

CHAPTER 4

Think & Discuss (p. 197)

1.
$$\begin{array}{r} 216.46 \\ - 204.96 \\ \hline \end{array}$$
 \$11.50 billion increase
- $$\begin{array}{r} 168.36 \\ - 167.14 \\ \hline \end{array}$$
 \$1.22 billion decrease

2. **Change in Sales from 1997 to 1998**

Age	Sales (billions)
10-14	16.09
15-19	11.50
20-24	-1.22
25-29	13.44
30-34	21.98
35-39	31.00
40-44	6.35
45+	46.67

Study Guide (p. 198)

1. $4 + (-5) = -1$ 2. $-10 - 3 = -13$
3. $(-1)6 - 4(2) = -6 - 8 = -14$
4. $12(5) + 2(-10) = 60 - 20 = 40$
5. commutative property of multiplication
6. commutative property of addition
7. distributive property of subtraction
8. $2x = 30$ 9. $x = 7 + y$
 $x = 15$ $3(7 + y) - 7y = 61$
 $x + 4y = 27$ $21 + 3y - 7y = 61$
 $15 + 4y = 27$ $-4y = 40$
 $4y = 12$ $y = -10$
 $y = 3$ $x = 7 - 10 = -3$
 (15, 3) (-3, -10)
10. $-x + 2y = -24$ 11. $9x + 3y = -24$
 $x + 3y = 20$ $8x - 3y = -10$
 $0 + 5y = -4$ $17x = -34$
 $y = -\frac{4}{5}$ $x = -2$
 $x - 2(-\frac{4}{5}) = 24$ $3(-2) + y = -8$
 $x = \frac{112}{5}$ $-6 + y = -8$
 $(\frac{112}{5}, -\frac{4}{5})$ $y = -2$
 (-2, -2)

Lesson 4.1

4.1 Guided Practice (p. 203)

1. A matrix is a rectangular arrangement of numbers in rows and columns.

A row matrix is a matrix with only 1 row: $[3 \ -2 \ 0 \ 4]$

A column matrix is a matrix with only 1 column: $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

A square matrix is a matrix with the same number of rows as columns: $\begin{bmatrix} 1 & 2 & 3 \\ 8 & 0 & 1 \\ 2 & -1 & 2 \end{bmatrix}$

2. Yes; the dimensions of both matrices are 3×2 and all the corresponding entries are equal.
3. The matrices must have the same dimensions.

$$4. -2 \left(\begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix} \right) = -2 \begin{bmatrix} -3 & 3 \\ 6 & -5 \\ -6 & 11 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -12 & 10 \\ 12 & -22 \end{bmatrix};$$

no; part (b) in Example 3 is $-2A + B$, which is not the same as $-2(A + B)$

5.

	This Year		
	Comprehensive	HMO Standard	HMO Plus
Individual	\$694.32	\$451.80	\$489.48
Family	\$1725.36	\$1187.76	\$1248.12

	Next Year		
	Comprehensive	HMO Standard	HMO Plus
Individual	\$683.91	\$463.10	\$499.27
Family	\$1699.48	\$1217.45	\$1273.08

6. $\begin{bmatrix} 20 \\ -22 \\ 9 \end{bmatrix} - \begin{bmatrix} -11 \\ -10 \\ -6 \end{bmatrix} = \begin{bmatrix} (20 + 11) \\ (-22 + 10) \\ (9 + 6) \end{bmatrix} = \begin{bmatrix} 31 \\ -12 \\ 15 \end{bmatrix}$
7. $\begin{bmatrix} -6 & -7 & 4 \\ -4 & 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -5 & 8 \\ 9 & 12 & -9 \end{bmatrix}$
 $= \begin{bmatrix} (-6 - 1) & (-7 - 5) & (4 + 8) \\ (-4 + 9) & (0 + 12) & (-1 - 9) \end{bmatrix}$
 $= \begin{bmatrix} -7 & -12 & 12 \\ 5 & 12 & -10 \end{bmatrix}$
8. $-4 \begin{bmatrix} 2 & 0 \\ -4 & -5 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 16 & 20 \end{bmatrix}$

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$$9. 6 \begin{bmatrix} -5 & -1 \\ 2 & 0 \end{bmatrix} - 5 \begin{bmatrix} -1 & 0 \\ 4 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} -30 & -6 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -5 & 0 \\ 20 & -15 \end{bmatrix} = \begin{bmatrix} (-30+5) & (-6-0) \\ (12-20) & (0+15) \end{bmatrix}$$

$$= \begin{bmatrix} -25 & -6 \\ -8 & 15 \end{bmatrix}$$

$$10. 1.04 \begin{bmatrix} \$683.91 & \$1699.48 \\ \$463.10 & \$1217.45 \\ \$499.27 & \$1273.08 \end{bmatrix} = \begin{bmatrix} \$711.27 & \$1767.46 \\ \$481.62 & \$1266.15 \\ \$519.24 & \$1324.00 \end{bmatrix}$$

Monthly payments:

$$\frac{1}{12} \begin{bmatrix} \$711.27 & \$1767.46 \\ \$481.62 & \$1266.15 \\ \$519.24 & \$1324.00 \end{bmatrix} = \begin{bmatrix} \$59.27 & \$147.29 \\ \$40.14 & \$105.51 \\ \$43.27 & \$110.33 \end{bmatrix}$$

4.1 Practice and Applications (pp. 203–206)

11. not equal 12. equal 13. not equal 14. equal

$$15. \begin{bmatrix} (1+3) & (-4+5) \\ (-7-5) & (2+2) \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ -12 & 4 \end{bmatrix}$$

16. Not possible; the two matrices do not have the same dimensions.

$$17. \begin{bmatrix} (-8+4) & (-2-5) \\ (6-1) & (-6+1) \end{bmatrix} = \begin{bmatrix} -4 & -7 \\ 5 & -5 \end{bmatrix}$$

$$18. \begin{bmatrix} (-3+2) & (5-7) \\ (0-4) & (-1+9) \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -4 & 8 \end{bmatrix}$$

$$19. \begin{bmatrix} (1.2+4.1) & (3.5+8.7) \\ (0.2+2.6) & (5.1+5.3) \end{bmatrix} = \begin{bmatrix} 5.3 & 12.2 \\ 2.8 & 10.4 \end{bmatrix}$$

$$20. \begin{bmatrix} (7-3) & (-1+6) & (4+3) \\ (11+10) & (-9+1) & (2-5) \end{bmatrix} = \begin{bmatrix} 4 & 5 & 7 \\ 21 & -8 & -3 \end{bmatrix}$$

21. Not possible; the two matrices do not have the same dimensions.

$$22. \begin{bmatrix} (\frac{1}{2}-2) & (\frac{1}{4}-\frac{3}{4}) \\ (3-\frac{1}{2}) & (8-5) \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & -\frac{1}{2} \\ \frac{5}{2} & 3 \end{bmatrix}$$

$$23. \begin{bmatrix} 4 & 12 & -28 \\ 16 & 0 & -24 \end{bmatrix} \quad 24. \begin{bmatrix} -10 & -30 \\ 15 & 5 \end{bmatrix}$$

$$25. \begin{bmatrix} 4 & 12 & 36 \\ -20 & 20 & 60 \\ -12 & -20 & -44 \end{bmatrix} \quad 26. \begin{bmatrix} 0 & 0 \\ 18 & 18 \\ -3 & -4 \end{bmatrix}$$

$$27. \begin{bmatrix} -1 & -1 & 2 \\ \frac{1}{8} & \frac{3}{11} & -5 \end{bmatrix} \quad 28. \begin{bmatrix} -21.5 & 8.5 \\ 3 & -12.75 \\ -12 & 11 \\ 25 & -20 \end{bmatrix}$$

$$29. \begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 12 & -8 \\ -16 & 20 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & -3 \\ -16 & 23 \end{bmatrix}$$

30.

$$\begin{bmatrix} -12 & -20 & 4 \\ 8 & -14 & -8 \end{bmatrix} - \begin{bmatrix} -1 & 5 & 13 \\ -3 & -6 & 19 \end{bmatrix} = \begin{bmatrix} -11 & -25 & -9 \\ 11 & -8 & -27 \end{bmatrix}$$

$$31. \begin{bmatrix} 14 & -14 \\ -2 & 6 \end{bmatrix} + \begin{bmatrix} 8 & -16 \\ -20 & -24 \end{bmatrix} = \begin{bmatrix} 22 & -30 \\ -22 & -18 \end{bmatrix}$$

$$32. \begin{bmatrix} -21 & 3 & 0 \\ 24 & -18 & -6 \end{bmatrix} - \begin{bmatrix} 8 & -2 & -14 \\ -6 & -10 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} -29 & 5 & 14 \\ 30 & -8 & -16 \end{bmatrix}$$

$$33. -2x = 6 \quad 34. 3x - 4 = -16 \quad -1 - 7 = y$$

$$x = -3 \quad 3x = -12 \quad -8 = y$$

$$-8 = y \quad x = -4$$

$$35. \begin{bmatrix} -6x & 8x \\ -22x & 10x \end{bmatrix} = \begin{bmatrix} 12 & -16 \\ y & -20 \end{bmatrix} \quad -6x = 12$$

$$x = -2$$

$$-22x = y$$

$$-22(-2) = y$$

$$44 = y$$

$$36. -3 + x = -8 \quad 8 - 7 = y$$

$$x = -5 \quad y = 1$$

37–41. Matrices can also be written with the rows and columns switched.

	Before All-Stars		After All-Stars	
	Wins	Losses	Wins	Losses
Braves	59	29	47	27
Mariners	37	51	39	34
Cubs	48	39	42	34

	Totals	
	Wins	Losses
Braves	106	56
Mariners	76	85
Cubs	90	73

	1996		1997	
	Number shipped (millions)	Value (millions)	Number shipped (millions)	Value (millions)
CDs	20,779	\$268,441	26,277	\$344,697
Cassettes	15,299	\$122,329	17,799	\$144,645
Videos	45	\$916	70	\$1260

	Total		41. Changes from 1996 to 1997	
	Number shipped (millions)	Value (millions)	Number shipped (millions)	Value (millions)
	47,056	\$613,138	5498	\$76,256
	33,098	\$266,974	2500	\$22,316
	115	\$2176	25	\$344

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42. Total Cost

	1995	1996	1997
Public 2 yr college	4,136	4,217	4,411
Public 4 yr college	6,671	7,014	7,331
Private 2 yr college	11,170	11,563	11,889
Private 4 yr college	16,602	17,611	18,475

43. Total Scores

	Sophomores	Juniors
1993	146.8	148.4
1994	146.1	147.8
1995	146.8	148.4
1996	146.2	148.1

$2V + M$

44. $\frac{148.4 + 147.8 + 148.4 + 148.1}{4} = 148.175$

45. 1991

	0-17	18-65	over 65
Northeast	4.8	12.6	2.8
Midwest	6.3	14.5	3.1
South	8.9	21.2	4.3
Mountain	1.6	3.4	0.6
Pacific	4.2	9.9	1.7

2010

	0-17	18-65	over 65
Northeast	4.2	11.4	2.5
Midwest	5.3	13.8	3.0
South	8.5	22.6	5.0
Mountain	1.7	4.2	0.9
Pacific	4.6	10.5	1.9

46. Change (percentage)

-0.6	-1.2	-0.3
-1.0	-0.7	-0.1
-0.4	1.4	0.7
0.1	0.8	0.3
0.4	0.6	0.2

47. South: 18-65, over 65

Mountain: 0-17, 18-65, over 65

Pacific: 0-17, 18-65, over 65

48. a. $B - A = \begin{bmatrix} -46,000 & 12.17 \\ 111,000 & 15.42 \\ 2,000 & -4.06 \\ -20,000 & -4.38 \end{bmatrix}$

111,000 more; \$4.06 less

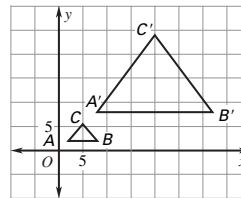
b. Volumes Average Price

Art	2,186,000	94.63
Law	1,543,000	161.60
Music	504,000	82.48
Travel	378,000	72.22

yes; no; For each subject area, adding the volumes sold in 1995 and 1996 gives the total of volumes sold over the two year period, but adding the price does not give the total average price over the two year period.

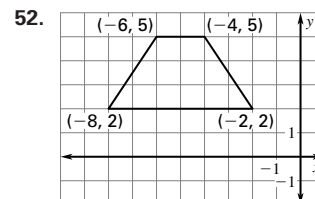
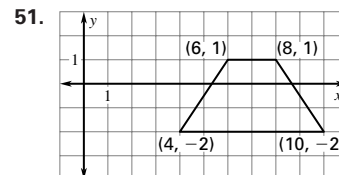
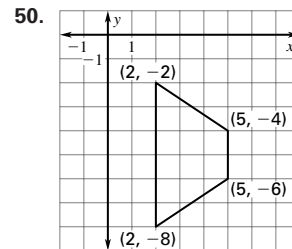
c. The number of art volumes sold has decreased and the price has increased over the two year period. The number of law volumes has increased and so has their average price. The number of music volumes sold has increased slightly, but the price has decreased. The number of travel volumes sold has decreased and so has the average price per volume. There is no direct relationship between the number of volumes sold and the average price per volume.

49. $4 \cdot \begin{matrix} A \\ B \\ C \end{matrix} \cdot \begin{matrix} x \\ y \end{matrix} = \begin{bmatrix} 8 & 8 \\ 32 & 8 \\ 20 & 24 \end{bmatrix}$



The "new" triangle has the vertices $A' = (8, 8)$, $B' = (32, 8)$, and $C' = (20, 24)$; the two triangles are similar with each side of $\Delta A'B'C'$ being parallel to and 4 times as long as the corresponding side of ΔABC .

4.1 Mixed Review (p. 206)



53. 20 54. -16 55. 7

56. $-\frac{7}{2}$ 57. $\frac{5}{14}$ 58. $3.2(2.4 + 8.1) = 3.2(10.5) = 33.6$

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59. $0 + 6 \leq -3$; no
 $-5 + 2 \leq -3$; yes
60. $-25 - 0 > 2$; no
 $25 - 23 > 2$; no
61. $8 - 3 < 5$; no
 $-24 + 27 < 5$; yes
62. $42 - 30 > 4$; yes
 $-21 - 0 > 4$; no
63. Sample answer: (1, 2) 64. Sample answer: (0, -3)
65. Sample answer: (5, 5)

Technology Activity (p. 207)

1. $\begin{bmatrix} 6.6 & -6.1 \\ 15.33 & 1.72 \end{bmatrix}$ 2. $\begin{bmatrix} -94 & 59 \\ 24 & 268 \\ -589 & -153 \end{bmatrix}$

3. $\begin{bmatrix} 6.4666 & 1.6688 \\ 23.0503 & 7.301 \end{bmatrix}$ 4. $\begin{bmatrix} 0.23417 & 6.34636 \\ 16.0816 & -2.3397 \end{bmatrix}$

5. Change

R C J E

CDs $\begin{bmatrix} -8 & -1 & 0 & -1 \\ -3 & -2 & -1 & 0 \end{bmatrix}$; none; Rock CDs, Country
 Tapes
 CDs, Easy Listening CDs, Rock tapes, Country tapes, Jazz
 tapes

Section 4.2

4.2 Guided Practice (p. 211)

1. columns; rows
2. AB is defined because the number of columns in A(1) is equal to the number of rows in B(1). BA is not defined because the number of columns in B(2) is not equal to the number of rows in A(6).

3. False; matrix multiplication is not commutative. If you multiply out both sides of the equation you get

$$\begin{bmatrix} 10 & 3 \\ -6 & 5 \end{bmatrix} = \begin{bmatrix} 10 & 6 \\ -3 & 5 \end{bmatrix}$$

4. defined; 3×3 5. defined; 3×3 6. not defined

7. $\begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1(2) + 0(1) & 1(0) + 0(3) \\ (-2)(2) + (-1)(1) & (-2)(0) + (-1)(3) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ -5 & -3 \end{bmatrix}$$

8. $[-20]$

9. $\begin{bmatrix} -3 & 3 \\ 3 & -2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$

$$= \begin{bmatrix} (-3)(1) + 3(-2) & (-3)(0) + 3(-1) \\ 3(1) + (-2)(-2) & 3(0) + (-2)(-1) \\ 0(1) + (-1)(-2) & 0(0) + (-1)(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -9 & -3 \\ 7 & 2 \\ 2 & 1 \end{bmatrix}$$

10. Bats Balls Uniforms

Women $\begin{bmatrix} 16 & 42 & 16 \end{bmatrix}$ Bats $\begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix}$
 Men $\begin{bmatrix} 14 & 43 & 15 \end{bmatrix}$ Balls
 Uniforms

$$\begin{bmatrix} 16 & 42 & 16 \\ 14 & 43 & 15 \end{bmatrix} \begin{bmatrix} 21 \\ 4 \\ 30 \end{bmatrix} = \begin{bmatrix} 16(21) + 42(4) + 16(30) \\ 14(21) + 43(4) + 15(30) \end{bmatrix}$$

$$= \begin{bmatrix} 336 + 168 + 480 \\ 294 + 172 + 450 \end{bmatrix}$$

$$= \begin{bmatrix} \$984 \\ \$916 \end{bmatrix} \begin{matrix} \text{Women's team} \\ \text{Men's team} \end{matrix}$$

4.2 Practice and Applications (pp. 211–213)

11. defined; 1×2 12. defined; 2×3 13. not defined

14. defined; 5×4 15. defined; 3×1 16. not defined

17. $[-\frac{1}{6}(12) + \frac{1}{2}(0) - \frac{1}{3}(-12)] = [-2 + 4] = [2]$

18. Not defined; the number of columns in the first matrix is 2, and this is not equal to the number of rows in the second matrix.

19. $\begin{bmatrix} 1(4) - 4(0) & 1(-1) - 4(-3) \\ 3(4) - 2(0) & 3(-1) - 2(-3) \end{bmatrix} = \begin{bmatrix} 4 - 0 & -1 + 12 \\ 12 - 0 & -3 + 6 \end{bmatrix}$

$$= \begin{bmatrix} 4 & 11 \\ 12 & 3 \end{bmatrix}$$

20. $\begin{bmatrix} -6(-1) - 2(-5) & -6(4) - 2(3) \\ 0(-1) + 3(-5) & 0(4) + 3(3) \end{bmatrix}$

$$= \begin{bmatrix} 6 + 10 & -24 - 6 \\ -15 & 9 \end{bmatrix} = \begin{bmatrix} 16 & -30 \\ -15 & 9 \end{bmatrix}$$

21. Not defined; the number of columns in the first matrix (3) is not equal to the number of rows in the second matrix (2).

22. $\begin{bmatrix} 6(1) + 0(1.5) & 6(0) + 0(-0.5) \\ -0.2(1) + 0.2(1.5) & -0.2(0) + 0.2(-0.5) \\ 2.9(1) + 0.3(1.5) & 2.9(0) + 0.3(-0.5) \end{bmatrix}$

$$= \begin{bmatrix} 6 & 0 \\ -0.2 + 0.3 & -0.1 \\ 2.9 + 0.45 & -0.15 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0.1 & -0.1 \\ 3.35 & -0.15 \end{bmatrix}$$

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$$23. \begin{bmatrix} -1(1.2) - 0.5(0.2) + 1.25(0) \\ 1(1.2) - 1.5(0.2) - 0.25(0) \end{bmatrix} = \begin{bmatrix} -1.2 - 0.1 \\ 1.2 - 0.3 \end{bmatrix}$$

$$= \begin{bmatrix} -1.3 \\ 0.9 \end{bmatrix}$$

$$24. \begin{bmatrix} -6(0) + (-7) + (-1) & -6(-1) + (-2) + (3) & -6(3) + (4) + (4) \\ -2(0) + 3(-7) + 8(-1) & -2(-1) + 3(-2) + 8(3) & -2(3) + 3(4) + 8(4) \\ 0.1(0) + 7(-7) + (-1) & 0.1(-1) + 7(-2) + (3) & 0.1(3) + 7(4) + (4) \end{bmatrix}$$

$$= \begin{bmatrix} -7 - 1 & 6 - 2 + 3 & -18 + 8 \\ -21 - 8 & 2 - 6 + 24 & -6 + 12 + 32 \\ -49 - 1 & -0.1 - 14 + 3 & 0.3 + 28 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 7 & -10 \\ -29 & 20 & 38 \\ -50 & -11.1 & 32.3 \end{bmatrix}$$

$$25. \begin{bmatrix} 6(-4) - 2(4) & 6(-2) - 2(-6) & 6(5) - 2(-1) \\ 1(-4) + 4(4) & 1(-2) + 4(-6) & 1(5) + 4(-1) \\ 0(-4) + 5(4) & 0(-2) + 5(-6) & 0(5) + 5(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -24 - 8 & -12 + 12 & 30 + 2 \\ -4 + 16 & -2 - 24 & 5 - 4 \\ 20 & -30 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} -32 & 0 & 32 \\ 12 & -26 & 1 \\ 20 & -30 & -5 \end{bmatrix}$$

$$26. \begin{bmatrix} 0(5) + 1(3) + 0(-4) & 0(-7) + 1(12) + 0(-5) & 0(4) + 1(6) + 0(-12) \\ 6(5) - 3(3) - 1(-4) & 6(-7) - 3(12) - 1(-5) & 6(4) - 3(6) - 1(-12) \\ -2(5) + 5(3) + 3(-4) & -2(-7) + 5(12) + 3(-5) & -2(4) + 5(6) + 3(-12) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 & 6 \\ 30 - 9 + 4 & -42 - 36 + 5 & 24 - 18 + 12 \\ -10 + 15 - 12 & 14 + 60 - 15 & -8 + 30 - 36 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 12 & 6 \\ 25 & -73 & 18 \\ -7 & 59 & -14 \end{bmatrix}$$

$$27. 2AB = 2 \begin{bmatrix} 4 & -2 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 4(1) - 2(-2) & 4(0) - 2(4) \\ 6(1) - 1(-2) & 6(0) - 1(4) \end{bmatrix}$$

$$= 2 \begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -16 \\ 16 & -8 \end{bmatrix}$$

$$28. AC = \begin{bmatrix} 4 & -2 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4(-1) - 2(-2) & 4(3) - 2(1) \\ 6(-1) - 1(-2) & 6(3) - 1(1) \end{bmatrix}$$

$$= \begin{bmatrix} -4 + 4 & 12 - 2 \\ -6 + 2 & 18 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 4 & 13 \end{bmatrix}$$

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$$29. D + E = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 2 & 4 \\ -2 & -3 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 5 & 6 \\ -1 & 4 & 2 \\ 3 & 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix}$$

$$D(D + E) = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 2 & 4 \\ -2 & -3 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3(1) - 2(-2) + 1(1) & 3(3) - 2(6) + 1(-2) & 3(7) - 2(6) + 1(-1) \\ -1(1) + 2(-2) + 4(1) & -1(3) + 2(6) + 4(-2) & -1(7) + 2(6) + 4(-1) \\ -2(1) - 3(-2) + 3(1) & -2(3) - 3(6) + 3(-2) & -2(7) - 3(6) + 3(-1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 + 4 + 1 & 9 - 12 - 2 & 21 - 12 - 1 \\ -1 - 4 + 4 & -3 + 12 - 8 & -7 + 12 - 4 \\ -2 + 6 + 3 & -6 - 18 - 6 & -14 - 18 - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & -5 & 8 \\ -1 & 1 & 1 \\ 7 & -30 & -35 \end{bmatrix}$$

$$30. \begin{bmatrix} 1 & 3 & 7 \\ -2 & 6 & 6 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} -2 & 5 & 6 \\ -1 & 4 & 2 \\ 3 & 1 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 1(-2) + 3(-1) + 7(3) & 1(5) + 3(4) + 7(1) & 1(6) + 3(2) + 7(-4) \\ -2(-2) + 6(-1) + 6(3) & -2(5) + 6(4) + 6(1) & -2(6) + 6(2) + 6(-4) \\ 1(-2) - 2(-1) - 1(3) & 1(5) - 2(4) - 1(1) & 1(6) - 2(2) - 1(-4) \end{bmatrix}$$

$$= \begin{bmatrix} -2 - 3 + 21 & 5 + 12 + 7 & 6 + 6 - 28 \\ 4 - 6 + 18 & -10 + 24 + 6 & -12 + 12 - 24 \\ -2 + 2 - 3 & 5 - 8 - 1 & 6 - 4 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 24 & -16 \\ 16 & 20 & -24 \\ -3 & -4 & 6 \end{bmatrix}$$

$$31. -3(AC) = -3 \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix} = \begin{bmatrix} 0 & -30 \\ 12 & -51 \end{bmatrix}$$

$$32. 0.5AB + 2AC = 0.5 \begin{bmatrix} 8 & -8 \\ 8 & -4 \end{bmatrix} + 2 \begin{bmatrix} 0 & 10 \\ -4 & 17 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -4 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 0 & 20 \\ -8 & 34 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 16 \\ -4 & 32 \end{bmatrix}$$

$$33. \begin{bmatrix} -2(1) + 1(x) + 2(3) \\ 3(1) + 2(x) + 4(3) \\ 0(1) - 2(x) + 4(3) \end{bmatrix} = \begin{bmatrix} -2 + x + 6 \\ 3 + 2x + 12 \\ -2x + 12 \end{bmatrix}$$

$$= \begin{bmatrix} x + 4 \\ 2x + 15 \\ -2x + 12 \end{bmatrix} = \begin{bmatrix} 6 \\ 19 \\ y \end{bmatrix}$$

$$x + 4 = 6$$

$$-2x + 12 = y$$

$$x = 2$$

$$-2(2) + 12 = y$$

$$8 = y$$

Chapter 4 continued

$$\begin{aligned}
 34. \quad & \begin{bmatrix} 4 & 1 & 3 \\ -2 & x & 1 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 2 & 1 \\ -1 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 4(9) + 1(2) + 3(-1) & 4(-2) + 1(1) + 3(4) \\ -2(9) + x(2) + 1(-1) & -2(-2) + x(1) + 1(4) \end{bmatrix} \\
 &= \begin{bmatrix} 35 & 5 \\ 2x - 19 & x + 8 \end{bmatrix} \\
 &= \begin{bmatrix} y & 5 \\ -13 & 11 \end{bmatrix}
 \end{aligned}$$

$$y = 35 \quad 2x - 19 = -13$$

$$2x = 6$$

$$x = 3$$

35. Grain Production

	Wheat	Rice	Maize
China	0.201	0.348	0.180
India	0.22	0.215	0.017
CLS	0.073	0.001	0.005
US	0.113	0.014	0.405

$$36. \quad \begin{bmatrix} 0.201 & 0.348 & 0.180 \\ 0.220 & 0.215 & 0.017 \\ 0.073 & 0.001 & 0.005 \\ 0.113 & 0.014 & 0.405 \end{bmatrix} \begin{bmatrix} 608,846 \\ 570,906 \\ 586,923 \end{bmatrix}$$

$$= \begin{bmatrix} 122,378 & 198,675 & 105,646 \\ 133,946 & 122,745 & 9,978 \\ 44,446 & 571 & 2,935 \\ 68,800 & 7,993 & 237,704 \end{bmatrix}$$

Total Weight

China	426,699.474
India	266,668.601
C.I.S.	47,951.279
U.S.	314,496.097

$$37. \text{ Matrix B} \quad 38. \quad AB = \begin{bmatrix} 3(6) + 5(5) + 4(4) \\ 5(6) + 2(5) + 5(4) \\ 4(6) + 6(5) + 2(4) \end{bmatrix} = \begin{bmatrix} 59 \\ 60 \\ 62 \end{bmatrix}$$

$$\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

39. Team 3; 62 points

$$\begin{aligned}
 40. \quad & [2 \ 0.5 \ 3] \begin{bmatrix} 109 & 136 \\ 127 & 159 \\ 64 & 79 \end{bmatrix} \\
 &= [218 + 63.5 + 192 \quad 272 + 79.5 + 237] \\
 &= [473.5 \quad 588.5]
 \end{aligned}$$

The 120 lb person burned about 474 calories and the 150 lb person about 589 calories.

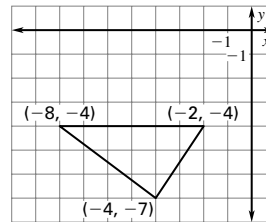
41. *Sample answer:* Check that the product is defined. To find the entry in the n^{th} row, m^{th} column, use the n^{th} row of the left matrix and the m^{th} column on the right matrix. Multiply each pair of corresponding entries and find the sum of these products.

$$42. \quad \begin{bmatrix} 0 & -1 \\ -4 & -2 \end{bmatrix} \begin{bmatrix} 7 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -28 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -26 & 8 \end{bmatrix}$$

C

43. B

$$\begin{aligned}
 44. \quad \text{a. } AB &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -7 & -4 & -4 \\ 4 & 8 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0(-7) + (-1)(4) & 0(-4) + (-1)(8) & 0(-4) + (-1)(2) \\ 1(-7) + 0(4) & 1(-4) + 0(8) & 1(-4) + 0(2) \end{bmatrix} \\
 &= \begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix}
 \end{aligned}$$



AB represents a 90° rotation of the original triangle.

$$\begin{aligned}
 \text{b. } & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -4 & -8 & -2 \\ -7 & -4 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} 0(-4) + (-1)(-7) & 0(-8) + (-1)(-4) & 0(-2) + (-1)(-4) \\ 1(-4) + 0(-7) & 1(-8) + 0(-4) & 1(-2) + 0(-4) \end{bmatrix} \\
 &= \begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix};
 \end{aligned}$$

vertices: $(7, -4)(4, -8)(4, -2)$

$$\begin{aligned}
 & \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 7 & 4 & 4 \\ -4 & -8 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 0(7) + (-1)(-4) & 0(4) + (-1)(-8) & 0(4) + (-1)(-2) \\ 1(7) + 0(-4) & 1(4) + 0(-8) & 1(4) + 0(-2) \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 8 & 2 \\ 7 & 4 & 4 \end{bmatrix};
 \end{aligned}$$

vertices: $(4, 7)(8, 4)(2, 4)$

4.2 Mixed Review (p. 213)

$$45. A = l \times w = 15 \times 12 = 180 \text{ m}^2$$

$$46. A = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 4 = 12 \text{ m}^2$$

$$47. A = \pi r^2 = \pi 9 \approx 28.3 \text{ ft}^2$$

$$\begin{aligned}
 48. \quad y - 8 &= \frac{1}{2}(x - 1) & 49. \quad y - 4 &= -\frac{1}{4}(x - 0) \\
 y &= \frac{1}{2}x + \frac{15}{2} & y &= -\frac{1}{4}x + 4
 \end{aligned}$$

$$\begin{aligned}
 50. \quad y + 6 &= \frac{10 + 6}{2 - 3}(x - 3) & 51. \quad y - 5 &= \frac{4 - 5}{4 - 1}(x - 1) \\
 y + 6 &= -16(x - 3) & y - 5 &= \frac{9}{3}(x - 1) \\
 y &= -16x + 42 & y &= 3x + 2
 \end{aligned}$$

Chapter 4 continued

52. $y + 5 = -\frac{5}{7}(x - 0)$
 $y = -\frac{5}{7}x - 5$
53. $y + 6 = \frac{-6}{4}(x - 0)$
 $y = \frac{3}{2}x - 6$
54. $y = 6 - 4x$
 $-3x - 2(6 - 4x) = 8$
 $-3x - 12 + 8x = 8$
 $5x = 20$
 $x = 4$
 $y = 6 - 16 = -10$
 $(4, -10)$
55. $y = -9 - 2x$
 $3x + 5(-9 - 2x) = 4$
 $3x - 10x = 49$
 $-7x = 49$
 $x = -7$
 $y = -9 + 14$
 $y = 5$
 $(-7, 5)$
56. $-9x + 5y = 1$
 $\frac{9x - 6y = 6}{-y = 7}$
 $y = -7$
 $3x - 2y = 2$
 $3x = 2 + 2y$
 $3x = 2 + 2(-7)$
 $3x = -12$
 $x = -4$
 $(-4, -7)$
57. $x - y = 1$
 $x = 1 + y$
 $2(1 + y) - 2y = 8$
 $2 + 2y - 2y = 8$
 $2 \neq 8$
 no solution
58. $-6x + 8y = -2$
 $\frac{6x + 2y = 7}{10y = 5}$
 $y = \frac{1}{2}$
 $6x + 1 = 7$
 $6x = 6$
 $x = 1$
 $(1, \frac{1}{2})$
59. $20x + 8y = -40$
 $\frac{-3x - 8y = 40}{17x = 0}$
 $x = 0$
 $-8y = 40$
 $y = -5$
 $(0, -5)$
60. $14x + 6y = 22$
 $\frac{-14x + 35y = 224}{41y = 246}$
 $y = 6$
 $7x + 3y = 11$
 $7x = 11 + (-3y)$
 $7x = 11 + (-3)(6)$
 $7x = 11 - 18$
 $7x = -7$
 $x = -1$
 $(-1, 6)$
61. $10x - 8y = -2$
 $\frac{-10x + 45y = -50}{37y = -52}$
 $y = \frac{-52}{37}$
 $2x - 9(\frac{-52}{37}) = 10$
 $2x = \frac{-98}{37}$
 $x = \frac{-49}{37}$
 $(\frac{-49}{37}, \frac{-52}{37})$

62. $x = 7y + 49$
 $12(7y + 49) + y = -24$
 $85y = -612$
 $y = -\frac{36}{5}$
 $x = 7(\frac{-36}{5}) + 49$
 $x = \frac{-252}{5} + \frac{245}{5} = \frac{-7}{5}$
 $(\frac{-7}{5}, \frac{-36}{5})$

Section 4.3

4.3 Guided Practice (p. 218)

1. *Sample answer:* Cramer's rule is a method of solving a system of linear equations by using the determinant of the coefficient matrix associated with the system. First find the determinant of the coefficient matrix and check that it does not equal 0. Each coordinate of the solution of the system is found as a fraction. The denominator of the fraction is the determinant of the coefficient matrix. The numerator of the fraction is the determinant of the matrix formed by using the coefficient matrix and replacing the column of coefficients of the corresponding variable with the column of constants from the system.

2. Yes; *Sample answer:* $\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} = \begin{vmatrix} 6 & 1 \\ 2 & 2 \end{vmatrix} = 10$

3. a. The calculation should be $6 - (-5) = 11$

b. The calculation should be $12 - (-2) = 14$

4. The determinant cannot be zero. 5. $0 - 6 = -6$

6. $1 - 20 = -19$ 7. $32 - 4 = 28$

8. $\begin{vmatrix} 6 & -8 \\ 4 & -5 \end{vmatrix} = -30 + 32 = 2$

$$x = \frac{\begin{vmatrix} 4 & -8 \\ -4 & -5 \end{vmatrix}}{2} = \frac{-20 - 32}{2} = \frac{-52}{2} = -26$$

$$y = \frac{\begin{vmatrix} 6 & 4 \\ 4 & -4 \end{vmatrix}}{2} = \frac{-24 - 16}{2} = \frac{-40}{2} = -20$$

$(-26, -20)$

9. $\begin{vmatrix} 2 & 7 \\ 3 & -8 \end{vmatrix} = -16 - 21 = -37$

$$x = \frac{\begin{vmatrix} -3 & 7 \\ -23 & -8 \end{vmatrix}}{-37} = \frac{24 + 161}{-37} = -5$$

$$y = \frac{\begin{vmatrix} 2 & -3 \\ 3 & -23 \end{vmatrix}}{-37} = \frac{-46 + 9}{-37} = \frac{-37}{-37} = 1$$

$(-5, 1)$

Chapter 4 continued

$$10. \begin{vmatrix} 12 & -2 \\ -14 & 11 \end{vmatrix} = 12 \cdot 11 - (-2 \cdot -14) \\ = 132 - 28 = 104$$

$$x = \frac{\begin{vmatrix} 2 & -2 \\ 51 & 11 \end{vmatrix}}{104} = \frac{22 + 102}{104} = \frac{124}{104} = \frac{31}{26}$$

$$y = \frac{\begin{vmatrix} 12 & 2 \\ -14 & 51 \end{vmatrix}}{104} = \frac{612 + 28}{104} = \frac{640}{104} = \frac{80}{13}$$

$$\left(\frac{31}{26}, \frac{80}{13}\right)$$

$$11. A = \frac{1}{2}(50 \cdot 70) = 1750 \text{ in.}^2$$

4.3 Practice and Applications (pp. 218–220)

$$12. 8 - 10 = -2 \quad 13. 24 - 0 = 24 \quad 14. 9 + 6 = 15$$

$$15. -14 + 77 = 63 \quad 16. 16 - 0 = 16$$

$$17. 9 - 40 = -31 \quad 18. -54 + 15 = -39$$

$$19. 0 + 24 = 24 \quad 20. 96 + 10 = 106$$

$$21. (36 + 40 + 16) - (-15 + 192 - 8) = 92 - 169 = -77$$

$$22. (10 + 0 + 16) - (0 + 5 - 36) = 26 + 31 = 57$$

$$23. (0 + 100 + 80) - (-130 + 0 - 50) = 180 + 180 = 360$$

$$24. (-16 + 224 - 40) - (-56 + 2 - 1280) = (168) - (-1334) \\ = 1502$$

$$25. (-64 + 0 + 0) - (0 - 180 + 0) = -64 + 180 = 116$$

$$26. (96 + 54 + 1404) - (81 + 936 + 96) = (1554) - (1113) = 441$$

$$27. (-54 + 0 - 60) - (45 + 0 - 240) = -114 + 195 = 81$$

$$28. (-324 + 396 - 3000) - (1980 - 810 - 240) = -2928 - 930 \\ = -3858$$

$$29. (0 - 192 + 200) - (0 + 180 + 560) = 8 - 740 = -732$$

$$30. A = \pm \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 2 & 7 & 1 \\ 5 & 5 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(0 + 5 + 10) - (35 + 0 + 2)]$$

$$= \pm \frac{1}{2} (15 - 37) = 11$$

$$31. A = \frac{1}{2} \begin{vmatrix} 3 & 6 & 1 \\ 3 & 0 & 1 \\ 1 & 3 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(0 + 6 + 9) - (0 + 9 + 18)] \\ = \pm \frac{1}{2} (15 - 27) = 6$$

$$32. A = \pm \frac{1}{2} \begin{vmatrix} 6 & -1 & 1 \\ 2 & 2 & 1 \\ 4 & 8 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(12 - 4 + 16) - (8 + 48 - 2)] \\ = \pm \frac{1}{2} (24 - 54) = 15$$

$$33. A = \pm \frac{1}{2} \begin{vmatrix} -4 & 2 & 1 \\ 3 & -1 & 1 \\ -2 & -2 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(4 - 4 - 6) - (2 + 8 + 6)] \\ = \pm \frac{1}{2} (-6 - 16) = 11$$

$$34. A = \pm \frac{1}{2} \begin{vmatrix} 2 & -6 & 1 \\ -1 & -4 & 1 \\ 0 & 2 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(-8 + 0 - 2) - (0 + 4 + 6)] \\ = \pm \frac{1}{2} (-10 - 10) = 10$$

$$35. A = \pm \frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ -2 & 6 & 1 \\ -1 & 1 & 1 \end{vmatrix} \\ = \pm \frac{1}{2} [(6 - 3 - 2) - (-6 + 1 - 6)] \\ = \pm \frac{1}{2} [1 + 11] = 6$$

$$36. \begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix} = 12 - 5 = 7 \\ x = \frac{\begin{vmatrix} 3 & 1 \\ 4 & 6 \end{vmatrix}}{7} = \frac{18 - 4}{7} = 2 \\ y = \frac{\begin{vmatrix} 2 & 3 \\ 5 & 4 \end{vmatrix}}{7} = \frac{8 - 15}{7} = -1 \\ (2, -1)$$

$$37. \begin{vmatrix} 7 & -5 \\ 3 & 10 \end{vmatrix} = 70 + 15 = 85 \\ x = \frac{\begin{vmatrix} 11 & -5 \\ -56 & 10 \end{vmatrix}}{85} = \frac{110 - 280}{85} = -2 \\ y = \frac{\begin{vmatrix} 7 & 11 \\ 3 & -56 \end{vmatrix}}{85} = \frac{-392 - 33}{85} = -5 \\ (-2, -5)$$

Chapter 4 *continued*

$$38. \begin{vmatrix} 9 & 2 \\ 4 & -3 \end{vmatrix} = -27 - 8 = -35$$

$$x = \frac{\begin{vmatrix} 7 & 2 \\ 42 & -3 \end{vmatrix}}{-35} = \frac{-21 - 84}{-35} = 3$$

$$y = \frac{\begin{vmatrix} 9 & 7 \\ 4 & 42 \end{vmatrix}}{-35} = \frac{378 - 28}{-35} = -10$$

(3, -10)

$$39. \begin{vmatrix} 1 & 7 \\ 3 & -5 \end{vmatrix} = -5 - 21 = -26$$

$$x = \frac{\begin{vmatrix} -3 & 7 \\ 17 & -5 \end{vmatrix}}{-26} = \frac{15 - 119}{-26} = 4$$

$$y = \frac{\begin{vmatrix} 1 & -3 \\ 3 & 17 \end{vmatrix}}{-26} = \frac{17 + 9}{-26} = -1$$

(4, -1)

$$40. \begin{vmatrix} -1 & -12 \\ 12 & -15 \end{vmatrix} = 15 + 144 = 159$$

$$x = \frac{\begin{vmatrix} 44 & -12 \\ -51 & -15 \end{vmatrix}}{159} = \frac{-660 - 612}{159} = -8$$

$$y = \frac{\begin{vmatrix} -1 & 44 \\ 12 & -51 \end{vmatrix}}{159}$$

$$y = \frac{51 - 528}{159} = -3$$

(-8, -3)

$$41. \begin{vmatrix} 4 & -3 \\ 8 & -7 \end{vmatrix} = -28 + 24 = -4$$

$$x = \frac{\begin{vmatrix} 18 & -3 \\ 34 & -7 \end{vmatrix}}{-4} = \frac{-126 + 102}{-4} = 6$$

$$y = \frac{\begin{vmatrix} 4 & 18 \\ 8 & 34 \end{vmatrix}}{-4} = \frac{136 - 144}{-4} = 2$$

(6, 2)

$$42. \begin{vmatrix} 4 & -5 \\ 2 & -7 \end{vmatrix} = -28 + 10 = -18$$

$$x = \frac{\begin{vmatrix} 13 & -5 \\ 24 & -7 \end{vmatrix}}{-18} = \frac{-91 + 120}{-18} = \frac{29}{-18}$$

$$y = \frac{\begin{vmatrix} 4 & 13 \\ 2 & 24 \end{vmatrix}}{-18} = \frac{96 - 26}{-18} = \frac{35}{-9}$$

$\left(-\frac{29}{18}, -\frac{35}{9}\right)$

$$43. \begin{vmatrix} 8 & -9 \\ -5 & 7 \end{vmatrix} = 56 - 45 = 11$$

$$x = \frac{\begin{vmatrix} 32 & -9 \\ 40 & 7 \end{vmatrix}}{11} = \frac{224 + 360}{11} = \frac{584}{11}$$

$$y = \frac{\begin{vmatrix} 8 & 32 \\ -5 & 40 \end{vmatrix}}{11} = \frac{320 + 160}{11} = \frac{480}{11}$$

$\left(\frac{584}{11}, \frac{480}{11}\right)$

$$44. \begin{vmatrix} 3 & 10 \\ 12 & 15 \end{vmatrix} = 45 - 120 = -75$$

$$x = \frac{\begin{vmatrix} 50 & 10 \\ 64 & 15 \end{vmatrix}}{-75} = \frac{750 - 640}{-75} = \frac{110}{-75} = -\frac{22}{15}$$

$$y = \frac{\begin{vmatrix} 3 & 50 \\ 12 & 64 \end{vmatrix}}{-75} = \frac{192 - 600}{-75} = \frac{136}{25}$$

$\left(-\frac{22}{15}, \frac{136}{25}\right)$

$$45. \begin{vmatrix} 1 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & -4 \end{vmatrix} = (4 + 6 - 12) - (9 + 4 - 8) = -7$$

$$x = \frac{\begin{vmatrix} -2 & 2 & -3 \\ -1 & -1 & 1 \\ 4 & 4 & -4 \end{vmatrix}}{-7} = \frac{(-8 + 8 + 12) - (12 - 8 + 8)}{-7}$$

$$= \frac{12 - 12}{-7} = 0$$

$$y = \frac{\begin{vmatrix} 1 & -2 & -3 \\ 1 & -1 & 1 \\ 3 & 4 & -4 \end{vmatrix}}{-7} = \frac{(4 - 6 - 12) - (9 + 4 + 8)}{-7}$$

$$= \frac{-14 - 21}{-7} = 5$$

$$z = \frac{\begin{vmatrix} 1 & 2 & -2 \\ 1 & -1 & -1 \\ 3 & 4 & 4 \end{vmatrix}}{-7} = \frac{(4 - 6 - 8) - (6 - 4 + 8)}{-7}$$

$$= \frac{-18 - 10}{-7} = 4$$

(0, 5, 4)

Chapter 4 continued

$$46. \begin{vmatrix} 1 & 3 & -1 \\ -2 & -6 & 1 \\ 3 & 5 & -2 \end{vmatrix} = (12 + 9 + 10) - (18 + 5 + 12) = -4$$

$$x = \frac{\begin{vmatrix} 1 & 3 & -1 \\ -3 & -6 & 1 \\ 4 & 5 & -2 \end{vmatrix}}{-4} = \frac{(12 + 12 + 15) - (24 + 5 + 18)39 - 47}{-4} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 1 & -1 \\ -2 & -3 & 1 \\ 3 & 4 & -2 \end{vmatrix}}{-4} = \frac{(6 + 3 + 8) - (9 + 4 + 4)}{-4} = \frac{17 - 17}{-4} = 0$$

$$z = \frac{\begin{vmatrix} 1 & 3 & 1 \\ -2 & -6 & -3 \\ 3 & 5 & 4 \end{vmatrix}}{-4} = \frac{(-24 - 27 - 10) - (-18 - 15 - 24)}{-4} = \frac{-61 + 57}{-4} = 1$$

(2, 0, 1)

$$47. \begin{vmatrix} 3 & 2 & -5 \\ 6 & 0 & -1 \\ 0 & -1 & 3 \end{vmatrix} = (0 + 0 + 30) - (0 + 3 + 36) = -9$$

$$x = \frac{\begin{vmatrix} -10 & 2 & -5 \\ 8 & 0 & -1 \\ -2 & -1 & 3 \end{vmatrix}}{-9} = \frac{(0 + 4 + 40) - (0 - 10 + 48)}{-9} = \frac{44 - 38}{-9}$$

$$= -\frac{6}{9} = -\frac{2}{3}$$

$$y = \frac{\begin{vmatrix} 3 & -10 & -5 \\ 6 & 8 & -1 \\ 0 & -2 & 3 \end{vmatrix}}{-9} = \frac{(72 + 0 + 60) - (0 + 6 - 180)}{-9} = \frac{132 + 174}{-9}$$

$$= \frac{306}{-9} = -34$$

$$z = \frac{\begin{vmatrix} 3 & 2 & -10 \\ 6 & 0 & 8 \\ 0 & -1 & -2 \end{vmatrix}}{-9}$$

—CONTINUED—

47. —CONTINUED—

$$= \frac{(0 + 0 + 60) - (0 - 24 - 24)}{-9} = \frac{60 + 48}{-9} = -12$$

$$\left(-\frac{2}{3}, -34, -12\right)$$

$$48. \begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 5 & 0 & -2 \end{vmatrix} = (-2 + 10 + 0) - (5 + 0 - 4) = 7$$

$$x = \frac{\begin{vmatrix} 9 & 2 & 1 \\ 3 & 1 & 1 \\ -1 & 0 & -2 \end{vmatrix}}{7} = \frac{(-18 - 2 + 0) - (-1 + 0 - 12)}{7} = \frac{-20 + 13}{7} = -1$$

$$y = \frac{\begin{vmatrix} 1 & 9 & 1 \\ 1 & 3 & 1 \\ 5 & -1 & -2 \end{vmatrix}}{7} = \frac{(-6 + 45 - 1) - (15 - 1 - 18)}{7} = \frac{38 + 4}{7} = 6$$

$$z = \frac{\begin{vmatrix} 1 & 2 & 9 \\ 1 & 1 & 3 \\ 5 & 0 & -1 \end{vmatrix}}{7} = \frac{(-1 + 30 + 0) - (45 + 0 - 2)}{7} = \frac{29 - 43}{7} = -2$$

(-1, 6, -2)

$$49. \begin{vmatrix} 4 & 1 & 6 \\ 3 & 3 & 2 \\ -1 & -1 & 1 \end{vmatrix} = (12 - 2 - 18) - (-18 - 8 + 3) = 15$$

$$x = \frac{\begin{vmatrix} 7 & 1 & 6 \\ 17 & 3 & 2 \\ -9 & -1 & 1 \end{vmatrix}}{15} = \frac{(21 - 18 - 102) - (-162 - 14 + 17)}{15}$$

$$= \frac{-99 + 159}{15} = 4$$

$$y = \frac{\begin{vmatrix} 4 & 7 & 6 \\ 3 & 17 & 2 \\ -1 & -9 & 1 \end{vmatrix}}{15} = \frac{(68 - 14 - 162) - (-102 - 72 + 21)}{15}$$

$$= \frac{-108 + 153}{15} = 3$$

—CONTINUED—

Chapter 4 continued

49. —CONTINUED—

$$z = \frac{\begin{vmatrix} 4 & 1 & 7 \\ 3 & 3 & 17 \\ -1 & -1 & -9 \end{vmatrix}}{15}$$

$$= \frac{(-108 - 17 - 21) - (-21 - 68 - 27)}{15}$$

$$= \frac{-146 + 116}{15} = -2$$

(4, 3, -2)

$$50. \begin{vmatrix} 1 & 4 & -1 \\ 2 & -1 & 2 \\ -3 & 1 & -3 \end{vmatrix} = (3 - 24 - 2) - (-3 + 2 - 24) = 2$$

$$x = \frac{\begin{vmatrix} -7 & 4 & -1 \\ 15 & -1 & 2 \\ -22 & 1 & -3 \end{vmatrix}}{2}$$

$$= \frac{(-21 - 176 - 15) - (-22 - 14 - 180)}{2}$$

$$= \frac{-212 + 216}{2} = 2$$

$$y = \frac{\begin{vmatrix} 1 & -7 & -1 \\ 2 & 15 & 2 \\ -3 & -22 & -3 \end{vmatrix}}{2}$$

$$= \frac{(-45 + 42 + 44) - (45 + -44 + 42)}{2}$$

$$= \frac{41 - 43}{2} = -1$$

$$z = \frac{\begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 15 \\ -3 & 1 & -22 \end{vmatrix}}{2}$$

$$= \frac{(22 - 180 - 14) - (-21 + 15 - 176)}{2}$$

$$= \frac{-172 + 182}{2} = 5$$

(2, -1, 5)

$$51. \begin{vmatrix} 2 & 1 & 1 \\ 1 & 4 & -2 \\ 6 & 5 & 0 \end{vmatrix} = (0 - 12 + 5) - (24 - 20 + 0) = -11$$

$$x = \frac{\begin{vmatrix} 5 & 1 & 1 \\ 9 & 4 & -2 \\ 16 & 5 & 0 \end{vmatrix}}{-11}$$

$$= \frac{(0 + 45 - 32) - (64 - 50 + 0)}{-11} = \frac{13 - 14}{-11} = \frac{1}{11}$$

51. —CONTINUED—

$$y = \frac{\begin{vmatrix} 2 & 5 & 1 \\ 1 & 9 & -2 \\ 6 & 16 & 0 \end{vmatrix}}{-11}$$

$$= \frac{(0 - 60 + 16) - (54 - 64 + 0)}{-11} = \frac{-44 + 10}{-11} = \frac{34}{11}$$

$$z = \frac{\begin{vmatrix} 2 & 1 & 5 \\ 1 & 4 & 9 \\ 6 & 5 & 16 \end{vmatrix}}{-11}$$

$$= \frac{(128 + 54 + 25) - (120 + 90 + 16)}{-11}$$

$$= \frac{207 - 226}{-11} = \frac{19}{11}$$

$$\left(\frac{1}{11}, \frac{34}{11}, \frac{19}{11}\right)$$

$$52. \begin{vmatrix} -1 & 2 & 7 \\ 2 & -1 & -2 \\ 3 & 5 & 2 \end{vmatrix} = (2 - 12 + 70) - (-21 + 10 + 8)$$

$$= 63$$

$$x = \frac{\begin{vmatrix} 13 & 2 & 7 \\ -2 & -1 & -2 \\ -14 & 5 & 2 \end{vmatrix}}{63}$$

$$= \frac{(-26 + 56 - 70) + (98 - 130 - 8)}{63} = \frac{-40 + 40}{63} = 0$$

$$y = \frac{\begin{vmatrix} -1 & 13 & 7 \\ 2 & -2 & -2 \\ 3 & -14 & 2 \end{vmatrix}}{63}$$

$$= \frac{(4 - 78 - 196) - (-42 - 28 + 52)}{63}$$

$$= \frac{-270 + 18}{63} = -4$$

$$z = \frac{\begin{vmatrix} -1 & 2 & 13 \\ 2 & -1 & -2 \\ 3 & 5 & -14 \end{vmatrix}}{63}$$

$$= \frac{(-14 - 12 + 130) - (-39 + 10 - 56)}{63}$$

$$= \frac{104 + 85}{63} = 3$$

(0, -4, 3)

—CONTINUED—

Chapter 4 continued

$$53. \begin{vmatrix} -3 & 1 & 2 \\ 9 & -1 & 2 \\ 8 & 5 & -4 \end{vmatrix} = (-12 + 16 + 90) - (-16) - 30 - 36$$

$$= 176$$

$$x = \frac{\begin{vmatrix} -14 & 1 & 2 \\ -8 & -1 & 2 \\ 6 & 5 & -4 \end{vmatrix}}{176}$$

$$= \frac{(-56 + 12 - 80) - (-12 - 140 + 32)}{176}$$

$$= \frac{-124 + 120}{176} = \frac{-4}{176} = -\frac{1}{44}$$

$$y = \frac{\begin{vmatrix} -3 & -14 & 2 \\ 9 & -8 & 2 \\ 8 & 6 & -4 \end{vmatrix}}{176}$$

$$= \frac{(-96 - 224 + 108) - (-128 - 36 + 504)}{176}$$

$$= \frac{-212 - 340}{176} = \frac{-552}{176} = -\frac{69}{22}$$

$$z = \frac{\begin{vmatrix} -3 & 1 & -14 \\ 9 & -1 & -8 \\ 8 & 5 & 6 \end{vmatrix}}{176}$$

$$= \frac{(18 - 64 - 630) - (112 + 120 + 54)}{176}$$

$$= \frac{-676 - 286}{176} = \frac{-962}{176} = -\frac{481}{88}$$

$$\left(-\frac{1}{44}, -\frac{69}{22}, -\frac{481}{88}\right)$$

$$54. A = \pm \frac{1}{2} \begin{vmatrix} 35 & 220 & 1 \\ 112 & 56 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(1960 + 0 + 0) - (0 + 0 + 24,640)]$$

$$= \pm \frac{1}{2} (-22,680) = 11,340 \text{ mi}^2$$

$$55. A = \pm \frac{1}{2} \begin{vmatrix} 0 & 2 & 1 \\ 12 & 2 & 1 \\ 12 & 26 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(0 + 24 + 312) - (24 + 0 + 24)]$$

$$= \pm \frac{1}{2} (336 - 48) = 144 \text{ ft}^2$$

$$56. A = \pm \frac{1}{2} \begin{vmatrix} 14 & 2 & 1 \\ 22 & 2 & 1 \\ 14 & 18 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(28 + 28 + 396) - (28 + 252 + 44)]$$

$$= \pm \frac{1}{2} (452 - 324) = 64 \text{ ft}^2$$

$$57. 144 \text{ ft}^2 \times \left(\frac{1 \text{ in.}}{6 \text{ ft}}\right)^2 = 144 \text{ ft}^2 \times \frac{1 \text{ in.}^2}{36 \text{ ft}^2} = 4 \text{ in.}^2$$

$$58. A = \pm \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 100 & 120 & 1 \\ 140 & 20 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(0 + 0 + 2000) - (16,800 + 0 + 0)]$$

$$= \pm \frac{1}{2} (2000 - 16,800) = 7400 \text{ mi}^2$$

$$59. 10x + 2y = 13.56 \quad \text{let } x = \text{price of premium gas/gal}$$

$$x = y + 0.12 \quad \text{let } y = \text{price of regular gas/gal}$$

$$\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix} = -10 - 2 = -12$$

$$x = \frac{\begin{vmatrix} 13.56 & 2 \\ 0.12 & -1 \end{vmatrix}}{-12} = \frac{-13.56 - 0.24}{-12} = \frac{-13.80}{-12} = \$1.15$$

\$1.15 premium gas per gallon

$$y = \frac{\begin{vmatrix} 10 & 13.56 \\ 1 & 0.12 \end{vmatrix}}{-12} = \frac{1.20 - 13.56}{-12} = \frac{-12.36}{-12} = \$1.03$$

\$1.03 regular gas per gallon

$$60. 4S + 4N = 184$$

$$S + 6F = 146$$

$$2N + 4F = 104$$

$$\begin{vmatrix} 4 & 4 & 0 \\ 1 & 0 & 6 \\ 0 & 2 & 4 \end{vmatrix} = (0 + 0 + 0) - (0 + 48 + 16) = -64$$

$$S = \frac{\begin{vmatrix} 184 & 4 & 0 \\ 146 & 0 & 6 \\ 104 & 2 & 4 \end{vmatrix}}{-64}$$

$$= \frac{(0 + 2496 + 0) - (0 + 2208 + 2336)}{-64}$$

$$= \frac{2496 - 4544}{-64} = 32$$

—CONTINUED—

Chapter 4 continued

60. —CONTINUED—

$$N = \frac{\begin{vmatrix} 4 & 184 & 0 \\ 1 & 146 & 6 \\ 0 & 104 & 4 \end{vmatrix}}{-64}$$

$$= \frac{(2336 + 0 + 0) - (0 + 2496 + 736)}{-64}$$

$$= \frac{2336 - 3232}{-64} = 14$$

$$F = \frac{\begin{vmatrix} 4 & 4 & 184 \\ 1 & 0 & 146 \\ 0 & 2 & 104 \end{vmatrix}}{-64}$$

$$= \frac{(0 + 0 + 368) - (0 + 1168 + 416)}{-64}$$

$$= \frac{368 - 1584}{-64} = 19$$

61. The determinant is multiplied by -1 . Proof for 2×2 matrices:

$$-1 \begin{vmatrix} a & b \\ c & d \end{vmatrix} = -(ad - bc) = bc - ad = 1 \begin{vmatrix} b & a \\ d & c \end{vmatrix}$$

62. A: $A = \pm \frac{1}{2} \begin{vmatrix} -3 & 4 & 1 \\ 4 & 2 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$$= \pm \frac{1}{2} [(-6 + 4 - 8) - (2 + 6 + 16)]$$

$$= \pm \frac{1}{2} (-10 - 24) = 17$$

$$A = \pm \frac{1}{2} \begin{vmatrix} 4 & 2 & 1 \\ 1 & -2 & 1 \\ 3 & -4 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(-8 + 6 - 4) - (-6 - 16 + 2)]$$

$$= \pm \frac{1}{2} (-6 + 20) = 7$$

63. C: $\begin{vmatrix} -5 & 6 \\ -2 & 10 \end{vmatrix} = -50 + 12 = -38$

$$\begin{vmatrix} -7 & 1 \\ 3 & 5 \end{vmatrix} = -35 - 3 = -38$$

64. a. $\det AB = (\det A)(\det B)$

b. $\det kA = k^2 \det A$

4.3 Mixed Review (p. 221)

65. $f(7) = 7 - 10 = -3$

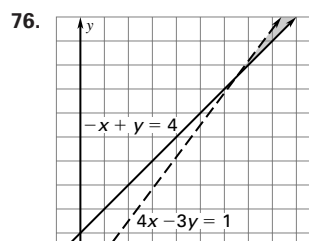
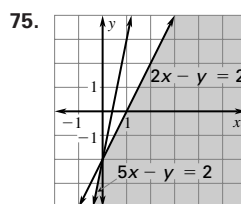
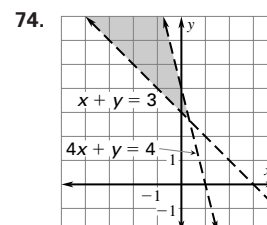
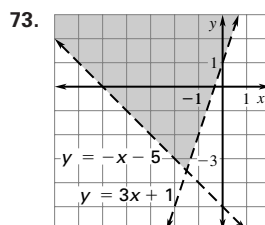
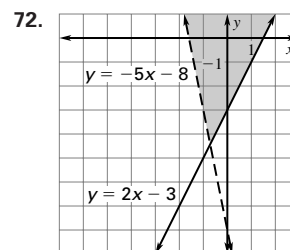
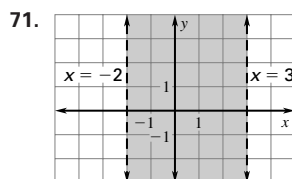
66. $f(-2) = 3(-2) + 7 = -6 + 7 = 1$

67. $f(-1) = -(-1)^2 + 5 = -1 + 5 = 4$

68. $f(7) = (7)^2 - 2(7) - 4 = 49 - 14 - 4 = 31$

69. $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) - 1 = \frac{1}{4} + 2 - 1 = 1\frac{1}{4}$

70. $f(3) = 3^5 - 2 \cdot 3 - 10 = 243 - 6 - 10 = 227$



77. $\begin{bmatrix} -2(6) - 4(3) & -2(-1) - 4(-3) \\ 5(6) + 1(3) & 5(-1) + 1(-3) \end{bmatrix} = \begin{bmatrix} -24 & 14 \\ 33 & -8 \end{bmatrix}$

78. $\begin{bmatrix} 7(0) - 1(4) & 7(-3) - 1(8) \\ 4(0) - 10(4) & 4(-3) - 10(8) \end{bmatrix} = \begin{bmatrix} -4 & -29 \\ -40 & -92 \end{bmatrix}$

79. $\begin{bmatrix} 11(-8) - 2(8) & 11(3) - 2(-1) \\ 0(-8) + 4(8) & 0(3) + 4(-1) \end{bmatrix} = \begin{bmatrix} -104 & 35 \\ 32 & -4 \end{bmatrix}$

80. $\begin{bmatrix} 3(10) - 5(12) & 3(9) - 5(16) \\ -7(10) + 2(12) & -7(9) + 2(16) \end{bmatrix} = \begin{bmatrix} -30 & -53 \\ -46 & -31 \end{bmatrix}$

81. $\begin{bmatrix} 0.5(0) + 3(4) & 0.5(0.6) + 3(0.8) \\ 0.2(0) + 1(4) & 0.2(0.6) + 1(0.8) \end{bmatrix} = \begin{bmatrix} 12 & 2.7 \\ 4 & 0.92 \end{bmatrix}$

82. $\begin{bmatrix} -2(1.6) + 1.3(-4) & -2(6) + 1.3(1.9) \\ 1.5(1.6) - 3(-4) & 1.5(6) - 3(1.9) \end{bmatrix}$

$$= \begin{bmatrix} -8.4 & -9.53 \\ 14.4 & 3.3 \end{bmatrix}$$

Quiz (p. 221)

1. $\begin{bmatrix} -5 & 4 & 15 \\ 2 & -14 & 1 \end{bmatrix}$ 2. $\begin{bmatrix} -5 & -7 \\ 0 & -1 \end{bmatrix}$

3. $\begin{bmatrix} -14 & 4 \\ -8 & -18 \end{bmatrix} + \begin{bmatrix} 12 & -6 \\ -10 & +6 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -18 & -12 \end{bmatrix}$

4. $\begin{bmatrix} 4 & -6 & 10 \\ 3 & 6 & 0 \\ 9 & -4 & 5 \end{bmatrix} + \begin{bmatrix} -8 & 4 & 12 \\ 0 & -24 & 20 \\ 8 & 0 & -4 \end{bmatrix} = \begin{bmatrix} -4 & -2 & 22 \\ 3 & -18 & 20 \\ 17 & -4 & 1 \end{bmatrix}$

Chapter 4 continued

$$5. \begin{bmatrix} 8(3) - (-2) & 8(7) - (0) \\ 6(3) - 2(-2) & 6(7) - 2(0) \end{bmatrix} = \begin{bmatrix} 26 & 56 \\ 22 & 42 \end{bmatrix}$$

$$6. \begin{bmatrix} 2(1) - 1(9) + 3(4) & 2(0) - 1(-3) + 3(-6) \\ 2(1) + 4(9) + 0(4) & 2(0) + 4(-3) + 0(-6) \end{bmatrix} = \begin{bmatrix} 5 & -15 \\ 38 & -12 \end{bmatrix}$$

$$7. -8 + 18 = 10 \quad 8. -18 + 18 = 0$$

$$9. (0 - 24 + 120) - (0 + 16 + 10) = 96 - 26 = 70$$

$$10. (-72 + 15 + 0) - (-12 + 0 - 30) = -57 + 42 = -15$$

$$11. \begin{vmatrix} -8 & 1 \\ -5 & 4 \end{vmatrix} = -32 + 5 = -27$$

$$x = \frac{\begin{vmatrix} -6 & 1 \\ 3 & 4 \end{vmatrix}}{-27} = \frac{-24 - 3}{-27} = 1$$

$$y = \frac{\begin{vmatrix} -8 & -6 \\ -5 & 3 \end{vmatrix}}{-27} = \frac{-24 - 30}{-27} = 2$$

(1, 2)

$$12. \begin{vmatrix} 3 & -2 \\ -6 & 1 \end{vmatrix} = 3 - 12 = -9$$

$$x = \frac{\begin{vmatrix} 10 & -2 \\ -7 & 1 \end{vmatrix}}{-9} = \frac{10 - 14}{-9} = \frac{4}{9}$$

$$y = \frac{\begin{vmatrix} 3 & 10 \\ -6 & -7 \end{vmatrix}}{-9} = \frac{-21 + 60}{-9} = \frac{39}{-9} = -\frac{13}{3}$$

$\left(\frac{4}{9}, -\frac{13}{3}\right)$

$$13. \begin{vmatrix} 5 & 4 \\ 3 & -6 \end{vmatrix} = -30 - 12 = -42$$

$$x = \frac{\begin{vmatrix} 12 & 4 \\ 3 & -6 \end{vmatrix}}{-42} = \frac{-72 - 12}{-42} = 2$$

$$y = \frac{\begin{vmatrix} 5 & 12 \\ 3 & 3 \end{vmatrix}}{-42} = \frac{15 - 36}{-42} = \frac{1}{2}$$

$\left(2, \frac{1}{2}\right)$

$$14. \begin{vmatrix} 4 & 1 & 6 \\ 2 & 2 & 4 \\ -1 & -1 & 1 \end{vmatrix} = (8 - 4 - 12) - (-12 - 16 + 2) = 18$$

$$x = \frac{\begin{vmatrix} 2 & 1 & 6 \\ 1 & 2 & 4 \\ -5 & -1 & 1 \end{vmatrix}}{18} = \frac{(4 - 20 - 6) - (-60 - 8 + 1)}{18} = \frac{-22 + 67}{18} = \frac{5}{2}$$

$$y = \frac{\begin{vmatrix} 4 & 2 & 6 \\ 2 & 1 & 4 \\ -1 & -5 & 1 \end{vmatrix}}{18} = \frac{(4 - 8 - 60) - (-6 - 80 + 4)}{18} = \frac{-64 + 82}{18} = 1$$

$$z = \frac{\begin{vmatrix} 4 & 1 & 2 \\ 2 & 2 & 1 \\ -1 & -1 & -5 \end{vmatrix}}{18} = \frac{(-40 - 1 - 4) - (-4 - 4 - 10)}{18} = \frac{-45 + 18}{18} = -\frac{3}{2}$$

$\left(\frac{5}{2}, 1, -\frac{3}{2}\right)$

$$15. \begin{vmatrix} 1 & 1 & 4 \\ 2 & -3 & -1 \\ -4 & 2 & 2 \end{vmatrix} = (-6 + 4 + 16) - (48 - 2 + 4) = -36$$

$$x = \frac{\begin{vmatrix} 7 & 1 & 4 \\ -24 & -3 & -1 \\ 8 & 2 & 2 \end{vmatrix}}{-36} = \frac{(-42 - 8 - 192) - (-96 - 14 - 48)}{-36} = \frac{-242 + 158}{-36} = \frac{7}{3}$$

$$y = \frac{\begin{vmatrix} 1 & 7 & 4 \\ 2 & -24 & -1 \\ -4 & 8 & 2 \end{vmatrix}}{-36} = \frac{(-48 + 28 + 64) - (384 - 8 + 28)}{-36} = \frac{44 - 404}{-36} = 10$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 7 \\ 2 & -3 & -24 \\ -4 & 2 & 8 \end{vmatrix}}{-36} = \frac{(-24 + 96 + 28) - (84 - 48 + 16)}{-36} = \frac{100 - 52}{-36} = -\frac{4}{3}$$

$\left(\frac{7}{3}, 10, -\frac{4}{3}\right)$

Chapter 4 continued

$$16. \begin{vmatrix} 3 & 3 & -2 \\ -5 & -2 & -3 \\ 7 & 1 & 6 \end{vmatrix} = (-36 - 63 + 10) - (28 - 9 - 90) \\ = -89 + 71 = -18$$

$$x = \frac{\begin{vmatrix} -18 & 3 & -2 \\ -1 & -2 & -3 \\ 14 & 1 & 6 \end{vmatrix}}{-18}$$

$$= \frac{(216 - 126 + 2) - (56 + 54 - 18)}{-18}$$

$$= \frac{92 - 92}{-18} = 0$$

$$y = \frac{\begin{vmatrix} 3 & -18 & -2 \\ -5 & -1 & -3 \\ 7 & 14 & 6 \end{vmatrix}}{-18}$$

$$= \frac{(-18 + 378 + 140) - (14 - 126 + 540)}{-18}$$

$$= \frac{500 - 428}{-18} = \frac{72}{-18} = -4$$

$$z = \frac{\begin{vmatrix} 3 & 3 & -18 \\ -5 & -2 & -1 \\ 7 & 1 & 14 \end{vmatrix}}{-18}$$

$$= \frac{(-84 - 21 + 90) - (252 - 3 - 210)}{-18}$$

$$= \frac{-15 - 39}{-18} = \frac{-54}{-18} = 3$$

$(0, -4, 3)$

$$17. A = \pm \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 5 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(0 + 0 + 30) - (6 + 0 + 0)]$$

$$= \pm \frac{1}{2} (30 - 6) = \pm \frac{1}{2} (24) = 12 \text{ ft}^2$$

Lesson 4.4

Developing Concepts Activity 4.4 (p. 222)

Exploring the Concept

$$1. AI = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$BI = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$$

$$CI = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$$

When the matrix is multiplied by the identity matrix, the result is the original matrix.

$$2. IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$

$$IB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ -7 & 6 \end{bmatrix}$$

$$IC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix} = \begin{bmatrix} 0.1 & 0.8 \\ 0.6 & 0.3 \end{bmatrix};$$

Yes

$$3. DE = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{Each product is the identity matrix.}$$

$$ED = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$4. \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Drawing Conclusions (p. 222)

$$1. AI = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{Multiplication by the identity matrix is commutative.}$$

$$IA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Any matrix times the identity will result in the original matrix.

2. Multiplying a 2×2 matrix by $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ results in the original matrix, just as multiplying a real number by 1 results in the original number.

$$3. \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

$$\text{Sample answer: } \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

4. The product of a matrix and its inverse is the identity matrix; the product of a nonzero real number and its reciprocal notes

5. No; $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$; you can never multiply zero by any number to get the number 1 needed for the identity matrix.

Chapter 4 continued

$$6. \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2a + 7c & 2b + 7d \\ a + 4c & b + 4d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{array}{l} a + 4c = 0 \\ a = -4c \\ 2a + 7c = 1 \\ 2(-4c) + 7c = 1 \\ -8c + 7c = 1 \\ -c = 1 \\ c = -1 \\ b + 4d = 1 \\ b = 1 - 4d \\ 2(1 - 4d) + 7d = 0 \\ -8d + 7d = -2 \\ -d = -2 \\ d = 2 \end{array}$$

$$b = 1 - 4 \cdot 2$$

$$b = -7$$

$$(4, -7, -1, 2) \quad \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$$

4.4 Guided Practice (p. 227)

$$1. \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 2. AB = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. *Sample answer:* To find the inverse of a 2×2 matrix A , first calculate $\det A$ and check that it is not zero. Then switch the upper left and lower right entries of A . Negate the lower left and the upper right entries of A . Multiply this new matrix by the reciprocal of the $\det A$.

4. *Sample answer:* In order for AX to be defined, the number of rows in X must also equal the number of columns in A , so X has 2 rows. The number of columns in the product B must be equal to the number of columns in X , so X has 2 columns.

5. $\det B = (8 - 8) = 0$; No

$$6. \frac{1}{-8 + 9} \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}$$

$$7. \frac{1}{3} \begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{2}{3} \\ 0 & -1 \end{bmatrix}$$

$$8. -\frac{1}{4} \begin{bmatrix} 4 & 0 \\ -6 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ \frac{3}{2} & \frac{1}{4} \end{bmatrix}$$

$$9. \frac{1}{\frac{65}{8}} \begin{bmatrix} \frac{1}{4} & -4 \\ 2 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{65} & -\frac{32}{65} \\ \frac{16}{65} & \frac{4}{65} \end{bmatrix}$$

$$10. \frac{1}{-5.5} \begin{bmatrix} 4 & -3 \\ -2.5 & 0.5 \end{bmatrix} = \begin{bmatrix} -\frac{8}{11} & \frac{6}{11} \\ \frac{5}{11} & -\frac{1}{11} \end{bmatrix}$$

$$11. \frac{1}{-6.08} \begin{bmatrix} 0.2 & -2 \\ -3.2 & 1.6 \end{bmatrix} = \begin{bmatrix} -0.0329 & 0.3289 \\ 0.5263 & -0.2632 \end{bmatrix}$$

12. $\det D = -5 + 6 = 1$

$$D^{-1} = \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$

$$[-71 \quad 39] \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [7 \quad 18] \quad \text{GR}$$

$$[-35 \quad 20] \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [5 \quad 5] \quad \text{EE}$$

$$[-118 \quad 69] \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [20 \quad 9] \quad \text{TI}$$

$$[-84 \quad 49] \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [14 \quad 7] \quad \text{NG}$$

$$[-95 \quad 57] \begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix} = [19 \quad 0] \quad \text{S—}$$

GREETINGS

4.4 Practice and Applications (pp. 227-229)

$$13. \frac{1}{1} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$$

$$14. \frac{1}{2} \begin{bmatrix} 3 & -2 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -1 \\ -4 & 3 \end{bmatrix}$$

$$15. \frac{1}{-1} \begin{bmatrix} 7 & -8 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 1 & -1 \end{bmatrix}$$

$$16. \frac{1}{1} \begin{bmatrix} -3 & -17 \\ -1 & -6 \end{bmatrix} = \begin{bmatrix} -3 & -17 \\ -1 & -6 \end{bmatrix}$$

$$17. \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -3 & 7 \end{bmatrix}$$

$$18. \frac{1}{-15} \begin{bmatrix} 1 & 2 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} -\frac{1}{15} & -\frac{2}{15} \\ -\frac{4}{15} & \frac{7}{15} \end{bmatrix}$$

$$19. \frac{1}{2} \begin{bmatrix} 2 & 7 \\ -2 & -6 \end{bmatrix} = \begin{bmatrix} 1 & \frac{7}{2} \\ -1 & -3 \end{bmatrix}$$

$$20. \frac{1}{4} \begin{bmatrix} 4 & 4 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & \frac{5}{4} \end{bmatrix} \quad 21. \frac{1}{6} \begin{bmatrix} 3 & 3 \\ 9 & 11 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{3}{2} & \frac{11}{6} \end{bmatrix}$$

$$22. \frac{1}{\frac{5}{2}} \begin{bmatrix} 1 & -\frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}$$

$$23. \frac{1}{2} \begin{bmatrix} 10 & -2.5 \\ -8 & 2.2 \end{bmatrix} = \begin{bmatrix} 5 & -1.25 \\ -4 & 1.1 \end{bmatrix}$$

$$24. \frac{1}{\frac{11}{4}} \begin{bmatrix} \frac{5}{2} & -\frac{3}{4} \\ 1 & \frac{4}{5} \end{bmatrix} = \begin{bmatrix} \frac{10}{11} & -\frac{3}{11} \\ \frac{4}{11} & \frac{16}{55} \end{bmatrix}$$

Chapter 4 continued

$$25. A^{-1} = -\frac{1}{25} \begin{bmatrix} 5 & 13 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} -5 & -13 \\ 0 & 5 \end{bmatrix} x = \begin{bmatrix} -\frac{1}{5} & -\frac{13}{25} \\ 0 & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{37}{25} & -\frac{1}{5} \\ -\frac{4}{5} & 0 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{37}{25} & -\frac{1}{5} \\ -\frac{4}{5} & 0 \end{bmatrix}$$

$$26. A^{-1} = \frac{1}{18} \begin{bmatrix} 2 & 1 \\ -8 & 5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{18} \\ -\frac{4}{9} & \frac{5}{18} \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 8 & 2 \end{bmatrix} x = \begin{bmatrix} \frac{1}{9} & \frac{1}{18} \\ -\frac{4}{9} & \frac{5}{18} \end{bmatrix} \begin{bmatrix} 17 & 20 \\ 26 & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{10}{3} & \frac{10}{3} \\ -\frac{1}{3} & -\frac{10}{3} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{10}{3} & \frac{10}{3} \\ -\frac{1}{3} & -\frac{10}{3} \end{bmatrix}$$

$$27. A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{1}{2} & -2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 6 \\ 3 & -1 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} -4 & 2 & -7 \\ 3 & -1 & 5 \end{bmatrix}$$

$$x = \begin{bmatrix} -4 & 2 & -7 \\ 3 & -1 & 5 \end{bmatrix}$$

$$28. A^{-1} = \frac{1}{7} \begin{bmatrix} 1 & 3 \\ -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{5}{7} \end{bmatrix} \begin{bmatrix} -5 & -3 \\ 4 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{1}{7} & \frac{3}{7} \\ -\frac{4}{7} & -\frac{5}{7} \end{bmatrix}$$

$$\begin{bmatrix} -12 & -5 & 18 \\ 4 & -3 & -13 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 0 & -2 & -3 \\ 4 & 5 & -1 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 & -2 & -3 \\ 4 & 5 & -1 \end{bmatrix}$$

$$29. A^{-1} = \frac{1}{5} \begin{bmatrix} 4 & -7 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

$$\begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 1 & 4 \end{bmatrix} x = \begin{bmatrix} \frac{4}{5} & -\frac{7}{5} \\ -\frac{1}{5} & \frac{3}{5} \end{bmatrix} \begin{bmatrix} -1 & -8 \\ -3 & -24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} \frac{17}{5} & \frac{136}{5} \\ -\frac{8}{5} & -\frac{64}{5} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{17}{5} & \frac{136}{5} \\ -\frac{8}{5} & -\frac{64}{5} \end{bmatrix}$$

$$30. A^{-1} = \frac{1}{1} \begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix} \begin{bmatrix} -7 & -9 \\ 4 & 5 \end{bmatrix} x = \begin{bmatrix} 5 & 9 \\ -4 & -7 \end{bmatrix} \begin{bmatrix} -2 & 5 \\ 2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 8 & -2 \\ -6 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 8 & -2 \\ -6 & 1 \end{bmatrix}$$

$$31. A^{-1} = \frac{1}{2} \begin{bmatrix} 6 & -2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -4 & 6 \end{bmatrix} x = \begin{bmatrix} 3 & -1 \\ 2 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 11 & -2 \\ 8 & -\frac{3}{2} \end{bmatrix}$$

$$x = \begin{bmatrix} 11 & -2 \\ 8 & -\frac{3}{2} \end{bmatrix}$$

$$32. A^{-1} = \frac{1}{10} \begin{bmatrix} -2 & 3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 4 & -3 \\ 6 & -2 \end{bmatrix} x = \begin{bmatrix} -0.2 & 0.3 \\ -0.6 & 0.4 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 13 & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 3.3 & 1.3 \\ 3.4 & -0.6 \end{bmatrix}$$

$$x = \begin{bmatrix} 3.3 & 1.3 \\ 3.4 & -0.6 \end{bmatrix}$$

$$33. A^{-1} = -1 \begin{bmatrix} -1 & 3 \\ -2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & -10 \end{bmatrix}$$

$$B^{-1} = -\frac{1}{19} \begin{bmatrix} -10 & -3 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{10}{19} & \frac{3}{19} \\ \frac{3}{19} & -\frac{1}{19} \end{bmatrix}$$

not inverses of each other

34.

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 5 & 2 & 3 \\ 7 & 3 & 4 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -1 & -11 & 8 \\ 1 & 7 & -5 \\ 1 & 14 & -10 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & -10 & 8 \\ 11 & 7 & -5 \\ 1 & 12 & -10 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 0.333 & 0.133 & 0.2 \\ -3.5 & -0.4 & -2.6 \\ -4.167 & -0.467 & -3.2 \end{bmatrix}$$

not inverses of each other

$$35. C = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix} \quad C^{-1} = \begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & -4 & 2 \\ 0 & 2 & 1 \\ 4 & -5 & 3 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} 11 & 2 & -8 \\ 4 & 1 & -3 \\ -8 & -1 & 6 \end{bmatrix}$$

inverses of each other

Chapter 4 continued

$$36. E = \begin{bmatrix} 10 & 2 & -25 \\ 4 & 1 & -10 \\ -9 & -2 & 23 \end{bmatrix} \quad E^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ -2 & 5 & 0 \\ 1 & 2 & 2 \end{bmatrix}$$

$$H = \begin{bmatrix} 3 & 4 & 5 \\ -2 & 5 & 0 \\ 1 & 2 & 2 \end{bmatrix} \quad H^{-1} = \begin{bmatrix} 10 & 2 & -25 \\ 4 & 1 & -10 \\ -9 & -2 & 23 \end{bmatrix}$$

inverses of each other

37.

$$A = \begin{bmatrix} -3 & 4 & 5 \\ 1 & 5 & 0 \\ 5 & 2 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -0.065 & -0.013 & 0.163 \\ 0.013 & 0.203 & -0.033 \\ 0.15 & -0.17 & 0.124 \end{bmatrix}$$

$$A \cdot A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

38.

$$B = \begin{bmatrix} -7 & 0 & -6 \\ -4 & 1 & 3 \\ 11 & -3 & -9 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 0 & -3 & -1 \\ 0.5 & -21.5 & -7.5 \\ -0.167 & 3.5 & 1.167 \end{bmatrix}$$

$$B \cdot B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

39.

$$A = \begin{bmatrix} 2 & 1 & -2 \\ 5 & 3 & 0 \\ 4 & 3 & 8 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 12 & -7 & 3 \\ -20 & 12 & -5 \\ 1.5 & -1 & 0.5 \end{bmatrix}$$

$$A^{-1} \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

40. JOB WELL DONE

$$[10 \ 15][2 \ 0][23 \ 5][12 \ 12][0 \ 4][15 \ 14][5 \ 0]$$

$$[10 \ 15] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-20 \ 55]$$

$$[2 \ 0] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [2 \ -4]$$

$$[23 \ 5] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [13 \ -21]$$

$$[12 \ 12] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-12 \ 36]$$

$$[0 \ 4] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-8 \ 20]$$

$$[15 \ 14] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [-13 \ 40]$$

$$[5 \ 0] \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = [5 \ -10]$$

-20, 55, 2, -4, 13, -21, -12, 36, -8, 20, -13, 40, 5, -10

41. STAY THERE

$$[19 \ 20][1 \ 25][0 \ 20][8 \ 5][18 \ 5]$$

$$[19 \ 20] \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [39 \ 98]$$

$$[1 \ 25] \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [26 \ 77]$$

$$[0 \ 20] \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [20 \ 60]$$

$$[8 \ 5] \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [13 \ 31]$$

$$[18 \ 5] \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = [23 \ 51]$$

39, 98, 26, 77, 20, 60, 13, 31, 23, 51

42. COME TO DINNER

$$[3 \ 15][13 \ 5][0 \ 20][15 \ 0][4 \ 9][14 \ 14][5 \ 18]$$

$$[3 \ 15] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-33 \ 12]$$

$$[13 \ 5] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [37 \ -8]$$

$$[0 \ 20] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-60 \ 20]$$

$$[15 \ 0] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [60 \ -15]$$

$$[4 \ 9] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-11 \ 5]$$

$$[14 \ 14] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [14 \ 0]$$

$$[5 \ 18] \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = [-34 \ 13]$$

-33, 12, 37, -8, -60, 20, 60, -15, -11, 5, 14, 0, -34, 13

43. HAPPY BIRTHDAY

$$[8 \ 1][16 \ 16][25 \ 0][2 \ 9][18 \ 20][8 \ 4][1 \ 25]$$

$$[8 \ 1] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [36 \ -14]$$

$$[16 \ 16] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [16 \ 0]$$

$$[25 \ 0] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [125 \ -50]$$

$$[2 \ 9] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [-26 \ -50]$$

$$[18 \ 20] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [10 \ 4]$$

$$[8 \ 4] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [24 \ -8]$$

$$[1 \ 25] \begin{bmatrix} 5 & -2 \\ -4 & 2 \end{bmatrix} = [-95 \ 48]$$

36, -14, 16, 0, 125, -50, -26, 14, 10, 4, 24, -8, -95, 48

Chapter 4 *continued*

$$44. D = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \quad D^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$(-1 \ 4) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [2 \ 5] \quad \text{BE}$$

$$(30 \ -41) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [19 \ 8] \quad \text{SH}$$

$$(39 \ -58) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \ 1] \quad \text{TA}$$

$$(22 \ -33) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [11 \ 0] \quad \text{K}_-$$

$$(31 \ -46) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [16 \ 1] \quad \text{PA}$$

$$(23 \ -34) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [12 \ 1] \quad \text{LA}$$

$$(1 \ 1) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [3 \ 5] \quad \text{CE}$$

BESHTAK PALACE

$$45. (21 \ -31) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [11 \ 1] \quad \text{KA}$$

$$(22 \ -26) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [18 \ 14] \quad \text{RN}$$

$$(-9 \ 19) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \ 11] \quad \text{AK}$$

$$(-20 \ 40) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \ 11] \quad \text{T}_-$$

$$(-3 \ 11) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \ 13] \quad \text{EM}$$

$$(20 \ -24) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [16 \ 12] \quad \text{PL}$$

$$(10 \ -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \ 0] \quad \text{E}_-$$

KARNAK TEMPLE

$$46. (39 \ -58) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \ 1] \quad \text{TA}$$

$$(-2 \ 12) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [8 \ 18] \quad \text{HR}$$

$$(0 \ 9) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [9 \ 18] \quad \text{IR}$$

$$(-19 \ 38) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [0 \ 19] \quad \text{S}_-$$

$$(13 \ -9) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [17 \ 21] \quad \text{QU}$$

$$(-16 \ 33) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [1 \ 18] \quad \text{AR}$$

$$(10 \ -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \ 0] \quad \text{E}_-$$

TAHRIR SQUARE

$$47. (32 \ -44) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \ 1] \quad \text{TH}$$

$$(10 \ -15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \ 0] \quad \text{E}_-$$

$$(-4 \ 15) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [7 \ 18] \quad \text{GR}$$

$$(9 \ -13) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [5 \ 1] \quad \text{EA}$$

$$(40 \ -60) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [20 \ 0] \quad \text{T}_-$$

$$(22 \ -25) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [19 \ 16] \quad \text{SP}$$

$$(7 \ -6) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [8 \ 9] \quad \text{HI}$$

$$(4 \ 6) \cdot \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = [14 \ 24] \quad \text{NX}$$

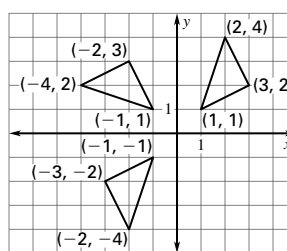
THE GREAT SPHINX

48. Egypt

$$49. \text{ a. } AT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix}$$

$$AAT = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \end{bmatrix}$$



90° rotation

b. Sample answer: Find A^{-1} and then multiply AAT by A^{-1} on the left. $A^{-1}AAT = IAT = AT$. Now multiply AT by A^{-1} on the left: $A^{-1}AT = IT = T$.

50. *Sample answer:* Write the equation in the form $AX = B$. Find A^{-1} and multiply both sides of the equation by A^{-1} on the left.

$$51. \left. \begin{array}{c} 1 \\ 2 \end{array} \right| \begin{array}{cc} 6 & 2 \\ -7 & -2 \end{array} = \left. \begin{array}{c} 3 \\ -\frac{1}{2} \end{array} \right| \begin{array}{c} 1 \\ -1 \end{array}$$

D

Chapter 4 continued

$$8. A = \begin{bmatrix} -1 & -2 \\ 2 & 8 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-4} \begin{bmatrix} 8 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

$$x = \begin{bmatrix} -2 & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{13}{2} \\ \frac{7}{4} \end{bmatrix}$$

$(-\frac{13}{2}, \frac{7}{4})$

$$9. A = \begin{bmatrix} 4 & 3 \\ 6 & -2 \end{bmatrix} \quad A^{-1} = \frac{1}{-26} \begin{bmatrix} -2 & -3 \\ -6 & 4 \end{bmatrix} = \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{1}{13} & \frac{3}{26} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix} \begin{bmatrix} 6 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{21}{13} \\ -\frac{2}{13} \end{bmatrix}$$

$(\frac{21}{13}, -\frac{2}{13})$

$$10. \quad s + b + m = 60,000$$

$$0.12s + 0.8b + 0.5m = 5400$$

$$s = b + m$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0.12 & 0.8 & 0.5 \\ 1 & -1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.5 & 0 & 0.5 \\ -2.8 & 33.3 & -1.17 \\ 3.3 & -33.3 & 0.67 \end{bmatrix}$$

$$x = \begin{bmatrix} 0.5 & 0 & 0.5 \\ -2.8 & 33.3 & -1.17 \\ 3.3 & -33.3 & 0.7 \end{bmatrix} \begin{bmatrix} 60,000 \\ 5,400 \\ 0 \end{bmatrix} = \begin{bmatrix} 30,000 \\ 10,000 \\ 20,000 \end{bmatrix}$$

Stock mutual fund: \$30,000

Bond mutual fund: \$10,000

Money market fund: \$20,000

4.5 Practice and Applications (pp. 233-235)

$$11. \begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \quad 12. \begin{bmatrix} 1 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$13. \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 10 \end{bmatrix} \quad 14. \begin{bmatrix} 2 & -5 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 15 \end{bmatrix}$$

$$15. \begin{bmatrix} 1 & 8 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -11 \end{bmatrix} \quad 16. \begin{bmatrix} 2 & -5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$17. \begin{bmatrix} 1 & -4 & 5 \\ 2 & 1 & -7 \\ -4 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ -23 \\ 38 \end{bmatrix}$$

$$18. \begin{bmatrix} 3 & -1 & 4 \\ 2 & 4 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 10 \\ 31 \end{bmatrix}$$

$$19. \begin{bmatrix} 0.5 & 3.1 & -0.2 \\ 1.2 & -2.5 & 0.7 \\ 0.3 & 4.8 & -4.3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5.9 \\ 2.2 \\ 4.8 \end{bmatrix}$$

$$20. \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 2 \\ 2 & 7 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ -4 \end{bmatrix}$$

$$21. \begin{bmatrix} 0 & 8 & -10 \\ 0 & 6 & -12 \\ -9 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -23 \\ 14 \\ 0 \end{bmatrix}$$

$$22. \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$23. A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

$(5, -7)$

$$24. A = \begin{bmatrix} 1 & 1 \\ 11 & 12 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 12 & -1 \\ -11 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 12 & -1 \\ -11 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 8 \end{bmatrix} = \begin{bmatrix} -20 \\ 19 \end{bmatrix}$$

$(-20, 19)$

$$25. A = \begin{bmatrix} 2 & 7 \\ 1 & 3 \end{bmatrix} \quad A^{-1} = -\begin{bmatrix} 3 & -7 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix}$$

$$x = \begin{bmatrix} -3 & 7 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -53 \\ -22 \end{bmatrix} = \begin{bmatrix} 5 \\ -9 \end{bmatrix}$$

$(5, -9)$

$$26. A = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$$

$(4, -4)$

$$27. A = \begin{bmatrix} 5 & -7 \\ 2 & -4 \end{bmatrix} \quad A^{-1} = \frac{1}{-6} \begin{bmatrix} -4 & 7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{7}{6} \\ \frac{1}{3} & -\frac{5}{6} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{2}{3} & -\frac{7}{6} \\ \frac{1}{3} & -\frac{5}{6} \end{bmatrix} \begin{bmatrix} 54 \\ 30 \end{bmatrix} = \begin{bmatrix} 1 \\ -7 \end{bmatrix}$$

$(1, -7)$

$$28. A = \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix} \quad A^{-1} = -\begin{bmatrix} 3 & 7 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix}$$

$$x = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} -9 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

$(6, -3)$

$$29. A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} -3 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -9 \\ 14 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \end{bmatrix}$$

$(-1, -4)$

Chapter 4 *continued*

$$30. A = \begin{bmatrix} 2 & 4 \\ 2 & 5 \end{bmatrix} \quad A^{-1} = \frac{1}{2} \begin{bmatrix} 5 & -4 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & -2 \\ -1 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{5}{2} & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -26 \\ -31 \end{bmatrix} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$$

$(-3, -5)$

$$31. A = \begin{bmatrix} 9 & -5 \\ -2 & 2 \end{bmatrix} \quad A^{-1} = \frac{1}{8} \begin{bmatrix} 2 & 5 \\ 2 & 9 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{9}{8} \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{1}{4} & \frac{5}{8} \\ \frac{1}{4} & \frac{9}{8} \end{bmatrix} \begin{bmatrix} 43 \\ -22 \end{bmatrix} = \begin{bmatrix} -3 \\ -14 \end{bmatrix}$$

$(-3, -14)$

$$32. x = \begin{bmatrix} -1 & -11 & 8 \\ 1 & 7 & -5 \\ 1 & 14 & -10 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -82 \\ 51 \\ 104 \end{bmatrix}$$

$(-82, 51, 104)$

$$33. x = \begin{bmatrix} 1 & 3 & 5 \\ -3 & -9 & -16 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} 9 \\ -30 \\ 4 \end{bmatrix} = \begin{bmatrix} -61 \\ 179 \\ -83 \end{bmatrix}$$

$(-61, 179, -83)$

$$34. A = \begin{bmatrix} 3 & 2 & 0 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 5 & -6 & 2 \\ -7 & 9 & -3 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -6 & 2 \\ -7 & 9 & -3 \\ -1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 13 \\ 13 \\ 9 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix} \quad (5, -1, 0)$$

$$35. A = \begin{bmatrix} -1 & 1 & -3 \\ 3 & -2 & 8 \\ 2 & -2 & 5 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 6 & 1 & 2 \\ 1 & 1 & -1 \\ -2 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 1 & 2 \\ 1 & 1 & -1 \\ -2 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ 14 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix} \quad (4, 3, 1)$$

$$36. A = \begin{bmatrix} 3 & 5 & -5 \\ -4 & 8 & -5 \\ 2 & -5 & 6 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 0.19 & -0.04 & 0.13 \\ 0.12 & 0.24 & 0.29 \\ 0.03 & 0.21 & 0.37 \end{bmatrix}$$

$$\begin{bmatrix} 0.19 & -0.04 & 0.13 \\ 0.12 & 0.24 & 0.29 \\ 0.03 & 0.21 & 0.37 \end{bmatrix} \cdot \begin{bmatrix} 21 \\ 1 \\ -16 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -5 \end{bmatrix} \quad (2, -2, -5)$$

$$37. A = \begin{bmatrix} 2 & 0 & 1 \\ 5 & -1 & 1 \\ -1 & 2 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -4 & 2 & 1 \\ -11 & 5 & 3 \\ 9 & -4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 2 & 1 \\ -11 & 5 & 3 \\ 9 & -4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix} \quad (2, 3, -2)$$

$$38. A = \begin{bmatrix} 4 & 3 & 1 \\ 6 & 1 & 0 \\ 3 & 5 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -0.2 & 0.27 & 0.07 \\ 1.2 & -0.6 & -0.4 \\ -1.8 & 0.73 & 0.93 \end{bmatrix}$$

$$\begin{bmatrix} -0.2 & 0.27 & 0.07 \\ 1.2 & -0.6 & -0.4 \\ -1.8 & 0.73 & 0.93 \end{bmatrix} \cdot \begin{bmatrix} 14 \\ 9 \\ 21 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \quad (1, 3, 1)$$

$$39. A = \begin{bmatrix} 1 & 1 & -3 \\ 2 & 0 & 1 \\ -7 & -2 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 0.4 & 1 & 0.2 \\ -1.8 & -4 & -1.4 \\ -0.8 & -1 & -0.4 \end{bmatrix}$$

$$\begin{bmatrix} 0.4 & 1 & 0.2 \\ -1.8 & -4 & -1.4 \\ -0.8 & -1 & -0.4 \end{bmatrix} \cdot \begin{bmatrix} -17 \\ 12 \\ -11 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} \quad (3, -2, 6)$$

$$40. A = \begin{bmatrix} 1 & 1 \\ 3.5 & 2.25 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -1.8 & 0.8 \\ 2.8 & -0.8 \end{bmatrix}$$

$$\begin{bmatrix} -1.8 & 0.8 \\ 2.8 & -0.8 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 50 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \end{bmatrix}$$

4 adults and 16 children

$$41. A = \begin{bmatrix} 0.7 & 0.72 & 0.73 \\ 0.26 & 0.25 & 0.27 \\ 0.04 & 0.03 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -16.2 & 43.8 & 23.8 \\ 21.6 & -58.4 & 1.6 \\ -4.4 & 15.6 & -24.4 \end{bmatrix}$$

$$\begin{bmatrix} -16.2 & 43.8 & 23.8 \\ 21.6 & -58.4 & 1.6 \\ -4.4 & 15.6 & -24.4 \end{bmatrix} \cdot \begin{bmatrix} 5483 \\ 2009 \\ 129 \end{bmatrix} = \begin{bmatrix} 2239.8 \\ 1313.6 \\ 4067.6 \end{bmatrix}$$

2239.8 g of A 1313.6 g of B 4067.6 g of C

$$42. B = \begin{bmatrix} 0.75 & 0.75 & 0.75 \\ 1 & -1 & -1 \\ 6.5 & 4.5 & 5.5 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} 0.667 & 0.5 & 0 \\ 8 & 0.5 & -1 \\ -7.333 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0.667 & 0.5 & 0 \\ 8 & 0.5 & -1 \\ -7.333 & -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 0 \\ 45 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

4 sheets of iridescent, 3 sheets of red, and 1 sheet of blue

$$43. \quad T + 25W + 5L = 20$$

$$T + 50W + 15L = 35$$

$$T + 100W + 20L = 50$$

$$A = \begin{bmatrix} 1 & 25 & 5 \\ 1 & 50 & 15 \\ 1 & 100 & 20 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1.333 & 0 & -0.333 \\ 0.013 & -0.04 & 0.027 \\ -0.133 & 0.2 & -0.067 \end{bmatrix}$$

$$\begin{bmatrix} 1.333 & 0 & -0.333 \\ 0.013 & -0.04 & 0.027 \\ -0.133 & 0.2 & -0.067 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 35 \\ 50 \end{bmatrix} = \begin{bmatrix} 10 \\ 0.2 \\ 1 \end{bmatrix}$$

Transformers are \$10.00, the wire cost \$0.20 per foot, and the light cost \$1.00.

Chapter 4 continued

44. $S + E + G = 200,000$

$$S = 5E$$

$$0.1(S + E) = G$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -5 & 0 \\ 0.1 & 0.1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.758 & 0.167 & 0.758 \\ 0.152 & -0.167 & 0.152 \\ 0.091 & 0 & -0.909 \end{bmatrix}$$

$$\begin{bmatrix} 0.758 & 0.167 & 0.758 \\ 0.152 & -0.167 & 0.152 \\ 0.091 & 0 & -0.909 \end{bmatrix} \cdot \begin{bmatrix} 200,000 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 151,515 \\ 30,303 \\ 18,182 \end{bmatrix}$$

Salaries: \$151,515

Equipment Maintenance: \$30,303

General Expenses: \$18,182

45. a. $2C + 3M = \$15$

$$3C + 5M = \$24$$

$$7C + 10M = \$50$$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -3 \\ -3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 15 \\ 24 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

The average unit price for each choice of meat and cheese is \$3.

b. $B = \begin{bmatrix} 3 & 5 \\ 7 & 10 \end{bmatrix} \quad B^{-1} = \begin{bmatrix} -2 & 1 \\ 1.4 & -0.6 \end{bmatrix}$

$$\begin{bmatrix} -2 & 1 \\ 1.4 & -0.6 \end{bmatrix} \cdot \begin{bmatrix} 24 \\ 50 \end{bmatrix} = \begin{bmatrix} 2 \\ 3.6 \end{bmatrix}$$

The average unit price for each choice of cheese is \$2 and for each choice of meat is \$3.60.

c. *Sample answer:* The average unit price for the cheese and for the meat is different in parts (a) and (b); perhaps the super basket has a more expensive assortment of meat and a less expensive assortment of cheese than the other baskets.

46. $A^{-1} = \begin{bmatrix} 40 & -3 & -33 & 9 \\ 1 & 0 & -1 & 0 \\ -39 & 3 & 33 & -8 \\ -24 & 2 & 20 & -5 \end{bmatrix}$

$$\begin{bmatrix} 40 & -3 & -33 & 9 \\ 1 & 0 & -1 & 0 \\ -39 & 3 & 33 & -8 \\ -24 & 2 & 20 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 5 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 20 \\ -1 \\ -12 \\ -8 \end{bmatrix}$$

(20, -1, -12, -8)

4.5 Mixed Review (p. 235)

47. $f(8) = \frac{3}{4}(8) - 8 = -2$ 48. $f(11) = -11 + 6 = -5$

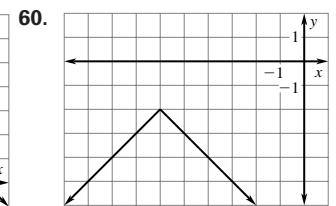
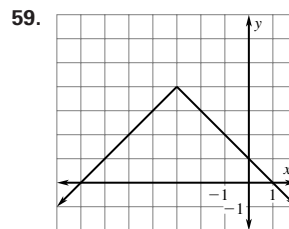
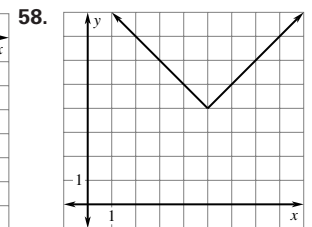
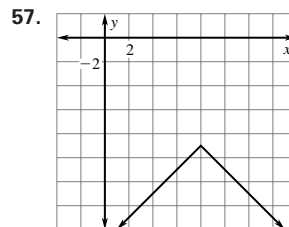
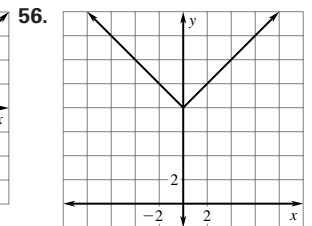
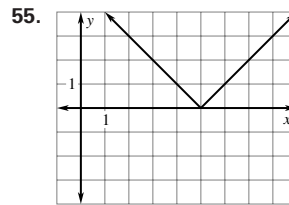
49. $f(-2) = \frac{3}{4}(-2) - 8 = -\frac{19}{2}$

50. $f(0) = \frac{3}{4}(0) - 8 = -8$ 51. $g(3) = 2(3) - 1 = 5$

52. $g(0) = 2(0) - 1 = -1$

53. $g(-1) = 2(-1) - 1 = -3$

54. $g(-3) = \frac{1}{8}(-3) - 8 = \frac{61}{8}$



61. $A^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$

62. $A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$

63. $A^{-1} = \frac{1}{1} \begin{bmatrix} -2 & -17 \\ 1 & 8 \end{bmatrix} = \begin{bmatrix} -2 & -17 \\ 1 & 8 \end{bmatrix}$

64. $A^{-1} = \frac{1}{4} \begin{bmatrix} -1 & 5 \\ -3 & 11 \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{5}{4} \\ -\frac{3}{4} & \frac{11}{4} \end{bmatrix}$

65. $A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ -3 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -\frac{3}{2} & \frac{7}{2} \end{bmatrix}$

66. $A^{-1} = \frac{1}{2} \begin{bmatrix} -2 & 2 \\ -7 & 6 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -\frac{7}{2} & 3 \end{bmatrix}$

Quiz 2 (p. 236)

1. $A^{-1} = \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$

2. $A^{-1} = \frac{1}{1} \begin{bmatrix} -3 & -5 \\ -4 & -7 \end{bmatrix} = \begin{bmatrix} -3 & -5 \\ -4 & -7 \end{bmatrix}$

3. $A^{-1} = \frac{1}{9} \begin{bmatrix} -3 & -1 \\ -9 & -6 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & -\frac{1}{9} \\ -1 & -\frac{2}{3} \end{bmatrix}$

4. $A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & -5 \\ -8 & 6 \end{bmatrix} = \begin{bmatrix} \frac{7}{2} & -\frac{5}{2} \\ -4 & 3 \end{bmatrix}$

Chapter 4 continued

$$5. A = \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix} x = \begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 24 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$6. A = \begin{bmatrix} -9 & 13 \\ 2 & -3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -3 & -13 \\ -2 & -9 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -13 \\ -2 & -9 \end{bmatrix} \begin{bmatrix} -9 & 13 \\ 2 & -3 \end{bmatrix} x = \begin{bmatrix} -3 & -13 \\ -2 & -9 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$x = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$7. A = \begin{bmatrix} 8 & 7 \\ -2 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -2 & -7 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 1 & \frac{7}{2} \\ -1 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{7}{2} \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 8 & 7 \\ -2 & -2 \end{bmatrix} x = \begin{bmatrix} 1 & \frac{7}{2} \\ -1 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

$$8. \quad 4s = 142$$

$$8s + x = 351$$

$$s = 35.5 \quad \text{A place setting costs } \$35.50.$$

$$8(35.5) + x = 351$$

$$x = 67 \quad \text{A serving set costs } \$67.00.$$

Math and History (p. 236)

$$1. \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 39 \\ 34 \\ 26 \end{bmatrix}$$

$$\det A = (27 + 2 + 4) - (3 + 6 + 12) = 33 - 21 = 12$$

$$x = \frac{\begin{vmatrix} 39 & 2 & 1 \\ 34 & 3 & 1 \\ 26 & 2 & 3 \end{vmatrix}}{12} = \frac{(351 + 52 + 68) - (78 + 78 + 204)}{12}$$

$$= \frac{471 - 360}{12} = \frac{111}{12} = 9.25$$

—CONTINUED—

1. —CONTINUED—

$$y = \frac{\begin{vmatrix} 3 & 39 & 1 \\ 2 & 34 & 1 \\ 1 & 26 & 3 \end{vmatrix}}{12} = \frac{(306 + 39 + 52) - (34 + 78 + 234)}{12}$$

$$= \frac{397 - 346}{12} = \frac{51}{12} = 4.25$$

$$z = \frac{\begin{vmatrix} 3 & 2 & 39 \\ 2 & 3 & 34 \\ 1 & 2 & 26 \end{vmatrix}}{12} = \frac{(234 + 68 + 156) - (117 + 204 + 104)}{12}$$

$$= \frac{458 - 425}{12} = \frac{33}{12} = 2.75$$

$$(9.25, 4.25, 2.75)$$

9.25 dou in a bundle of top-grade rice

4.25 dou in a bundle of medium-grade rice

2.75 dou in a bundle of low-grade rice

2. The arrangement is alike in that it looks like a coefficient matrix laid on its side. It is arranged in rows and columns like the modern matrix.

It is different in the fact that it is not set with a variable matrix so that the exact amount could be found.

Chapter 4 Extension (p. 238)

$$1. \begin{bmatrix} 6 & 4 & \vdots & 8 \\ 3 & 3 & \vdots & 9 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 4 & \vdots & 8 \\ 0 & 1 & \vdots & 5 \end{bmatrix} \quad -\frac{1}{2}R_1 + R_2 \rightarrow R_2$$

$$y = 5$$

$$6x + 4(5) = 8$$

$$6x = -12$$

$$x = -2$$

$$(-2, 5)$$

$$2. \begin{bmatrix} 1 & 1 & \vdots & 2 \\ 7 & 8 & \vdots & 21 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & \vdots & 2 \\ 0 & 1 & \vdots & 7 \end{bmatrix} \quad (-7)R_1 + R_2 \rightarrow R_2$$

$$y = 7$$

$$x + 7 = 2$$

$$x = -5$$

$$(-5, 7)$$

Chapter 4 continued

$$3. \begin{bmatrix} 1 & 2 & \vdots & -9 \\ -2 & -3 & \vdots & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & \vdots & -9 \\ 0 & 1 & \vdots & -4 \end{bmatrix} \quad 2R_1 + R_2 \rightarrow R_2$$

$$y = -4$$

$$x + 2(-4) = -9$$

$$x = -1$$

$$(-1, -4)$$

$$4. \begin{bmatrix} 1 & -3 & \vdots & 5 \\ -2 & -4 & \vdots & 20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & \vdots & 5 \\ 0 & -10 & \vdots & 30 \end{bmatrix} \quad 2R_1 + R_2 \rightarrow R_2$$

$$-10y = 30$$

$$y = -3$$

$$x - 3(-3) = 5$$

$$x = -4$$

$$(-4, -3)$$

$$5. \begin{bmatrix} 3 & 2 & \vdots & 2 \\ 5 & -6 & \vdots & 50 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & \vdots & 2 \\ 14 & 0 & \vdots & 56 \end{bmatrix} \quad 3R_1 + R_2 \rightarrow R_2$$

$$14x = 56$$

$$x = 4$$

$$3(4) + 2y = 2$$

$$2y = -10$$

$$y = -5$$

$$(4, -5)$$

$$6. \begin{bmatrix} 1 & 1 & \vdots & -1 \\ 7 & 9 & \vdots & -19 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & \vdots & -1 \\ 0 & 2 & \vdots & -12 \end{bmatrix} \quad -7R_1 + R_2 \rightarrow R_2$$

$$2y = -12$$

$$y = -6$$

$$x - 6 = -1$$

$$x = 5$$

$$(5, -6)$$

$$7. \begin{bmatrix} -2 & -1 & \vdots & -5 \\ 6 & 5 & \vdots & 17 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & \vdots & -5 \\ 0 & 2 & \vdots & 2 \end{bmatrix} \quad 3R_1 + R_2 \rightarrow R_2$$

$$2y = 2$$

$$y = 1$$

$$-2x - 1 = -5$$

$$-2x = -4$$

$$x = 2$$

$$(2, 1)$$

$$8. \begin{bmatrix} 9 & -4 & \vdots & 2 \\ -6 & -16 & \vdots & -6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{4}{9} & \vdots & \frac{2}{9} \\ -6 & -16 & \vdots & -6 \end{bmatrix} \quad \frac{1}{9}R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & -\frac{4}{9} & \vdots & \frac{2}{9} \\ 0 & -\frac{56}{3} & \vdots & -\frac{14}{3} \end{bmatrix} \quad 6R_1 + R_2 \rightarrow R_2$$

$$-\frac{56}{3}y = -\frac{14}{3}$$

$$y = \frac{1}{4}$$

$$9x - 4\left(\frac{1}{4}\right) = 2$$

$$9x = 3$$

$$x = \frac{1}{3}$$

$$\left(\frac{1}{3}, \frac{1}{4}\right)$$

$$9. \begin{bmatrix} -12 & 15 & \vdots & 3 \\ -7 & -20 & \vdots & -4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{15}{12} & \vdots & -\frac{1}{4} \\ -7 & -20 & \vdots & -4 \end{bmatrix} \quad -\frac{1}{12}R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & -\frac{15}{12} & \vdots & -\frac{1}{4} \\ 0 & -\frac{115}{4} & \vdots & -\frac{23}{4} \end{bmatrix} \quad 7R_1 + R_2 \rightarrow R_2$$

$$-\frac{115}{4}y = -\frac{23}{4}$$

$$y = \frac{1}{5}$$

$$-12x + 15\left(\frac{1}{5}\right) = 3$$

$$-12x = 0$$

$$x = 0$$

$$\left(0, \frac{1}{5}\right)$$

$$10. \begin{bmatrix} 2 & 6 & 3 & \vdots & 2 \\ 1 & 3 & 1 & \vdots & 1 \\ 1 & 5 & 2 & \vdots & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 & \vdots & 1 \\ 1 & 3 & 1 & \vdots & 1 \\ 1 & 5 & 2 & \vdots & -1 \end{bmatrix} \quad -R_2 + R_1 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 3 & 2 & \vdots & 1 \\ 0 & 0 & -1 & \vdots & 0 \\ 0 & 2 & 0 & \vdots & -2 \end{bmatrix} \quad \begin{array}{l} R_2 - R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array}$$

$$-z = 0 \quad 2y = -2$$

$$z = 0 \quad y = -1$$

$$2x + 6(-1) + 3(0) = 2$$

$$2x = 8$$

$$x = 4$$

$$(4, -1, 0)$$

Chapter 4 continued

$$11. \begin{bmatrix} 2 & 6 & 3 & \vdots & 8 \\ 1 & 5 & 5 & \vdots & 1 \\ 1 & 3 & 1 & \vdots & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & \vdots & 2 \\ 1 & 5 & 5 & \vdots & 1 \\ 1 & 3 & 1 & \vdots & 3 \end{bmatrix} R_1 - 2R_3 \rightarrow R_1$$

$$\begin{bmatrix} 0 & 0 & 1 & \vdots & 2 \\ 0 & 2 & 4 & \vdots & -2 \\ 1 & 3 & 1 & \vdots & 3 \end{bmatrix} R_2 - R_3 \rightarrow R_2$$

$z = 2$
 $2y + 4(2) = -2 \quad x + 3(-5) + 2 = 3$
 $2y = -10 \quad x = 3 - 2 + 15$
 $y = -5 \quad x = 16$
 $(16, -5, 2)$

$$12. \begin{bmatrix} 2 & 10 & 0 & \vdots & 28 \\ 1 & 3 & 4 & \vdots & 22 \\ 1 & 5 & -1 & \vdots & 10 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 2 & \vdots & 8 \\ 0 & -2 & 5 & \vdots & 12 \\ 1 & 5 & -1 & \vdots & 10 \end{bmatrix} R_1 - 2R_3 \rightarrow R_1$$

$$R_2 - R_3 \rightarrow R_2$$

$2z = 8 \quad -2y + 5(4) = 12$
 $z = 4 \quad -2y = 12 - 20$
 $y = 4$
 $x + 5(4) - 1(4) = 10$
 $x = 10 - 20 + 4$
 $x = -6$

$$13. \begin{bmatrix} 1 & 4 & -2 & \vdots & 3 \\ 1 & 3 & 7 & \vdots & 1 \\ 2 & 9 & 1 & \vdots & 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2 & \vdots & 3 \\ 0 & -1 & 9 & \vdots & -2 \\ 2 & 9 & 1 & \vdots & 8 \end{bmatrix} R_2 - R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & 4 & -2 & \vdots & 3 \\ 0 & -1 & 9 & \vdots & -2 \\ 0 & 1 & 5 & \vdots & 2 \end{bmatrix} R_3 - 2R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & -22 & \vdots & -5 \\ 0 & -1 & 9 & \vdots & -2 \\ 0 & 1 & 5 & \vdots & 2 \end{bmatrix} R_1 - 4R_3 \rightarrow R_1$$

$$\begin{bmatrix} 1 & 0 & -22 & \vdots & -5 \\ 0 & 0 & 14 & \vdots & 0 \\ 0 & 1 & 5 & \vdots & 2 \end{bmatrix} R_2 + R_3 \rightarrow R_2$$

$14z = 0 \quad y + 5(0) = 2 \quad x - 22z = -5$
 $z = 0 \quad y = 2 \quad x = -5$
 $(-5, 2, 0)$

$$14. \begin{bmatrix} 1 & -1 & 3 & \vdots & 6 \\ 1 & -2 & 0 & \vdots & 5 \\ 2 & -2 & 5 & \vdots & 9 \end{bmatrix} \text{interchange } R_1 + R_2$$

$$\begin{bmatrix} 1 & -2 & 0 & \vdots & 5 \\ 1 & -1 & 3 & \vdots & 6 \\ 2 & -2 & 5 & \vdots & 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 & \vdots & 5 \\ 0 & 1 & 3 & \vdots & 1 \\ 2 & -2 & 5 & \vdots & 9 \end{bmatrix} R_2 - R_1 \rightarrow R_2$$

$$\begin{bmatrix} 1 & -2 & 0 & \vdots & 5 \\ 0 & 1 & 3 & \vdots & 1 \\ 0 & 2 & 5 & \vdots & -1 \end{bmatrix} R_3 - 2R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & -2 & 0 & \vdots & 5 \\ 0 & 1 & 3 & \vdots & 1 \\ 0 & 0 & -1 & \vdots & -3 \end{bmatrix} R_3 - 2R_2 \rightarrow R_3$$

$-z = -3$
 $z = 3$
 $y + 3(3) = 1 \quad x - 2(-8) = 5$
 $y = 1 - 9 \quad x = 5 - 16$
 $y = -8 \quad x = -11$
 $(-11, -8, 3)$

$$15. \begin{bmatrix} 1 & 0 & 2 & \vdots & 4 \\ 1 & 1 & 1 & \vdots & 6 \\ 3 & 3 & 4 & \vdots & 28 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 2 & \vdots & 4 \\ 0 & 1 & -1 & \vdots & 2 \\ 0 & 3 & -2 & \vdots & 16 \end{bmatrix} R_2 - R_1 \rightarrow R_2$$

$$R_3 - 3R_1 \rightarrow R_3$$

$$\begin{bmatrix} 1 & 0 & 2 & \vdots & 4 \\ 0 & 1 & -1 & \vdots & 2 \\ 0 & 0 & 1 & \vdots & 10 \end{bmatrix} R_3 - 3R_2 \rightarrow R_3$$

$z = 10$
 $y = 2 + 10 = 12$
 $x = 4 - 20 = -16$
 $(-16, 12, 10)$

$$16. \begin{bmatrix} 1 & -2 & 7 & \vdots & 6 \\ 5 & -10 & 35 & \vdots & 30 \\ 3 & -6 & 21 & \vdots & 18 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 7 & \vdots & 6 \\ 0 & 0 & 0 & \vdots & 0 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix} R_2 - 5R_1 \rightarrow R_2$$

$$R_3 - 3R_1 \rightarrow R_3$$

All entries in R_2 and R_3 become zero; the system has an infinite number of solutions. The solution of the equation $x - 2y + 7z = 6$.

Chapter 4 continued

Chapter 4 Review (p. 240–242)

1. $\begin{bmatrix} 15 & -5 \\ 1 & 5 \end{bmatrix}$

2. Not possible; the matrices have different dimensions.

3. $\begin{bmatrix} 8 & 11 \\ 9 & 13 \\ 8 & 6 \end{bmatrix}$

4. Not possible; the matrices have different dimensions.

5. $\begin{bmatrix} 8 & 12 & -2 \\ 20 & -10 & 4 \\ 0 & 22 & 2 \end{bmatrix}$ 6. $\begin{bmatrix} -1 & 0 \\ 2 & 4 \\ -3 & 1 \end{bmatrix}$

7. $1 = y - 9$

$$10 = y$$

$$-5x = 5$$

$$x = -1$$

$$(-1, 10)$$

8. $4y + 5 = 7$

$$4y = -12$$

$$y = -3$$

$$-1 + 8 = x$$

$$x = 7$$

$$(7, -3)$$

9. $3y - 4 = 11$

$$3y = 15$$

$$y = 5$$

$$4 + x = 3$$

$$x = -1$$

$$(-1, 5)$$

10. $7y - 1 = 6$

$$7y = 7$$

$$y = 1$$

$$-3 - x = -2$$

$$-x = 1$$

$$(-1, 1)$$

11. $\begin{bmatrix} -120 & -84 \\ 40 & 28 \end{bmatrix}$ 12. $\begin{bmatrix} 5 & 24 \\ 25 & -36 \end{bmatrix}$ 13. $\begin{bmatrix} 17 & -29 & 64 \\ 18 & -36 & 72 \end{bmatrix}$

14. $(-18 - 3) = -21$ 15. $(6 + 6) = 12$

16. $(12 + 0 + 0) - (0 - 9 + 8) = 12 - 17 = -5$

17. $(-6 + 6 + 0) - (4 - 8 + 0) = 4$

18. $A = \pm \frac{1}{2} \begin{vmatrix} 0 & 1 & 1 \\ 2 & 4 & 1 \\ 1 & 8 & 1 \end{vmatrix} = \pm \frac{1}{2} [(0 + 1 + 16)]$

$$= \pm \frac{1}{2} (17 - 6) = \frac{11}{2} \text{ unit}^2$$

19. $\begin{vmatrix} 7 & -4 \\ 2 & 5 \end{vmatrix} = (35 + 8) = 43$

$$x = \frac{\begin{vmatrix} -3 & -4 \\ -7 & 5 \end{vmatrix}}{43} = \frac{(-15 - 28)}{43} = -1$$

$$y = \frac{\begin{vmatrix} 7 & -3 \\ 2 & -7 \end{vmatrix}}{43} = -\frac{49 + 6}{43} = -1$$

$$(-1, -1)$$

20. $\begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -4 - 1 = -5$

$$x = \frac{\begin{vmatrix} -2 & 1 \\ 19 & -2 \end{vmatrix}}{-5} = \frac{4 - 19}{-5} = 3$$

$$y = \frac{\begin{vmatrix} 2 & -2 \\ 1 & 19 \end{vmatrix}}{-5} = \frac{38 + 2}{-5} = -8$$

$$(3, -8)$$

21. $\begin{vmatrix} 5 & -4 & 4 \\ -1 & 3 & -2 \\ 4 & 2 & 7 \end{vmatrix} = (105 + 32 + 8) - (48 + 20 + 28)$

$$= 145 - 96 = 49$$

$$x = \frac{\begin{vmatrix} 18 & -4 & 4 \\ 0 & 3 & -2 \\ 3 & -2 & 7 \end{vmatrix}}{49} = \frac{(378 + 24 + 0) - (36 + 72 + 0)}{49}$$

$$= \frac{402 - 108}{49} = \frac{294}{49} = 6$$

$$y = \frac{\begin{vmatrix} 5 & 18 & 4 \\ -1 & 0 & -2 \\ 4 & 3 & 7 \end{vmatrix}}{49} = \frac{(0 - 144 - 12) - (0 - 30 - 126)}{49}$$

$$= -\frac{156 + 156}{49} = 0$$

$$z = \frac{\begin{vmatrix} 5 & -4 & 18 \\ -1 & 3 & 0 \\ 4 & -2 & 3 \end{vmatrix}}{49} = \frac{(45 + 0 + 36) - (216 + 0 + 12)}{49}$$

$$= \frac{81 - 228}{49} = -\frac{149}{49} = -3$$

$$(6, 0, -3)$$

22. $\begin{bmatrix} 11 & -3 \\ -7 & 2 \end{bmatrix}$ 23. $\frac{1}{4} \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} \end{bmatrix}$

24. det is zero; no inverse 25. $\begin{bmatrix} 1 & 1 \\ 5 & 6 \end{bmatrix}$

26. $X = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 0 & 9 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ -5 & -7 \end{bmatrix}$

27. $X = \begin{bmatrix} -3 & -5 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 4 & 3 \end{bmatrix}$

28. $\begin{bmatrix} 9 & 8 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$

$$A^{-1} = -\frac{1}{1} \begin{bmatrix} -1 & -8 \\ 1 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ -1 & -9 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 8 \\ -1 & -9 \end{bmatrix} \begin{bmatrix} -6 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}; (2, -3)$$

Chapter 4 continued

$$29. \begin{bmatrix} 1 & -3 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 17 \end{bmatrix}$$

$$A^{-1} = \frac{1}{18} \begin{bmatrix} 3 & 3 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ -\frac{5}{18} & \frac{1}{18} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ -\frac{5}{18} & \frac{1}{18} \end{bmatrix} \begin{bmatrix} -2 \\ 17 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{3}{2} \end{bmatrix}; \left(\frac{5}{2}, \frac{3}{2} \right)$$

$$30. \begin{bmatrix} 4 & -14 \\ 18 & -12 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ 9 \end{bmatrix}$$

$$A^{-1} = \frac{1}{204} \begin{bmatrix} -12 & 14 \\ -18 & 4 \end{bmatrix}$$

$$X = \frac{1}{204} \begin{bmatrix} -12 & 14 \\ -18 & 4 \end{bmatrix} \begin{bmatrix} -15 \\ 9 \end{bmatrix} = \frac{1}{204} \begin{bmatrix} 306 \\ 306 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{3}{2} \end{bmatrix}; \left(\frac{3}{2}, \frac{3}{2} \right)$$

$$31. A = \begin{bmatrix} 1 & -1 & -4 \\ -1 & 3 & -1 \\ 1 & -1 & 5 \end{bmatrix} \quad A^{-1} = \frac{1}{9} \begin{bmatrix} 7 & \frac{9}{2} & \frac{13}{2} \\ 2 & \frac{9}{2} & \frac{5}{2} \\ -1 & 0 & 1 \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} 7 & \frac{9}{2} & \frac{13}{2} \\ 2 & \frac{9}{2} & \frac{5}{2} \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 36 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}; (4, 1, 0)$$

$$32. A = \begin{bmatrix} 4 & 10 & -1 \\ 11 & 28 & -4 \\ -6 & -15 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -4 & -5 & -12 \\ 2 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -4 & -5 & -12 \\ 2 & 2 & 5 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 19 \\ -9 \\ -11 \end{bmatrix}; (19, -9, -11)$$

$$33. A = \begin{bmatrix} 5 & -3 & 5 \\ 3 & 2 & 4 \\ 2 & -1 & 3 \end{bmatrix} \quad A^{-1} = \frac{1}{9} \begin{bmatrix} 5 & 2 & -11 \\ \frac{1}{2} & \frac{5}{2} & -\frac{5}{2} \\ \frac{7}{2} & \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

$$X = \frac{1}{9} \begin{bmatrix} 5 & 2 & -11 \\ \frac{1}{2} & \frac{5}{2} & -\frac{5}{2} \\ \frac{7}{2} & \frac{1}{2} & \frac{3}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 11 \\ 4 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -27 \\ 18 \\ 36 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}; (-3, 2, 4)$$

Chapter 4 Test (p. 243)

$$1. \begin{bmatrix} 5 & 7 & 3 \\ 1 & -5 & 5 \end{bmatrix} \quad 2. \begin{bmatrix} 2 & 5 & -3 \\ -2 & -1 & 9 \end{bmatrix}$$

$$3. -4 \left(\begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} \right) = \begin{bmatrix} 12 & -8 \\ 4 & -8 \end{bmatrix} \quad 4. \begin{bmatrix} 18 \\ 0 \\ -2 \end{bmatrix}$$

$$5. \begin{bmatrix} -23 & 4 \\ 17 & 8 \end{bmatrix} \quad 6. \begin{bmatrix} 4 & 6 & 0 \\ -5 & -2 & 1 \\ 7 & 12 & -1 \end{bmatrix}$$

$$7. y + 6 = 8 \quad x - 4 = -9$$

$$y = 2 \quad x = -5$$

$$(-5, 2)$$

$$8. -22 = 2x \quad -y = 4 \quad 9. 3x = -15 \quad 24 = y$$

$$-11 = x \quad y = 4 \quad x = -5 \quad y = 24$$

$$(-11, -4) \quad (-5, 24)$$

$$10. \det A = 28 - (27) = 1 \quad 11. 2 + 1 = 3$$

$$12. (0 + 0 + 2) - (10 + 24 + 0) = -32$$

$$13. (0 + 0 - 120) - (0 - 10 + 18) = -128$$

$$14. A = \pm \frac{1}{2} \begin{vmatrix} 2 & 1 & 1 \\ 5 & 3 & 1 \\ 7 & 1 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(6 + 7 + 5) - (21 + 2 + 5)] = 5$$

$$15. A = \pm \frac{1}{2} \begin{vmatrix} -1 & 0 & 1 \\ -3 & 3 & 1 \\ 0 & 4 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(-3 + 0 - 12) - (0 - 4 + 0)] = \frac{11}{2}$$

$$16. A = \pm \frac{1}{2} \begin{vmatrix} -3 & 2 & 1 \\ -1 & 4 & 1 \\ -4 & 3 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(-12 - 8 - 3) - (-16 - 9 - 2)] = 2$$

$$17. A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \det A = 6 - 5 = 1$$

$$x = \frac{\begin{vmatrix} 12 & 1 \\ 27 & 3 \end{vmatrix}}{1} = 36 - 27 = 9$$

$$y = \frac{\begin{vmatrix} 2 & 12 \\ 5 & 27 \end{vmatrix}}{1} = 54 - 60 = -6$$

$$(9, -6)$$

$$18. A = \begin{bmatrix} -4 & 5 \\ 5 & -6 \end{bmatrix} \quad \det A = 24 - 25 = -1$$

$$x = \frac{\begin{vmatrix} -10 & 5 \\ 13 & -6 \end{vmatrix}}{-1} = -(60 - 65) = 5$$

$$y = \frac{\begin{vmatrix} -4 & -10 \\ 5 & 13 \end{vmatrix}}{-1} = -(-52 + 50) = 2$$

$$(5, 2)$$

Chapter 4 continued

$$19. A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\det A = (2 + 1 + 0) - (0 + 1 - 0) = 2$$

$$x = \frac{\begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & -1 \\ -1 & -1 & 1 \end{vmatrix}}{2} = \frac{(4 - 1 + 0) - (-2 + 2 + 0)}{2} = \frac{3}{2}$$

$$y = \frac{\begin{vmatrix} 1 & 2 & 0 \\ 0 & 0 & -1 \\ -1 & -1 & 1 \end{vmatrix}}{2} = \frac{(0 + 2 + 0) - (0 + 1 + 0)}{2} = \frac{1}{2}$$

$$z = \frac{\begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ -1 & -1 & -1 \end{vmatrix}}{2} = \frac{(-2 + 0 + 0) - (-4 + 0 + 0)}{2}$$

$$= 1$$

$$\left(\frac{3}{2}, \frac{1}{2}, 1\right)$$

$$20. A = \begin{bmatrix} 5 & -2 & 7 \\ 2 & 5 & 3 \\ 3 & -1 & 4 \end{bmatrix}$$

$$\det A = (100 - 18 - 14) - (105 - 15 - 16)$$

$$= 68 - 74 = -6$$

$$x = \frac{\begin{vmatrix} 12 & -2 & 7 \\ 10 & 5 & 3 \\ 8 & -1 & 4 \end{vmatrix}}{-6}$$

$$= \frac{(240 - 48 - 70) - (280 - 36 - 80)}{-6}$$

$$= \frac{122 - 164}{-6} = 7$$

$$y = \frac{\begin{vmatrix} 5 & 12 & 7 \\ 2 & 10 & 3 \\ 3 & 8 & 4 \end{vmatrix}}{-6}$$

$$= \frac{(200 + 108 + 112) - (210 + 120 + 96)}{-6}$$

$$= \frac{420 - 426}{-6} = 1$$

$$z = \frac{\begin{vmatrix} 5 & -2 & 12 \\ 2 & 5 & 10 \\ 3 & -1 & 8 \end{vmatrix}}{-6}$$

$$= \frac{(200 - 60 - 24) - (180 - 50 - 32)}{-6} = \frac{116 - 98}{-6} = -3$$

$$(7, 1, -3)$$

$$21. \frac{1}{36 - 15} \begin{bmatrix} 9 & -5 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{7} & -\frac{5}{21} \\ -\frac{1}{7} & \frac{4}{21} \end{bmatrix}$$

$$22. \frac{1}{-1 + 2} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$$

$$23. \frac{1}{30 - 24} \begin{bmatrix} -5 & -4 \\ -6 & -6 \end{bmatrix} = \begin{bmatrix} -\frac{5}{6} & -\frac{2}{3} \\ -1 & -1 \end{bmatrix}$$

$$24. \frac{1}{-5 + 0} \begin{bmatrix} -5 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{5} \end{bmatrix}$$

$$25. A = \begin{bmatrix} 8 & 7 \\ 1 & 1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & -7 \\ -1 & 8 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & -7 \\ -1 & 8 \end{bmatrix} \begin{bmatrix} 3 & -6 \\ -2 & 9 \end{bmatrix} = \begin{bmatrix} 17 & -69 \\ -19 & 78 \end{bmatrix}$$

$$26. A = \begin{bmatrix} 2 & 5 \\ 2 & 6 \end{bmatrix} \quad A^{-1} = \frac{1}{12 - 10} \begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix}$$

$$x = \frac{1}{2} \begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & -5 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -\frac{5}{2} \\ -1 & 1 \end{bmatrix}$$

$$27. A = \begin{bmatrix} 1 & 0 \\ -6 & 2 \end{bmatrix} \quad A^{-1} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 6 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 & 0 \\ 3 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 10 & 6 & 8 \\ 4 & 12 & 2 \end{bmatrix} = \begin{bmatrix} 10 & 6 & 8 \\ 32 & 24 & 25 \end{bmatrix}$$

$$28. A = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -9 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

$$(6, 1)$$

$$29. A = \begin{bmatrix} 3 & 2 \\ -2 & 5 \end{bmatrix} \quad A^{-1} = \frac{1}{19} \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}$$

$$x = \frac{1}{19} \begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -8 \\ 18 \end{bmatrix} = \frac{1}{19} \begin{bmatrix} -76 \\ 38 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$(-4, 2)$$

$$30. A = \begin{bmatrix} 2 & -7 \\ -3 & 11 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 11 & 7 \\ 3 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} 11 & 7 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -10 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

$$(-4, -2)$$

$$31. A = \pm \frac{1}{2} \begin{vmatrix} 1 & 7 & 1 \\ 4 & 5 & 1 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(5 + 14 + 8) - (10 + 2 + 28)]$$

$$= \pm \frac{1}{2} [27 - 40] = \frac{13}{2}$$

Chapter 4 continued

$$32. A = \begin{bmatrix} 2 & -1 \\ 3 & -1 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix}$$

$$[44 \ -15] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [1 \ 14] \quad \text{AN}$$

$$[3 \ -1] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [0 \ 1] \quad \text{—A}$$

$$[80 \ -32] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [16 \ 16] \quad \text{PP}$$

$$[39 \ -17] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [12 \ 5] \quad \text{LE}$$

$$[3 \ -1] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [0 \ 1] \quad \text{—A}$$

$$[12 \ -4] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [0 \ 4] \quad \text{—D}$$

$$[77 \ -26] \begin{bmatrix} -1 & 1 \\ -3 & 2 \end{bmatrix} = [1 \ 25] \quad \text{AY}$$

AN APPLE A DAY

$$33. \quad x + y = 5 \quad A = \begin{bmatrix} 1 & 1 \\ 1.5 & 5 \end{bmatrix} \quad \det A = 5 - 1.5 = 3.5$$

$$1.5x + 5y = 18$$

$$x = \frac{\begin{vmatrix} 5 & 1 \\ 18 & 5 \end{vmatrix}}{3.5} = \frac{25 - 18}{3.5} = \frac{7}{3.5} = 2$$

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 1.5 & 18 \end{vmatrix}}{3.5} = \frac{18 - 7.5}{3.5} = \frac{10.5}{3.5} = 3$$

You should make your lunch 2 times and buy your lunch 3 times a week.

Chapter 4 Standardized Test (p. 244–245)

$$1. \quad 2 \left(\begin{bmatrix} 2 & -12 \\ -7 & 9 \end{bmatrix} \right) = \begin{bmatrix} 4 & -12 \\ -14 & 18 \end{bmatrix} \quad \text{B}$$

$$2. \quad -4x = -16 \quad -20y(4) = y$$

$$x = 4 \quad y = -80$$

D

$$3. \quad \begin{bmatrix} 19 & -2 \\ 13 & 2 \\ -2 & -3 \end{bmatrix} \quad \text{A} \quad 4. \quad (4 + 0 - 60) - (0 + 16 + 6)$$

$$= (-56 - 22)$$

$$= -78$$

A

$$5. \quad \frac{1}{-36 + 35} \begin{bmatrix} -4 & 5 \\ -7 & 9 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ 7 & -9 \end{bmatrix}$$

D

$$6. \quad A = \begin{bmatrix} 4 & 6 \\ -6 & -9 \end{bmatrix} \quad \det A = -36 + 36 = 0$$

no inverse

B

$$7. \quad A = \pm \frac{1}{2} \begin{vmatrix} -3 & 4 & 1 \\ 6 & 3 & 1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \pm \frac{1}{2} [(-9 + 8 - 6) - (6 + 3 + 24)]$$

$$= \pm \frac{1}{2} [-7 - 33] = 20$$

D

$$8. \quad A = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix}$$

$$x = \begin{bmatrix} 3 & -5 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \end{bmatrix}$$

C

$$9. \quad \det A = (0 + 8) = 8$$

$$\det B(-10 + 3) = -7$$

A

	T-Shirts Sold	Sweatshirts Sold	T-Shirts left	Sweatshirts left
S	4	0	21	20
M	5	3	20	17
L	6	5	94	35
XL	2	4	48	16

$$b. \quad [17 \ 12]; \begin{bmatrix} 2 \\ 5 \end{bmatrix}; [94]$$

The groups profit from the first basketball game was \$94.

$$11. \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$a. \quad SA = [19 \ 1] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [20 \ 21]$$

$$TU = [20 \ 21] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [41 \ 62]$$

$$RD = [18 \ 4] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [22 \ 26]$$

$$AY = [1 \ 25] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [26 \ 51]$$

20, 21, 41, 62, 22, 26, 51

$$b. \quad [20 \ 21] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [19 \ 1]$$

$$[41 \ 62] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [20 \ 21]$$

$$[22 \ 26] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [18 \ 4]$$

SATURDAY

Chapter 4 *continued*

$$\text{c. } [34 \ 49] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 19 \\ 15 \end{bmatrix} \quad \text{SO}$$

$$[6 \ 9] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [3 \ 3] \quad \text{CC}$$

$$[23 \ 41] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [5 \ 18] \quad \text{ER}$$

$$[6 \ 12] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [0 \ 6] \quad \text{—F}$$

$$[14 \ 19] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [9 \ 5] \quad \text{IE}$$

$$[16 \ 20] \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} = [12 \ 4] \quad \text{LD}$$

SOCCER FIELD

$$[19 \ 15] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [34 \ 49]$$

$$[3 \ 3] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [6 \ 9]$$

$$[5 \ 18] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [23 \ 41]$$

$$[0 \ 6] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [6 \ 12]$$

$$[9 \ 5] \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = [14 \ 19]$$

$$[12 \ 4] \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = [16 \ 20]$$

$$12. \ A = \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix} \quad \det A = -5 + 6 = 1$$

$$A^{-1} = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}$$

$$\text{a. } x = \frac{\begin{vmatrix} 1 & -2 \\ 4 & -5 \end{vmatrix}}{1} = \frac{-5 + 8}{1} = 3$$

$$y = \frac{\begin{vmatrix} 1 & 1 \\ 3 & 4 \end{vmatrix}}{1} = \frac{4 - 3}{1} = 1$$

(3, 1)

$$\text{b. } x = \begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(3, 1)

$$\text{c. } x = 1 + 2y$$

$$3(1 + 2y) - 5y = 4$$

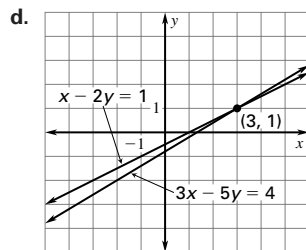
$$3 + 6y - 5y = 4$$

$$y = 1$$

$$x = 1 + 2(1)$$

$$x = 3$$

(3, 1)



e. *Sample answer:* Solving it with the inverse matrix was a direct way to find x and y .