $y = 0.007t^3 + 0.740t^2 + 49t - 236$

$4400 = 0.007(t)^3 - 0.740(t)^2 + 49(t) - 236$

$0.007t^3 - 0.74t^2 + 49t - 4636 = 0$

$t \approx 101$

about 101 seconds

50. a. Dog-walking: $y = 7.5x^2 - 20.5x + 16$

Lawn-care: $y = 0.833x^3 - 4x^2 + 24.17x - 18$

b. *Sample answer:* Solve for y in both equations when $x = 12$. Dog walking profits in December equal \$850 while lawn care profits are \$1135.

51. a.

$f(x)$	$ax^3 + bx^2 + cx + d$			
		$3ax^2 + 3ax + a + 2bx + b + c$		
$f(x+1)$	$ax^3 + 3ax^2 + 3ax + a + bx^2 + 2bx + b + cx + c + d$		$6ax + 6a + 2b$	
		$3ax^2 + 9ax + 7a + 2bx + 3b + c$		$6a$
$f(x+2)$	$ax^3 + 6ax^2 + 12ax + 8a + bx^2 + 4bx + 4b + cx + 2c + d$		$6ax + 12a + 2b$	
		$3ax^2 + 15a + 19a + 2bx + 5b + c$		$6a$
$f(x+3)$	$ax^3 + 9ax^2 + 27ax + 27a + bx^2 + 6bx + 9b + cx + 3c + d$		$6ax + 18a + 2b$	
		$3ax^2 + 21ax + 37a + 2bx + 7b + c$		$6a$
$f(x+4)$	$ax^3 + 12ax^2 + 48a + 64a + bx^2 + 8b + 16b + cx + 4c + d$		$6ax + 24a + 2b$	
		$3ax^2 + 27ax + 61a + 2bx + 9b + c$		
$f(x+5)$	$ax^3 + 15ax^2 + 75ax + 125a + bx^2 + 10xb + 25b + cx + 5c + d$			

b. $f(x) = x^3 - 9x^2 + 22x - 15$

6.9 Mixed Review (p. 386)

52. $3x^2 = 6$

$x^2 = 2$

$x = \pm\sqrt{2}$

53. $16x^2 = 4$

$x^2 = \frac{1}{4}$

$x = \pm\frac{1}{2}$

54. $4x^2 = 14$

$x^2 = \frac{14}{4}$

$x = \pm\frac{\sqrt{14}}{2}$

55. $6x^2 = 13$

$x^2 = \frac{13}{6}$

$x = \pm\frac{\sqrt{78}}{6}$

56. $3x^2 = 15$

$x^2 = 5$

$x = \pm\sqrt{5}$

57. $2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \pm\frac{\sqrt{2}}{2}$

58. $x^2 + 12x + 36 = -27 + 36$

$(x + 6)^2 = 9$

$(x + 6) = \pm 3$

$x = -9, -3$

59. $x^2 + 6x + 9 = 24 + 9$

$(x + 3)^2 = 33$

$x + 3 = \pm\sqrt{33}$

$x = -3 \pm \sqrt{33}$

60. $x^2 - 3x + \frac{9}{4} = 18 + \frac{9}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{81}{4}$

$x - \frac{3}{2} = \pm\frac{9}{2}$

$x = 6, -3$

61. $x^2 + 4x + 4 = -\frac{11}{2} + 4$

$(x + 2)^2 = -\frac{3}{2}$

$x + 2 = \pm\frac{i\sqrt{6}}{2}$

$x = -2 \pm \frac{i\sqrt{6}}{2}$

62. $x^2 - 14x + 49 = 15 + 49$

$(x - 7)^2 = 64$

$x - 7 = \pm 8$

$x = 15, -1$

63. $x^2 - 6x + 9 = -\frac{32}{3} + \frac{27}{3}$

$(x - 3)^2 = -\frac{5}{3}$

$x - 3 = \pm\frac{i\sqrt{15}}{3}$

$x = 3 \pm \frac{i\sqrt{15}}{3}$

64. $(2x - 1)(4x^2 + 2x + 1)$ **65.** $(3x + 2)(9x^2 - 6x + 4)$

66. $8(3x + 2)(9x^2 - 6x + 4)$

67. $(2x - 5)(4x^2 + 10x + 25)$ **68.** $3(x - 2)(x^2 + 2x + 4)$

69. $8(x + 3)(x^2 - 3x + 9)$

70. $(3x + 10)(9x^2 - 30x + 100)$

71. $3(x + 3)(x^2 - 3x + 9)$

Quiz 3 (p. 386)

1. $f(x) = 2x^3 - x^2 - 22x - 15$

$x \approx -2.61, -0.74, 3.86$

2. $f(x) = x^3 + 3x^2 + 3x + 2$

$x = -2, \frac{-1 \pm i\sqrt{3}}{2}$

3. $f(x) = x^4 - 3x^3 - 2x^2 - 6x - 8$

$x = -1, 4, \pm i\sqrt{2}$

4. $f(x) = 2x^4 - x^3 - 8x^2 + x + 6$

$x = -\frac{3}{2}, -1, 1, 2$

5. $y = (x - 2)(x + 2)(x + 2)$

$= (x^2 - 4)(x + 2)$

$= x^3 + 2x^2 - 4x - 8$

6. $y = (x + 0)(x - 1)(x + 3)$

$= (x + 0)(x^2 + 2x - 3)$

$= x^3 + 2x^2 - 3x$

7. $y = (x - 4)(x - 2 - i)(x - 2 + i)$

$= (x - 4)(x - 2)^2 + 1$

$= x^3 - 8x^2 + 21x - 20$

8. $y = (x - 2)(x - 5)(x - i)(x + i)$

$= (x^2 - 7x + 10)(x^2 + 1)$

$= x^4 - 7x^3 + 11x^2 - 7x + 10$

9. $y = (x - 4)(x - 2 + 3i)(x - 2 - 3i)$

$= (x - 4)(x^2 - 4x + 13)$

$= x^3 - 8x^2 + 29x - 52$

10. $y = (x - 1 + i)(x - 1 - i)(x - 2 - 2i)(x - 2 + 2i)$

$= (x^2 - 2x + 2)(x^2 - 4x + 8)$

$= x^4 - 6x^3 + 18x^2 - 24x + 16$

11. local max: $(0.79, 8.21)$

local min: $(-2.12, -4.06)$

12. local max: $(-0.5, 0.56)$

local min: $(-1.62, -1), (0.62, -1)$

13. local max: $(2.42, 0.77)$

local min: $(3.58, -0.77)$

14. local max: $(-3, 0)$

local min: $(-1.67, -1.19)$

15. $y = a(x + 2)(x - 2)(x + 4)$

$3 = a(1)(-3)(3)$

$-\frac{1}{3} = a$

$y = -\frac{1}{3}(x + 2)(x - 2)(x + 4)$

16. $y = a(x + 1)(x - 4)(x - 2)$

$1 = a(-2)(-7)(-5)$

$-\frac{1}{70} = a$

$y = -\frac{1}{70}(x + 1)(x - 4)(x - 2)$

17. $y = a(x + 0)(x - 3)(x - 5)$

$6 = a(2)(-1)(-3)$

$1 = a$

$y = (x + 0)(x - 3)(x - 5)$

18. $y = a(x - 1)(x + 3)(x + 5)$

$10 = a(-5)(-1)(1)$

$2 = a$

$y = 2(x - 1)(x + 3)(x + 5)$

19. $f(x) = x^3 - 3x^2 + x - 4$ **20.** $f(x) = x^3 - 4x^2 + 2x$

21. $N = -3.75x^3 + 50.9x^2 - 97.3x + 3210$ where x is the number of years since 1988.

Chapter 6 Review *(pp. 388–390)*

1. $\frac{4}{9} \cdot \frac{216x^3}{y^3} = \frac{96x^3}{y^3}$; negative exponent, power of a quotient, power of a product, and power of a power properties

2. $\frac{x^4}{x^4} = 1$; negative exponent, product of powers, power of a power, and zero exponent properties

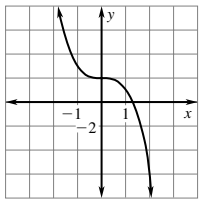
3. $\frac{-63xy^9}{18x^{-2}y^3} = -\frac{7}{2}x^3y^6$; quotient of powers property

4. $5x^2y^2 \cdot \frac{1}{25x^2y} = \frac{y}{5}$; negative exponent, quotient of powers, and zero exponent properties

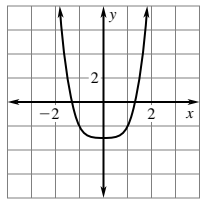
5.
$$\begin{array}{r|rrrr} 3 & 1 & 3 & -12 & 7 \\ & & 3 & 18 & 18 \\ \hline & 1 & 6 & 6 & 25 \end{array}$$

6.
$$\begin{array}{r|rrrrr} -1 & 1 & -5 & -3 & 1 & -5 \\ & & -1 & 6 & -3 & 2 \\ \hline & 1 & -6 & 3 & -2 & -3 \end{array}$$

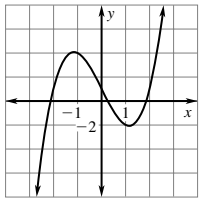
7. $f(x) = -x^3 + 2$



8. $f(x) = x^4 - 3$



9. $f(x) = x^3 - 4x + 1$



10. $(3x^3 + x^2 + 1) - (x^3 + 3) = 2x^3 + x^2 - 2$

11. $(x - 3)(x^2 + x - 7) = x^3 - 2x^2 - 10x + 21$

12. $(x + 3)(x - 5)(2x + 1) = (x^2 - 2x - 15)(2x + 1)$
$= 2x^3 - 3x^2 - 32x - 15$

13. $x^3 = -64$
$x = -4$

14. $x^4 - 6x^2 - 27 = 0$
$(x + 3)(x^3 - 3x^2 + 3x - 9) = 0$
$(x + 3)(x - 3)(x^2 + 3) = 0$
$x = -3, 3$

15. $x^2(x + 3) - (x + 3) = 0$
$(x + 3)(x^2 - 1) = 0$
$(x + 3)(x - 1)(x + 1) = 0$
$x = -3, -1, 1$

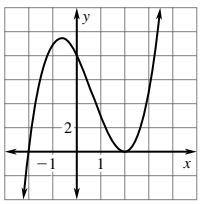
16.
$$x - 1 \overline{)\, x^4 + 5x^3 - x^2 - 3x - 1}$$
$$\begin{array}{r} x^3 + 6x^2 + 5x + 2 + \dfrac{1}{x-1} \\ \\ -x^4 + x^3 \\ \hline 6x^3 - x^2 \\ -6x^3 + 6x^2 \\ \hline 5x^2 - 3x \\ -5x^2 + 5x \\ \hline 2x - 1 \\ -2x + 2 \\ \hline 1 \end{array}$$

17.
$$2x - 5 \overline{)\, 2x^3 - 5x^2 + 5x + 4}$$
$$\begin{array}{r} x^2 + \dfrac{5}{2} + \dfrac{33}{2(2x-5)} \\ \\ -2x^3 + 5x^2 \\ \hline 5x \\ -5x + \frac{25}{2} \\ \hline \frac{33}{2} \end{array}$$

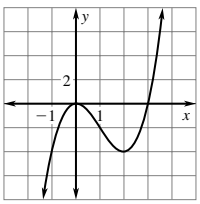
18. $f(x) = x^3 + 12x^2 + 21x + 10$
$= (x + 1)(x^2 + 11 + 10)$
$= (x + 1)^2(x + 10)$
$x = -1, -10$

19. $f(x) = x^4 + x^3 - x^2 + x - 2$
$= (x - 1)(x^3 + 2x^2 + x + 2)$
$= (x - 1)(x + 2)(x^2 + 1)$
$x = 1, -2$

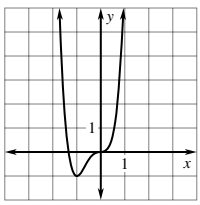
20. x-intercepts: $2, -2$
local max: $(-0.68, 9.5)$
local min: $(2, 0)$



21. x-intercepts: $0, 3$
local max: $(0, 0)$
local min: $(2, -4)$



22. x-intercepts: $0, -1.34$
local max: none
local min: $(-1, -1)$



23.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
2	9	28	65	126	217
	12	19	37	61	91
		12	18	24	30
			6	6	6

24. $y = a(x - 1)(x + 1)(x - 4)$

$-12 = a(1)(3)(-2)$

$2 = a$

$y = 2(x - 1)(x + 1)(x - 4)$

Chapter 6 Test *(p. 391)*

1. x^5; quotient of powers property

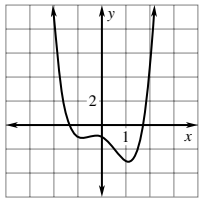
2. $729x^{18}$; power of a product and power of a power properties

3. x^{11}; quotient of powers property

4. $\frac{1}{512x^9y^6}$; power of a power, power of a product, and negative exponent properties

5. $\frac{3}{y^3}$; product of a power, quotient of a power, zero exponent, and negative exponent properties

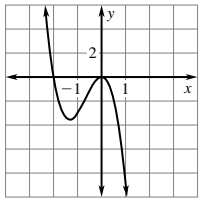
6. $y = x^4 - 2x^2 - x - 1$



$f(x) \to +\infty$ as $x \to -\infty$,

$f(x) \to +\infty$ as $x \to +\infty$

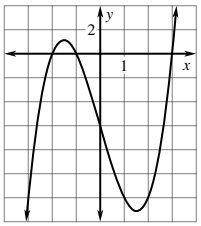
7. $y = -3x^3 - 6x^2$



$f(x) \to +\infty$ as $x \to -\infty$,

$f(x) \to -\infty$ as $x \to +\infty$,

8. $y = (x - 3)(x + 1)(x + 2)$



$f(x) \to -\infty$ as $x \to -\infty$,

$f(x) \to +\infty$ as $x \to +\infty$

9. $x^2 - 14x + 8$

10. $10x^3 - 17x^2 + 15x - 18$ **11.** $x^3 - 13x - 12$

12. $64x^3 + 343 = (4x + 7)(16x^2 - 28x + 49)$

13. $400x^2 - 25 = 25(4x - 1)(4x + 1)$

14. $x^4 + 8x^2 - 9 = (x^2 + 9)(x - 1)(x + 1)$

15. $2x^3 - 3x^2 + 4x - 6 = (2x - 3)(x^2 + 2)$

16. $3x^4 - 11x^2 - 20 = 0$

$(3x^2 + 4)(x^2 - 5) = 0$

$x = \pm\sqrt{5}, \pm\frac{2i\sqrt{3}}{3}$

17. $81x^4 - 16 = 0$

$(9x^2 - 4)(9x^2 + 4) = 0$

$(3x - 2)(3x + 2)(9x^2 + 4) = 0$

$x = \pm\frac{2}{3}, \pm\frac{2}{3}i$

18. $4x^3 - 8x^2 - x + 2 = 0$

$(x - 2)(4x^2 - 1) = 0$

$(x - 2)(2x - 1)(2x + 1) = 0$

$x = 2, \frac{1}{2}, -\frac{1}{2}$

19.

-1	8	5	4	-1	7
		-8	3	-7	8
	8	-3	7	-8	15

$8x^3 - 3x^2 + 7x - 8 + \frac{15}{x + 1}$

20.

-3	12	31	-17	-6
		-36	15	6
	12	-5	-2	0

$12x^2 - 5x - 2$

21. $0, \pm1, \pm2, \pm7, \pm14$;

$f(x) = x^3 - 5x^2 - 14$

$= (x + 0)(x^2 - 5x - 14)$

$= (x + 0)(x - 7)(x + 2)$

$x = 0, 7, -2$

22. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

$f(x) = x^3 + 4x^2 + 9x + 36$

$= (x + 4)(x^2 + 9)$

$x = -4, \pm3i$

23. $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$;

$f(x) = x^4 + x^3 - 2x^2 + 4x - 24$

$= (x + 3)(x^3 - 2x^2 + 4x - 8)$

$= (x + 3)(x - 2)(x^2 + 4)$

$x = -3, 2, \pm2i$

24. $f(x) = (x - 1)(x + 3)(x - 4)$

$= (x^2 + 2x - 3)(x - 4)$

$= x^3 - 2x^2 - 11x + 12$

25. $f(x) = (x - 2)^2(x + 1)x$

$= (x^2 - 4x + 4)(x^2 + x)$

$= x^4 - 3x^3 + 4x$

26. $f(x) = (x - 5)(x^2 + 4)$

$= x^3 - 5x^2 + 4x - 20$

27. $f(x) = (x^2 - 9)(x - 2 + i)(x - 2 - i)$

$= (x^2 - 9)(x^2 - 4x + 5)$

$= x^4 - 4x^3 - 4x^2 + 36x - 45$

28. $f(x) = 0.25x^3 - 7x^2 + 15$

$x = -1.428, 1.505, 27.923$

29. $f(x) = \frac{1}{9}(x - 3)^2(x + 3)^2$

x-intercepts: ± 3

local max: $(0, 9)$

local min: $(-3, 0), (3, 0)$

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

30.

$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
7	20	83	256	623	1292
	13	63	173	367	669
		50	110	194	302
			60	84	108
				24	24

31. $f(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$

32. $(7.5 \times 10^{13})(1.0 \times 10^{-3}) = 7.5 \times 10^{10}$ in. $\times \frac{1 \text{ mi}}{12 \text{ in.}} \times \frac{1 \text{ mi}}{5280 \text{ ft}}$

$= 1.1837 \times 10^6$ mi

Chapter 6 Standardized Test *(pp. 392–393)*

1. $-4^0 = -1$

D

2. $f(x) = 7x^4 - 3x^3 + 8x^2 + x - 9$

$f(-1) = 7 + 3 + 8 - 1 - 9$

$f(-1) = 8$

A

3. $f(x) = x^4 + 1$

$f(x) \to +\infty$ as $x \to -\infty$

A

4. $f(t) = 0.141t^4 - 5.577t^3 + 790.32t^2 + 5382.6t + 343{,}539$

$f(2) \approx 3.8 \times 10^5$

C

5. $(2x + 1)(4x^2 - 2x + 1) = 8x^3 + 1$

E

6. $x^5 = 246x$

$x = 0, \pm 4$

A

7.

4	4	−11	−9	−5
		16	20	44
	4	5	11	39

$x^2 + 5x + 11 + \frac{39}{x - 4}$

B

8. $f(x) = x^3 - 8x^2 + x + 42$

$f(x) = (x + 2)(x - 3)(x - 7)$

$x = -2, 3, 7$

E

9. $f(x) = -3x^4 + x + 2$

4 zeros

E

10. $y = a(x + 3)(x - 1)(x - 4)$

$7 = a(12)$

$\frac{7}{12} = a$

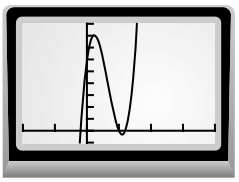
$y = \frac{7}{12}(x + 3)(x - 1)(x - 4)$

D

11. D **12.** A

13. a. $f(x) = x^3 - 10x^2 + 19x + 30$

b.



c. no; the local max occurs at about $(1.15, 40.15)$ and the local min occurs at about $(5.52, -1.63)$, but x must be greater than 6 for the side of length $x - 6$ to have a positive measure.

d. $0 = x^3 - 10x^2 + 19x - 190$

$x = 10$

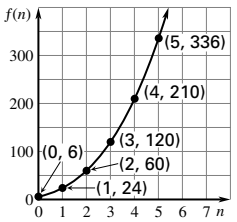
11 ft by 5 ft by 4 ft

14. a. 3 **b.** $f(n) = n^3 + 6n^2 + 11n + 6$

c. $f(n) = (n + 1)(n + 2)(n + 3)$; for prism n, the dimensions are $(n + 1)$ by $(n + 2)$ by $(n + 3)$

d. $f(49) = (50)(51)(52) = 132{,}600$

e.



The domain is all whole numbers.

Chapter 6 *continued*

Cumulative Practice, Chapters 1–6 (pp. 394–395)

1. $5x + 4 = -21$

$5x = -25$

$x = -5$

2. $3(2x + 5) = 69$

$6x + 15 = 69$

$6x = 54$

$x = 9$

3. $|x - 2| = 6$

$x = -4$ or $x = 8$

4. $|7 - 3x| = 23$

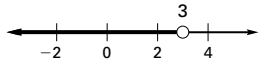
$-3x = -30$ or $-3x = 16$

$x = 10$ $\quad x = \frac{16}{-3}$

5. $10 - 4x > -2$

$-4x > -12$

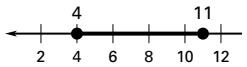
$x < 3$



6. $0 \le 2x - 8 \le 14$

$8 \le 2x \le 22$

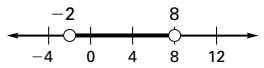
$4 \le x \le 11$



7. $|x - 3| < 5$

$-5 < x - 3 < 5$

$-2 < x < 8$



8. $|5x + 2| \ge 17$

$5x + 2 \le -17$ or $5x + 2 \ge 17$

$5x \le -19$ $\quad 5x \ge 15$

$x \le -\frac{19}{5}$ $\quad x \ge 3$

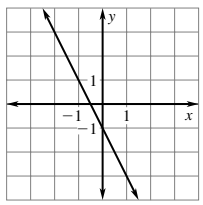
9. $m = \dfrac{1 - 1}{4 + 2} = 0$

10. $m = \dfrac{2 - 0}{0 + 3} = \dfrac{2}{3}$

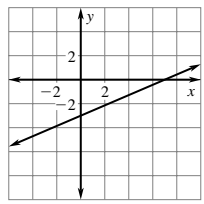
11. $m = \dfrac{-5 - 7}{-1 - 2} = \dfrac{-12}{3} = 4$

12. $m = \dfrac{4 + 3}{-4 - 1} = -\dfrac{7}{5}$

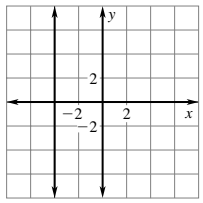
13. $y = -2x - 1$



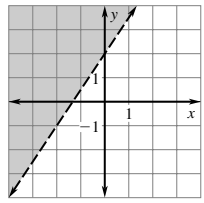
14. $3x - 7y = 21$



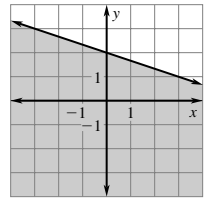
15. $x = -4$



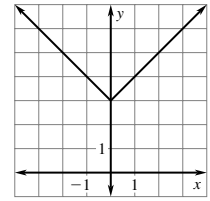
16. $y > \frac{3}{2}x + 2$



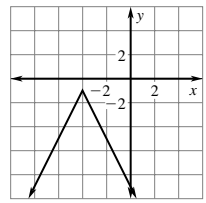
17. $2x + 6y \le 12$



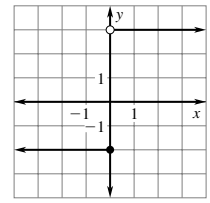
18. $y = |x| + 3$



19. $y = -2|x + 4| - 1$

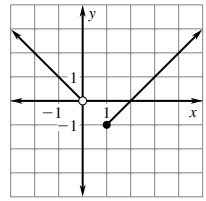


20. $f(x) = \begin{cases} -2, & \text{if } x \le 0 \\ 3, & \text{if } x > 0 \end{cases}$



21. $f(x) = \begin{cases} -x, & \text{if } x < 1 \\ x - 2, & \text{if } x \ge 1 \end{cases}$

22. $y = 3x - 2$



23. $y - 1 = \dfrac{1 - 9}{1 + 1}(x - 1)$

$y = -4x + 5$

24. $x = -8$

25. $y = 8 - x$

$2x - (8 - x) = 1$

$3x = 9$

$x = 3$

$y = 5$

26. $3x - 4y = 5$

$\underline{4x + 4y = 2}$

$7x = 7$

$x = 1$

$2 + 2y = 1$

$y = -\frac{1}{2}$

27. $\det A = -35$

$$x = \frac{\begin{vmatrix} 4 & 1 & 1 \\ 10 & -4 & 3 \\ -1 & 1 & 1 \end{vmatrix}}{-35} = \frac{-35}{-35} = 1,$$

$$y = \frac{\begin{vmatrix} 1 & 4 & 1 \\ 1 & 10 & 3 \\ -4 & -1 & 1 \end{vmatrix}}{-35} = \frac{0}{-35} = 0,$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 4 \\ 1 & -4 & 10 \\ -4 & 1 & -1 \end{vmatrix}}{-35} = \frac{-105}{-35} = 3$$

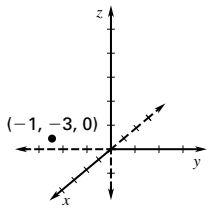
28. $\det A = -6$

$$x = \frac{\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 2 \\ -3 & 1 & 1 \end{vmatrix}}{-6} = \frac{12}{-6} = -2,$$

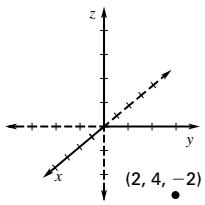
$$y = \frac{\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 2 \\ 1 & -3 & 1 \end{vmatrix}}{-6} = \frac{12}{-6} = -2,$$

$$z = \frac{\begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & 2 \\ 1 & 1 & -3 \end{vmatrix}}{-6} = \frac{-6}{-6} = 1$$

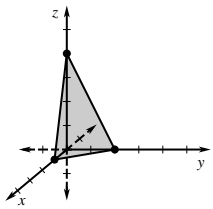
29.



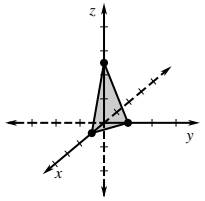
30.



31.



32.



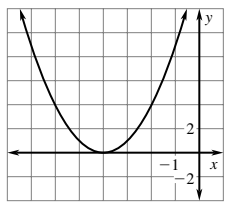
33. $\begin{bmatrix} -11 & 8 \\ 1 & -2 \end{bmatrix}$ **34.** $\begin{bmatrix} -12 & -18 \\ 24 & 12 \\ 30 & 6 \end{bmatrix}$

35. $\begin{bmatrix} 17 & -7 & -27 \\ 3 & -9 & 69 \end{bmatrix}$ **36.** 2 **37.** 3 **38.** 306 **39.** -55

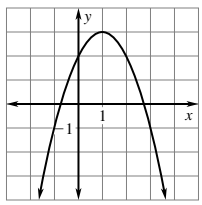
40. $\begin{bmatrix} -3 & 2 \\ -7 & 5 \end{bmatrix}$ **41.** $\begin{bmatrix} 7 & 2 \\ -4 & -1 \end{bmatrix}$ **42.** $\begin{bmatrix} -2 & \frac{9}{2} \\ 1 & -2 \end{bmatrix}$

43. no inverse; det = 0

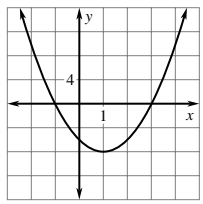
44. $y = x^2 + 8x + 16$



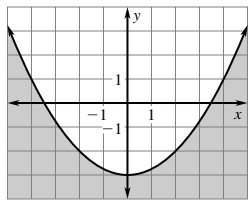
45. $y = -(x - 1)^2 + 3$



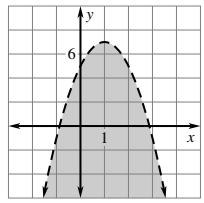
46. $y = 2(x + 1)(x - 3)$



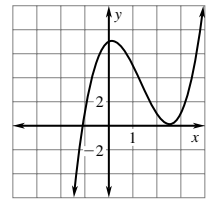
47. $y \leq \frac{1}{4}x^2 - 3$



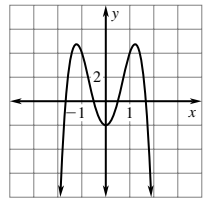
48. $y < -2x^2 + 4x + 5$



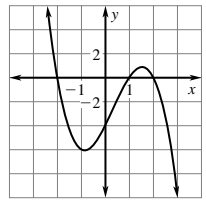
49. $y = x^3 - 4x^2 + x + 7$



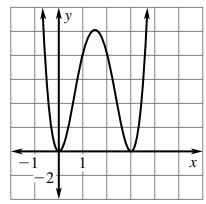
50. $y = -3x^4 + 9x^2 - 2$



51. $y = -(x + 2)(x - 1)(x - 2)$



52. $y = 2x^2(x - 3)^2$



53. $3x^2 - 7 = 2x^2 + 6$

$x^2 = 13$

$x = \pm\sqrt{13}$

54. $4x^2 + 12x + 9 = 0$

$(2x + 3) = 0$

$x = -\frac{3}{2}$

55. $x^2 = -64$

$x = \pm 8i$

56. $x^2 + 4x - 4 = 0$

$(x + 2)^2 = 8$

$x + 2 = \pm 2\sqrt{2}$

$x = -2 \pm 2\sqrt{2}$

57. $100 \geq x^2$

$-10 \leq x \leq 10$

58. $x^2 + 5x - 6 > 0$

$(x + 6)(x - 1) > 0$

$x < -6$ or $x > 1$

59. $x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$x = \pm 2$ or $x = \pm 1$

60. $3x^3(x - 5) = 0$

$x = 0, 5$

61. $2x^2(x + 2) - 3(x + 2) = 0$

$(x + 2)(2x^2 - 3) = 0$

$x = -2, \pm\frac{\sqrt{6}}{2}$

62. $\frac{7 + 3i}{4 - i}\left(\frac{4 + i}{4 + i}\right) = \frac{25 + 19i}{17}$

63. $4i(5 - 8i) = 32 + 20i$

64. $(9 + 5i)(9 - 5i) = 81 + 25 = 106$

65. $(6 - 2i) - (-3 - 4i) = 9 + 2i$

66. $y = a(x - 5)^2 + 3$

$11 = a(2)^2 + 3$

$8 = 4a$

$y = 2(x - 5)^2 + 3$

67. $y = a(x + 3)(x + 2)$

$-6 = a(6)$

$-1 = a$

$y = -(x + 3)(x + 2)$

68. $a + b + c = 4$

$9a + 3b + c = -4$

$36a + 6b + c = -61$

$y = -3x^2 + 8x - 1$

69. $36x^2y^6$

70. $\frac{7y^4}{x^{10}}$ **71.** $\frac{16}{25}$ **72.** $5x^3y^3$

73. $x^4 - 5x^3 + 11x^2 - 27x + 36$ **74.** $12x^3 + 2$

75.

$$-2 \left| \begin{array}{ccccc} 1 & -3 & 8 & 0 & -2 \\ & -2 & 10 & -36 & 72 \\ \hline 1 & -5 & 18 & -36 & 70 \end{array} \right.$$

$x^3 - 5x^2 + 18x - 36 + \frac{70}{x + 2}$

76. $f(x) = 2x^3 - 5x^2 - 4x + 3$

$= (x + 1)\left(x - \frac{1}{2}\right)(x - 3)$

$x = -1, \frac{1}{2}, 3$

77. $f(x) = x^4 - 25$

$= (x^2 - 5)(x^2 + 5)$

$x = \pm\sqrt{5}, \pm i\sqrt{5}$

78. $f(x) = x^3 + 11x^2 + x + 11$

$= (x^2 + 1)(x + 11)$

$x = \pm i, -11$

79. $y = a(x + 4)(x + 1)(x - 1)$

$6 = a(2)(-1)(-3)$

$1 = a$

$y = (x + 4)(x + 1)(x - 1)$

80. $y = a(x + 6)(x + 0)(x - 3)$

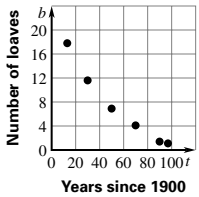
$-144 = a(12)(6)(3)$

$-\frac{2}{3} = a$

$y = \frac{-2}{3}(x + 6)(x + 0)(x - 3)$

81. $r = \frac{I}{Pt}$; $r = \frac{165}{3000} = 5.5\%$

82. negative correlation



83. after 8 minutes

84. $A = \begin{bmatrix} -2 & 5 \\ 1 & 8 \end{bmatrix}$

$E\ X\ [5 \quad 24]\begin{bmatrix} -2 & 5 \\ 1 & 8 \end{bmatrix} = 14,217$

$I\ T\ [9 \quad 20]\begin{bmatrix} -2 & 5 \\ 1 & 8 \end{bmatrix} = 2,205$

$\_\_\_\ N\ [0 \quad 14]\begin{bmatrix} -2 & 5 \\ 1 & 8 \end{bmatrix} = 14,112$

$O\ W\ [15 \quad 23]\begin{bmatrix} -2 & 5 \\ 1 & 8 \end{bmatrix} = -7,259$

85. $\frac{3.66 \times 10^9 \text{ mi}}{6.71 \times 10^8 \text{ mi/h}} = 5.45 \text{ h}$

Project Chapters 1–6 (pp. 396–397)

1. yes; The resulting matrix is a magic square with a magic constant of 21.
2. yes; The result is always a magic square. The magic constant in terms of a is $3(a + 5)$.
3. Answers may vary.
4. yes; The result is a magic square with a magic constant of 30.
5. The transpose is also a magic square with a magic constant of 15.
6. The diagonal gives us a magic constant of 34, so the magic square can be completed using trial and error until a match is found.

$$\begin{matrix} 7 & 2 & 11 & 14 \\ 12 & 13 & 8 & 1 \\ 6 & 3 & 10 & 15 \\ 9 & 16 & 5 & 4 \end{matrix}$$

7. The sum of the entries are 45 for the 3×3 magic square and 136 for the 4×4 magic square.
8. $S = \frac{1}{2}n^4 + \frac{1}{2}n^2$ is a quartic function.
9. $M = \frac{1}{2}n^3 + \frac{1}{2}n$ is a cubic function.

   Extension:

   $S = \frac{1}{2}n^4 + \left(a - \frac{1}{2}\right)n^2 \qquad M = \frac{1}{2}n^3 + \left(a - \frac{1}{2}\right)n$