CHAPTER 8

Lesson 8.1

Think & Discuss *(p.463)*

1. Atmospheric pressure decreases as altitude increases.
2. About 7 lb/in^2.

Skills Review *(p. 464)*

1. **2.** **3.**

4. **5.**

6. as

as

7. as

as

8. as

as

9. as

as

10.

***Activity Developing Concepts** (p. 465)*

1.

The graph passes through the point $\left(0, \frac{1}{3}\right)$. The x-axis is an asymptote of the graph. The domain is all real numbers. The range is $y > 0$.

Lesson 8.2

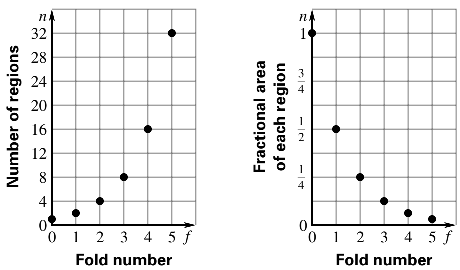
Developing Concepts Activity 8.2 (p. 473)
Exploring the Concept

2. Original piece of paper is folded into 4 regions; $\frac{1}{4}$ of the paper's area is in each region.

3.

Fold Number	0	1	2	3	4	5
Number of Regions	1	2	4	8	16	32
Fractional Area of each Region	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

4.



Drawing Conclusions (p. 473)

1. $y = 2^x$ **2.** $y = 2^8 = 256$ **3.** $y = \left(\frac{1}{2}\right)^x$

4. $y = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$

5. $(2^x)\left(\frac{1}{2}\right)^x = \left(\frac{2}{2}\right)^x = 1$

The original area is one. At each stage the number of regions times the area of each region must continue to equal one whole.

8.2 Guided Practice (p. 477)

1. $y = 1500(0.65)^t$

initial amount $= 1500$

decay factor $= 0.65$

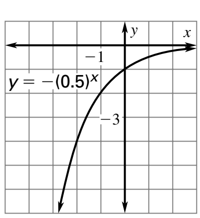
percent decrease is 35% $(1 - 0.65)$

2. $y = 2\left(\frac{1}{5}\right)^{x-2} + 3$

The asymptote is $y = 3$.

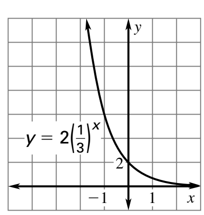
3. $0 < b < 1$

4.



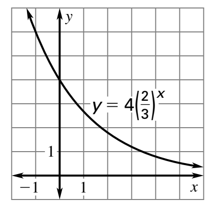
Domain: all real numbers
Range: $y < 0$

5.



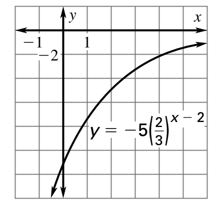
Domain: all real numbers
Range: $y > 0$

6.



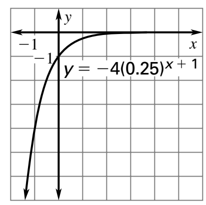
Domain: all real numbers
Range: $y > 0$

7.



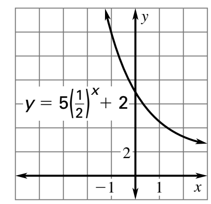
Domain: all real numbers
Range: $y < 0$

8.



Domain: all real numbers
Range: $y < 0$

9.



Domain: all real numbers
Range: $y > 2$

10. $y = 50(0.92)^t$

a. initial amount 50 g **b.** 8% $(1 - 0.08) = (0.92)$

8.2 Practice and Applications (pp. 477–479)

11. $f(x) = 4\left(\frac{3}{8}\right)^x$ exponential decay

12. $f(x) = 10 \cdot 3^x$ exponential growth

13. $f(x) = 8 \cdot 7^{-x} = 8 \cdot \left(\frac{1}{7}\right)^x$ exponential decay

14. $f(x) = 8 \cdot 7^x$ exponential growth

15. $f(x) = 5\left(\frac{1}{8}\right)^{-x} = 5 \cdot (8)^x$ exponential growth

16. $f(x) = 3\left(\frac{4}{3}\right)^x$ exponential growth

17. $f(x) = 8\left(\frac{2}{3}\right)^x$ exponential decay

18. $f(x) = 5(0.25)^{-x} = 5\left(\frac{1}{0.25}\right)^x = 5 \cdot (4)^x$

exponential growth

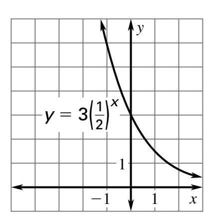
19. $y = (0.25)^x = \left(\frac{1}{4}\right)^x$ (F) **20.** $y = -3^{x-1} + 3$ (E)

21. $y = -\left(\frac{1}{3}\right)^{x-1} + 3$ (D) **22.** $y = \left(\frac{1}{2}\right)^{x-1}$ (B)

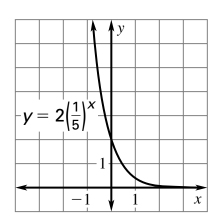
23. $y = -(0.25)^x = -\left(\frac{1}{4}\right)^x$ (C)

24. $y = (0.5)^x - 1$ (A)

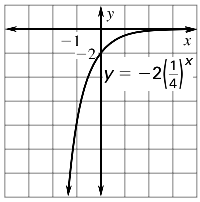
25. $y = 3\left(\frac{1}{2}\right)^x$



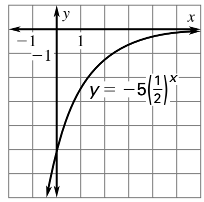
26. $y = 2\left(\frac{1}{5}\right)^x$



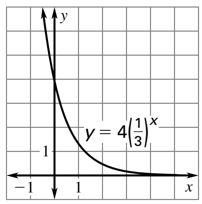
27. $y = -2\left(\frac{1}{4}\right)^x$



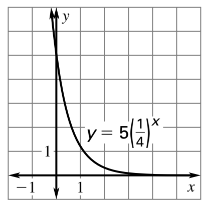
28. $y = -5\left(\frac{1}{2}\right)^x$



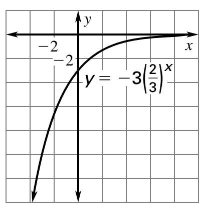
29. $y = 4\left(\frac{1}{3}\right)^x$



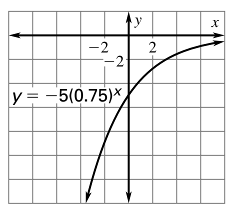
30. $y = 5\left(\frac{1}{4}\right)^x$



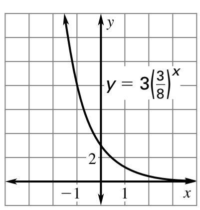
31. $y = -3\left(\frac{2}{3}\right)^x$



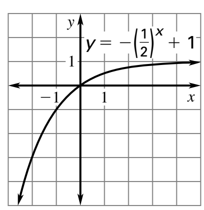
32. $y = -5(0.75)^x$



33. $y = 3\left(\frac{3}{8}\right)^x$

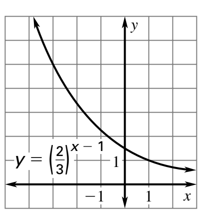


34. $y = -\left(\frac{1}{2}\right)^x + 1$



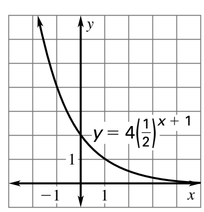
Domain: all real numbers
Range: $y < 1$

35. $y = \left(\frac{2}{3}\right)^{x-1}$



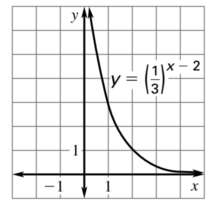
Domain: all real numbers
Range: $y > 0$

36. $y = 4\left(\frac{1}{2}\right)^{x+1}$



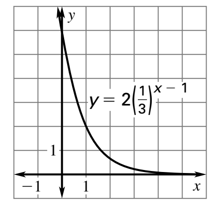
Domain: all real numbers
Range: $y > 0$

37. $y = \left(\frac{1}{3}\right)^{x-2}$



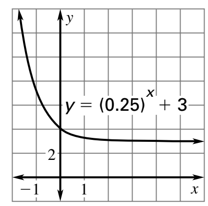
Domain: all real numbers
Range: $y > 0$

38. $y = 2\left(\frac{1}{3}\right)^{x-1}$



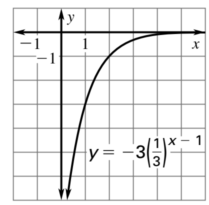
Domain: all real numbers
Range: $y > 0$

39. $y = (0.25)^x + 3$



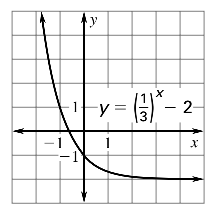
Domain: all real numbers
Range: $y > 3$

40. $y = -3\left(\frac{1}{3}\right)^{x-1}$



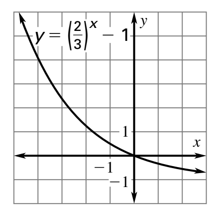
Domain: all real numbers
Range: $y < 0$

41. $y = \left(\frac{1}{3}\right)^x - 2$



Domain: all real numbers
Range: $y > -2$

42. $y = \left(\frac{2}{3}\right)^x - 1$



Domain: all real numbers
Range: $y > -1$

43. $V = 780(0.95)^t$ **44.** $c = 120(0.88)^h$

45. $i = 400(0.71)^h$

46. $P = 100(0.99997)^t$

$P = 100(0.99997)^{20{,}000}$

$P \approx 54.88$ g

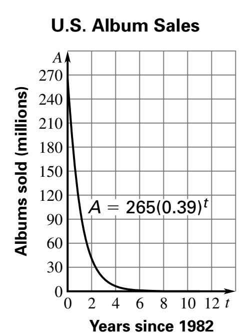
47. $A = 265(0.39)^t$

initial amount: 265

decay factor: 0.39

annual percent decrease: 61%

48.



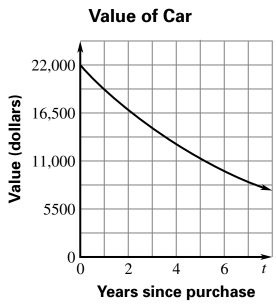
49. The graph shows 1 million albums sold in about 6 years since 1982, or 1988.

50. $V = 22{,}000(0.875)^t$

$= 22{,}000(0.875)^3$

$= \$14{,}738$

51.



52. Estimate value $= \$8{,}000$

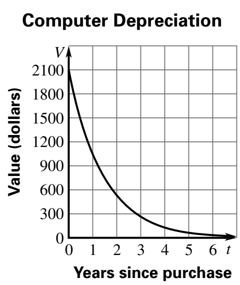
≈ 7.6 years

53. $V = \$2{,}100(0.5)^t$

$= \$2{,}100(0.5)^2$

$= \$525$

54.



55. Value $600 \approx 22$ months

56. $V = 500(0.88)^n$

$= 500(0.88)^{240}$

$= 2.4 \times 10^{-11}$ mL

57. a. $V = 18{,}354(0.83)^t$

b. $$A(n) = \left[18{,}354 - \frac{280}{\left(\frac{0.085}{12}\right)}\right]\left(1 + \frac{0.085}{12}\right)^n + \frac{280}{\left(\frac{0.085}{12}\right)}$$

$$= \left(18{,}354 - \frac{280}{0.0071}\right)(1 + 0.0071)^n + \frac{280}{0.0071}$$

$$= (18{,}354 - 39{,}437)(1.0071)^n + 39{,}437$$

$$= (-21{,}083)(1.007)^n + 39{,}437$$

c.

Value of the car	*Payoff*	*Years after purchase*
\$15,234	\$16,486	1
\$12,644	\$14,452	2
\$10,494	\$12,238	3
\$8,711	\$9,828	4
\$7,230	\$7,205	5

$A_{(12)} = (-21{,}083)(1.0071)^{12} + 39{,}437$

$= (-22{,}951) + 39{,}437$

$= \$16{,}486$

$A_{(24)} = (-21{,}083)(1.0071)^{24} + 39{,}437$

$= (-24{,}985) + 39{,}437$

$= \$14{,}452$

$A_{(36)} = (-21{,}083)(1.0071)^{36} + 39{,}437$

$= (-27{,}199) + 39{,}437$

$= \$12{,}238$

$A_{(48)} = (-21{,}083)(1.0071)^{48} + 39{,}437$

$= (-29{,}609) + 39{,}437$

$= \$9{,}828$

$A_{(60)} = (-21{,}083)(1.0071)^{60} + 39{,}437$

$= (-32{,}232) + 39{,}437$

$= \$7{,}205$

It would make the most sense to sell the car after the fifth year, when the value is more than the amount owed. You could sell the car for enough money to pay off the rest of the loan.

58. The product of two exponential decay functions is always another exponential decay function. Because b is less than one and greater than zero, then the products of two is another exponential decay function.

$y = ab^x,\ 0 < b < 1$ let $b = \dfrac{1}{c}$

$$y = a\left(\frac{1}{c}\right)^x \cdot a\left(\frac{1}{c}\right)^x$$

$$y = a^2\left(\frac{1}{c}\right)^{2x}$$

The quotient of two exponential decay function is not always another exponential decay function.

$y = ab^x,\ 0 < b < 1$

$$y = \frac{ab^x}{ab^x} = ab^x \cdot ab^{-x} = a^2b^{x-x} = a^2b^0 = a^2$$

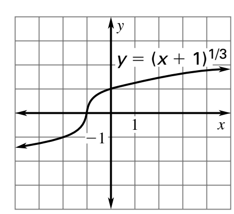
Example:

$$y = \frac{2\left(\frac{1}{3}\right)^x}{2\left(\frac{1}{3}\right)^x} = 2\left(\tfrac{1}{3}\right)^x \cdot 2\left(\tfrac{1}{3}\right)^{-x} = 4\left(\tfrac{1}{3}\right)^{x-x} = 4\left(\tfrac{1}{3}\right)^0 = 4$$

Mixed Review (p. 479)

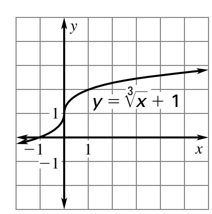
59. $y = (x + 1)^{1/3} = \sqrt[3]{(x + 1)}$

x	y
0	1
7	2
-2	-1
-9	-2



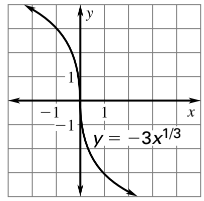
60. $y = \sqrt[3]{x} + 1$

x	y
1	2
8	3
0	1
-1	0
-8	-1



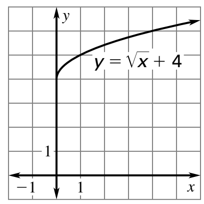
61. $y = -3x^{1/3} = -3\sqrt[3]{x}$

x	y
0	0
1	-3
-1	3
8	-6
-8	6



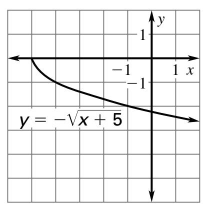
62. $y = \sqrt{x} + 4$

x	y
0	4
1	5
4	6
2	5.41
3	5.73



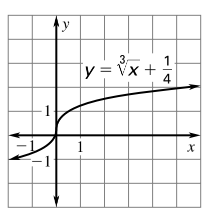
63. $y = -\sqrt{x+5}$

x	y
0	$-\sqrt{5} \approx -2.24$
1	$-\sqrt{6} \approx -2.45$
-1	-2
2	$-\sqrt{7} \approx -2.65$
-2	$-\sqrt{3} \approx -1.73$
3	$-\sqrt{8} \approx -2.83$
-3	$-\sqrt{2} \approx -1.41$
4	-3
-4	-1
-5	0



64. $y = \sqrt[3]{x} + \frac{1}{4}$

x	y
0	$\frac{1}{4}$
1	$\frac{5}{4}$
-1	$-\frac{3}{4}$
8	$\frac{9}{4}$
-8	$-\frac{7}{4}$



65. 11, 18, 13, 15, 17, 15, 23, 20, 12

Mean:

$11 + 18 + 13 + 15 + 17 + 15 + 23 + 20 + 12 = 144$

$144 \div 9 = 16$

Median: 11 12 13 15 <u>15</u> 17 18 20 23; 15

Mode: 15 appears twice; 15

Range: $23 - 11 = 12$

66. 25, 30, 32, 42, 31, 33, 36, 22

Mean:

$25 + 30 + 32 + 42 + 31 + 33 + 36 + 22 = 251$

$251 \div 8 = 31.375$

Median: 22 25 30 31 ↓ 32 33 36 42; 31.5

Mode: none

Range: $42 - 22 = 20$

67. a. $A = 2000\left(1 + \frac{0.07}{4}\right)^{4 \cdot 4}$

$= 2000(1.0175)^{16}$

$= \$2639.86$

b. $A = 2000\left(1 + \frac{0.05}{12}\right)^{12 \cdot 4}$

$= 2000\left(1 + \frac{0.05}{12}\right)^{48}$

$= \$2441.79$

Developing Concepts Activity (p. 480)

1.

n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.594	2.705	2.717	2.718	2.718	2.718

2. Yes; approaching the fixed decimal 2.718

Lesson 8.3

Guided Practice (p. 483)

1. The Euler number, e, is the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \to +\infty$; 2.718

2. $f(x) = \frac{1}{4}e^{2x}$ is an example of exponential growth, because $\frac{1}{4} > 0$ and $e^2 > 1$.

3. No, since e is irrational.

4. $e^2 \cdot e^6 = e^8$ **5.** $e^{-2} \cdot 3e^7 = 3e^{-2+7} = 3e^5$

6. $(2e^{5x})^2 = 4e^{10x}$ **7.** $(4e^{-2})^3 = 64e^{-6} = \frac{64}{e^6}$

8. $\left(\frac{1}{2}e^{-2}\right)^4 = \frac{1}{16}e^{-8} = \frac{1}{16e^8}$

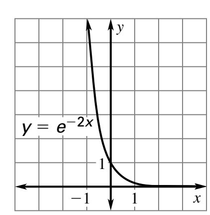
9. $\sqrt{36e^{4x}} = (36e^{4x})^{1/2} = 6e^{2x}$ **10.** $\frac{e^x}{e^{2x}} = \frac{1}{e^x}$

11. $\frac{12e^4}{36e^{-2}} = \frac{1}{3}e^{4+2} = \frac{e^6}{3}$

12. $y = -2$

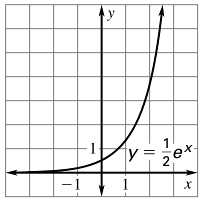
13. $y = e^{-2x}$

x	y
0	1
1	0.135
-1	7.39



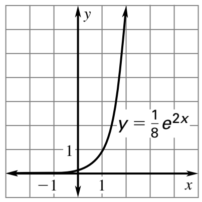
14. $y = \frac{1}{2}e^x$

x	y
-1	0.18
0	$\frac{1}{2}$
1	1.36
2	3.69



15. $y = \frac{1}{8}e^{2x}$

x	y
0	$\frac{1}{8}$
1	0.924
2	6.8



16. $s = 119.6e^{0.0917(26)} \approx 1298$

Practice and Applications (p. 483)

17. $e^2 \cdot e^4 = e^{2+4} = e^6$ **18.** $e^{-3} \cdot e^5 = e^{-3+5} = e^2$

19. $(3e^{-3x})^{-1} = \dfrac{1}{3}e^{3x} = \dfrac{e^{3x}}{3}$ **20.** $(3e^{4x})^2 = 9e^{8x}$

21. $3e^{-2} \cdot e^6 = 3e^{-2+6} = 3e^4$ **22.** $\left(\dfrac{1}{4}e^{-2}\right)^3 = \dfrac{1}{64e^6}$

23. $e^x \cdot e^{-3x} \cdot e^5 = e^{1x+3x+5} = e^{-2x+5}$

24. $\sqrt{4e^{2x}} = (4e^{2x})^{1/2} = 2e^{2x(1/2)} = 2e^x$

25. $(100e^{0.5x})^{-2} = \dfrac{1}{(100)^2}e^{0.5x(-2)} = \dfrac{1}{10{,}000}e^{-x} = \dfrac{1}{10{,}000e^x}$

26. $e^x \cdot 4e^{2x+1} = 4e^{x+2x+1} = 4e^{3x+1}$

27. $\dfrac{e^x}{2e} = \dfrac{e^xe^{-1}}{2} = \dfrac{e^{x-1}}{2}$

28. $\dfrac{5e^x}{e^{5x}} = 5e^x \cdot e^{-5x} = 5e^{x-5x} = 5e^{-4x} = \dfrac{5}{e^{4x}}$

29. $\sqrt[3]{27e^{6x}} = (27e^{6x})^{1/3} = 3e^{6x(1/3)} = 3e^{2x}$

30. $(32e^{-4x})^3 = (32)^3e^{-12x} = 32{,}768e^{-12x} = \dfrac{32{,}768}{e^{12x}}$

31. $\dfrac{6e^{3x}}{4e} = \dfrac{3}{2}e^{3x-1}$ **32.** $\sqrt[3]{64e^{9x}} = (64e^{9x})^{1/3} = 4e^{3x}$

33. $e^3 = 20.086$ **34.** $e^{-2/3} = 0.513$ **35.** $e^{1.7} = 5.474$

36. $e^{1/2} = 1.649$ **37.** $e^{-1/4} = 0.779$ **38.** $e^{3.2} = 24.533$

39. $e^8 = 2980.958$ **40.** $e^{-3} = 0.050$ **41.** $e^{-4} = 0.018$

42. $2e^{1/2} = 3.297$ **43.** $-4e^{-3} = -0.199$

44. $0.5e^{3.2} = 12.266$ **45.** $-1.2e^5 = -178.096$

46. $0.02e^{-0.3} = 0.015$ **47.** $225e^{-50} = 4.34 \times 10^{-20}$

48. $-8.95e^{1/5} = -10.932$

49. $f(x) = 5e^{-3x}$; exponential decay since $5 > 0$ and $-3 < 0$

50. $f(x) = \frac{1}{8}e^{5x}$; exponential growth since $\frac{1}{8} > 0$ and $5 > 0$

51. $f(x) = e^{-4x}$; exponential decay since $1 > 0$ and $-4 < 0$

52. $f(x) = \frac{1}{6}e^{2x}$; exponential growth since $\frac{1}{6} > 0$ and $2 > 0$

53. $f(x) = \frac{1}{4}e^{2x}$; exponential growth since $\frac{1}{4} > 0$ and $2 > 0$

54. $f(x) = e^{-8x}$; exponential decay since $1 > 0$ and $-8 < 0$

55. $f(x) = e^{3x}$; exponential growth since $1 > 0$ and $3 > 0$

56. $f(x) = \frac{1}{4}e^{-x}$; exponential decay since $\frac{1}{4} > 0$ and $-1 < 0$

57. $f(x) = e^{-6x}$; exponential decay since $1 > 0$ and $-6 < 0$

58. $f(x) = \frac{3}{8}e^{7x}$; exponential growth since $\frac{3}{8} > 0$ and $7 > 0$

59. $f(x) = e^{-9x}$; exponential decay since $1 > 0$ and $-9 < 0$

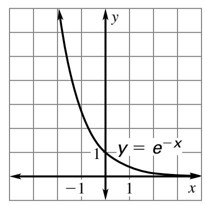
60. $f(x) = e^{8x}$; exponential growth since $1 > 0$ and $8 < 0$

61. $y = 3e^{0.5x}$ (C) **62.** $y = \frac{1}{3}e^{0.5x}$ (E)

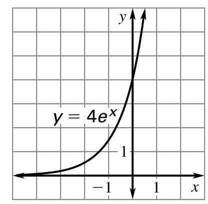
63. $y = \frac{1}{2}e^{-(x-1)}$ (F) **64.** $y = e^{-x} + 1$ (B)

65. $y = 3e^{-x} - 2$ (D) **66.** $y = 3e^x - 2$ (A)

67. $y = e^{-x}$ **68.** $y = 4e^x$

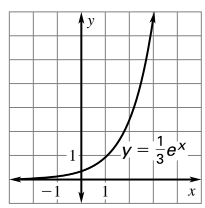


Domain: all real numbers
Range: $y > 0$



Domain: all real numbers
Range: $y > 0$

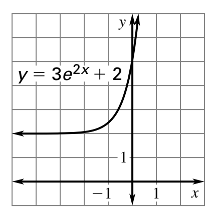
69. $y = \frac{1}{3}e^x$



Domain: all real numbers
Range: $y > 0$

70. $y = 3e^{2x} + 2$

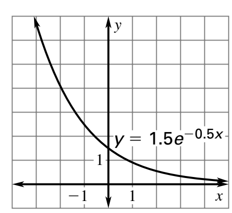
x	y
0	5
1	24.17
-1	2.41



Domain: all real numbers
Range: $y > 2$

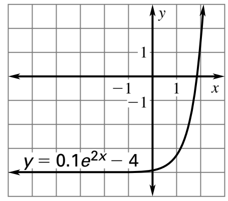
71. $y = 1.5e^{-0.5x}$

x	y
0	1.5
1	0.91
2	0.55
-1	2.47



Domain: all real numbers
Range: $y0$

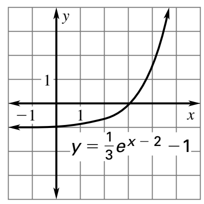
72. $y = 0.1e^{2x} - 4$



x	y
0	-3.9
1	-3.26
-1	-3.99

Domain: all real numbers
Range: $y > -4$

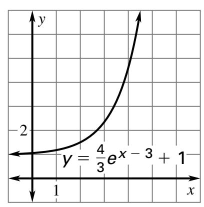
73. $y = \frac{1}{3}e^{x-2} - 1$



x	y
0	-0.95
1	-0.88
2	-0.67
-1	-0.98

Domain: all real numbers
Range: $y > -1$

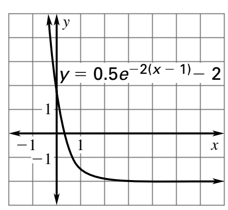
74. $y = \frac{4}{3}e^{x-3} + 1$



x	y
0	1.07
1	1.18
2	1.49
3	2.33
-1	1.02

Domain: all real numbers
Range: $y > 1$

75. $y = 0.5e^{-2(x-1)} - 2$



x	y
0	1.69
1	-1.5
2	-1.93
-1	25.29

Domain: all real numbers
Range: $y > -2$

76. $A = Pe^{rt}$ $P = 975$ $r = 0.055$ $t = 6$

$= 975e^{0.055(6)}$

$= \$1356.19$

77. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Annually $A = 2500\left(1 + \frac{0.06}{1}\right)^{1 \cdot 1}$

$= \$2650$

Semiannually $A = 2500\left(1 + \frac{0.06}{2}\right)^{2 \cdot 1}$

$= \$2652.25$

Quarterly $A = 2500\left(1 + \frac{0.06}{4}\right)^{4 \cdot 1}$

$= \$2653.41$

Monthly $A = 2500\left(1 + \frac{0.06}{12}\right)^{12 \cdot 1}$

$= \$2654.19$

Continuously $A = Pe^{rt}$

$= 2500e^{0.06(1)}$

$= \$2654.59$

The extra amount of interest earned with more and more compoundings decreases drastically. The difference between compounding monthly and continuously is only 40¢, 0.016% of the initial amount invested.

78. $A = Pe^{rt}$

$= 2500e^{0.06(1)}$

$= 2654.59$

$A = P\left(1 + \frac{r}{365}\right)^{365 \cdot t}$

$= 2500\left(1 + \frac{0.06}{365}\right)^{365 \cdot 1}$

$= \$2654.58$

The difference is 1¢.

79. $P = 14.7e^{-0.00004(29{,}028)}$

$\approx 4.603 \text{ lb/in.}^2$

80. $A = A_0e^{-0.05t}$

$= 4e^{-0.05(14)}$

$\approx 1.98 \text{ cm}^2$

$\approx 2 \text{ cm}^2$

81. $\sqrt[3]{\frac{8(81e^{11}x)}{3e^5x^{-2}}} = \left(\frac{8(81e^{11}x)}{3e^5x^{-2}}\right)^{1/3} = \left(\frac{648e^{11}x}{3e^5x^{-2}}\right)^{1/3}$

$= (216e^{11-5}x^{1+2})^{1/3}$

$= 6e^2x$ (E)

82. (B) $f(x) = 3e^x - 2$

x	y
0	1
-4	-1.95

83.

n	10^1	10^2	10^3
$\left(1 + \frac{1}{n}\right)^n$	2.59374246	2.70481383	2.716923933

n	10^4	10^5	10^6
$\left(1 + \frac{1}{n}\right)^n$	2.718145936	2.718268303	2.718281378

$n = 10^{10}$; *Sample answer:* From the table made in the activity on p. 480, I noticed that using a value of $n = 10^k$, the answer is accurate to $k - 1$ decimal places, with an error in the k^{th} decimal place.

8.2 Mixed Review (p. 485)

84.
$$f(x) = -3x$$
$$y = -3x$$
$$x = -3y$$
$$y = -\frac{x}{3}$$
$$f^{-1}(x) = -\frac{x}{3}$$

85.
$$f(x) = 6x + 7$$
$$y = 6x + 7$$
$$x = 6y + 7$$
$$6y = x - 7$$
$$y = \frac{x - 7}{6}$$
$$f^{-1}(x) = \frac{x - 7}{6}$$

86.
$$f(x) = -5x - 24$$
$$y = -5x - 24$$
$$x = -5y - 24$$
$$-5y = x + 24$$
$$\left(-\frac{1}{5}\right)(-5y) = (x + 24)\left(-\frac{1}{5}\right)$$
$$y = -\left(\frac{x + 24}{5}\right)$$
$$f^{-1}(x) = -\left(\frac{x + 24}{5}\right)$$

87.
$$f(x) = \tfrac{1}{2}x - 10$$
$$y = \tfrac{1}{2}x - 10$$
$$x = \tfrac{1}{2}y - 10$$
$$\tfrac{1}{2}y = x + 10$$
$$\tfrac{1}{2}y(2) = 2(x + 10)$$
$$y = 2x + 20$$
$$f^{-1}(x) = 2x + 20$$

88.
$$f(x) = -14x + 7$$
$$y = -14x + 7$$
$$x = -14y + 7$$
$$-14y = x - 7$$
$$-14y\left(-\frac{1}{14}\right) = \left(-\frac{1}{14}\right)(x - 7)$$
$$y = -\left(\frac{x - 7}{14}\right)$$
$$f^{-1}(x) = -\left(\frac{x - 7}{14}\right)$$

89.
$$f(x) = -\tfrac{1}{5}x - 13$$
$$y = -\tfrac{1}{5}x - 13$$
$$x = -\tfrac{1}{5}y - 13$$
$$-\tfrac{1}{5}y = x + 13$$
$$(-5)\left(-\tfrac{1}{5}y\right) = (-5)(x + 13)$$
$$y = -5x - 65$$
$$f^{-1}(x) = -5x - 65$$

90.
$$\sqrt{x} = 20$$
$$\left(\sqrt{x}\right)^2 = (20)^2$$
$$x = 400$$

91.
$$\sqrt[3]{5x - 4} + 7 = 10$$
$$(5x - 4)^{1/3} + 7 = 10$$
$$[(5x - 4)^{1/3}]^3 = (3)^3$$
$$5x - 4 = 27$$
$$5x = 27 + 4$$
$$5x = 31$$
$$x = \tfrac{31}{5}$$
$$x = 6.2$$

92.
$$2(x + 4)^{2/3} = 8$$
$$(x + 4)^{2/3} = 4$$
$$[(x + 4)^{2/3})]^{3/2} = (4)^{3/2}$$
$$x + 4 = \sqrt{(4)^3}$$
$$x = \sqrt{64} - 4$$
$$x = 8 - 4$$
$$x = 4$$

-12 is also an answer.
$$x = \sqrt{64} - 4$$
$$x = -8 - 4$$
$$x = -12$$

93.
$$\sqrt{x^2 - 4} = x - 2$$
$$(x^2 - 4)^{1/2} = x - 2$$
$$[(x^2 - 4)^{1/2}]^2 = (x - 2)^2$$
$$x^2 - 4 = (x - 2)^2$$
$$x^2 - 4 - [(x - 2)(x - 2)] = 0$$
$$x^2 - 4 - (x^2 - 4x + 4) = 0$$
$$x^2 - 4 - x^2 + 4x - 4 = 0$$
$$4x - 8 = 0$$
$$4x = 8$$
$$x = 2$$

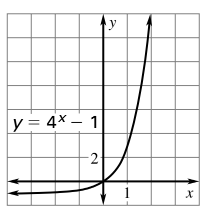
94.
$$\sqrt{x + 3} = \sqrt{2x - 1}$$
$$[(x + 3)^{1/2}]^2 = [(2x - 1)^{1/2}]^2$$
$$x + 3 = 2x - 1$$
$$-x = -4$$
$$x = 4$$

95.
$$\sqrt{3x - 5} - 3\sqrt{x} = 0$$
$$\left(\sqrt{3x - 5} - 3\sqrt{x}\right)^2 = (0)^2$$
$$3x - 5 - 9x = 0$$
$$-6x = 5$$
$$x = -\tfrac{5}{6} \text{ no solution}$$

Quiz 1 (p. 485)

1. $y = 4^x - 1$

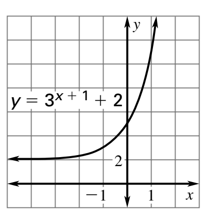
x	y
0	0
1	3
-1	$-\frac{3}{4}$



Domain: all real numbers
Range: $y > -1$

2. $y = 3^{x+1} + 2$

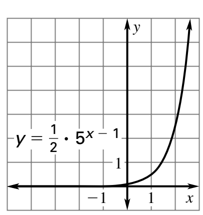
x	y
0	5
1	11
-1	3



Domain: all real numbers
Range: $y > 2$

3. $y = \frac{1}{2} \cdot 5^{x-1}$

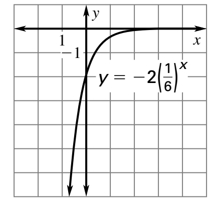
x	y
0	$\frac{1}{10}$
1	$\frac{1}{2}$
-1	$\frac{1}{25}$



Domain: all real numbers
Range: $y > 0$

4. $y = -2\left(\frac{1}{6}\right)^x$

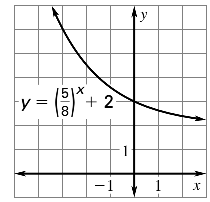
x	y
0	-2
1	$-\frac{1}{3}$
-1	-12



Domain: all real numbers
Range: $y < 0$

5. $y = \left(\frac{5}{8}\right)^x + 2$

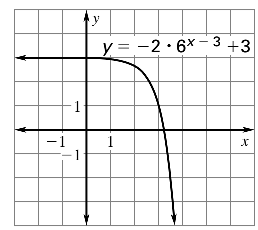
x	y
0	3
1	$2\frac{5}{8}$
-1	$3\frac{3}{8}$



Domain: all real numbers
Range: $y > 2$

6. $y = -2 \cdot 6^{x-3} + 3$

x	y
0	$2\frac{107}{108}$
3	1



Domain: all real numbers
Range: $y < 3$

7. $2e^3 \cdot e^4 = 2e^{3+4} = 2e^7$ **8.** $4e^{-5} \cdot e^7 = 4e^{-5+7} = 4e^2$

9. $(-3e^{2x})^2 = 9e^{4x}$

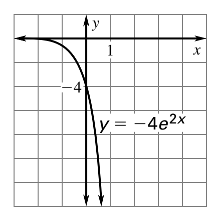
10. $(5e^{-3})^{-4x} = \dfrac{e^{12x}}{5^{4x}}$ **11.** $\dfrac{3e^x}{4e} = \dfrac{3}{4}e^{x-1}$

12. $\dfrac{6e^x}{e^{5x}} = 6e^{x-5x} = \dfrac{6}{e^{4x}}$ **13.** $\sqrt{16e^x x} = 4e\sqrt{x}$

14. $\sqrt[3]{125e^{6x}} = 5e^{2x}$

15. $f(x) = -4e^{2x}$

x	y
0	4
1	-29.56
-1	-0.54



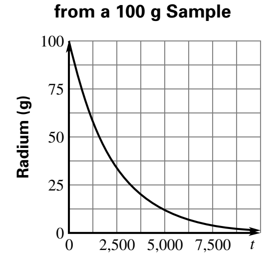
16. $R = 100e^{-0.00043t}$

$R = 100e^{-0.00043(10{,}000)}$

$R = 1.357$ g

x	y
0	100
1	99.96

Amount of Radium Left from a 100 g Sample



Lesson 8.4

8.4 Guided Practice (p. 490)

1. common logarithm

2. a. $\log_3(-1)$ is not defined because (-1) is not positive. $(-1) = y$ The logarithm of y with base b is denoted by $\log_b y$, where b and y are positive.

b. $\log_1 1$ is not defined because $b \neq 1(\log_b y)$

3. The expression $\log_b y$ is read as "log base b of y." Let b and y be positive numbers, $b \neq 1$. The logarithm of y with base b is denoted by $\log_b y$ and is defined as follows:

$\log_b y = x$ if and only if $b^x = y$.

4. $5^2 = 25$ is true but by definition $\log_b y = x$ if and only if $b^x = y$.

So, $\log_2 25 = x$ if and only if $2^x = 25$ (not $5^2 = 25$).

$2^4 = 16$ and $2^5 = 32$ so x is between 4 and 5.

5. $\log_3 9 = 2$ *is* $3^2 = 9$ **6.** $\log_5 5 = 1$ *is* $5^1 = 5$

7. $\log_{1/2} 4 = -2$ *is* $\frac{1}{2}^{-2} = 4$ **8.** $\log_{19} 1 = 0$ *is* $19^0 = 1$

9. $\log_2 64 \quad 2^6 = 64$ so $\log_2 64 = 6$

10. $\log_{25} 5 \quad 25^{1/2} = 5$ so $\log_{25} 5 = \frac{1}{2}$

11. $\log_6 1 \quad 6^0 = 1$ so $\log_6 1 = 0$

12. $10^{\log 4}$

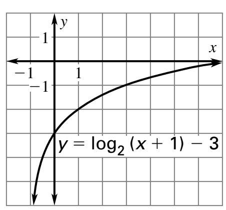
$g(x) = \log_b x \quad f(g(x)) = b^{\log_b x} = x$

$f(x) = b^x$

$f(g(x)) = 10^{\log 4} = 4$

13. $y = \log_2(x + 1) - 3$

x	y
0	-3
3	-1
$-\frac{1}{2}$	-4

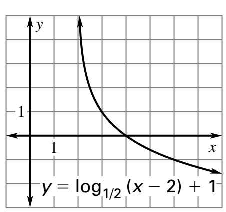


Domain: $x > -1$

Range: all real numbers

14. $y = \log_{1/2}(x - 2) + 1$

x	y
4	0
2.5	2
3	1
6	-1



Domain: $x > 2$

Range: all real numbers

15.

$$s = 0.159 + 0.118 \log d$$
$$0.1 = 0.159 + 0.118 \log d$$
$$0.1 - 0.159 = 0.118 \log d$$
$$\frac{0.1 - 0.159}{0.118} = \log d$$
$$\frac{-0.059}{0.118} = \log d$$
$$-0.5 = \log d$$
$$\log d = -0.5$$
$$10^{-0.5} = d$$
$$0.316 \text{ mm} \approx d$$

8.4 Practice and Applications (pp. 490–492)

16. $\log_4 1024 = 5$ *is* $4^5 = 1024$

17. $\log_5 \frac{1}{5} = -1$ *is* $5^{-1} = \frac{1}{5}$ **18.** $\log_{36} \frac{1}{6} = -\frac{1}{2}$ *is* $36^{-1/2} = \frac{1}{6}$

19. $\log_8 512 = 3$ *is* $8^3 = 512$

20. $\log_{12} 144 = 2$ *is* $12^2 = 144$

21. $\log_{14} 196 = 2$ *is* $14^2 = 196$

22. $\log_8 4096 = 4$ *is* $8^4 = 4096$

23. $\log_{105} 11{,}025 = 2$ *is* $105^2 = 11{,}025$

24. $\log_5 125 \quad 5^3 = 125$ so $\log_5 125 = 3$

25. $\log_7 343 \quad 7^3 = 343$ so $\log_7 343 = 3$

26. $\log_8 1 \quad 8^0 = 1$ so $\log_8 1 = 0$

27. $\log_{12} 12 \quad 12^1 = 12$ so $\log_{12} 12 = 1$

28. $\log_6 36 \quad 6^2 = 36$ so $\log_6 36 = 2$

29. $\log_4 16 \quad 4^2 = 16$ so $\log_4 16 = 2$

30. $\log_9 729 \quad 9^3 = 729$ so $\log_9 729 = 3$

31. $\log_7 2401 \quad 7^4 = 2401$ so $\log_7 2401 = 4$

32. $\log_{1/4} \frac{1}{4} \quad \frac{1}{4}^1 = \frac{1}{4}$ so $\log_{1/4} \frac{1}{4} = 1$

33. $\log_4 4^{-0.38} \quad 4^{-0.38} = 4^{-0.38}$ so $\log_4 4^{-0.38} = -0.38$

34. $\log_4 \frac{1}{2} \quad 4^{-1/2} = \frac{1}{4^{1/2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ so $\log_4 \frac{1}{2} = -\frac{1}{2}$

35. $\log_{1/5} 25$

$$\left(\frac{1}{5}\right)^{-2} = \frac{1}{\left(\frac{1}{5}\right)^2} = \frac{1}{1} \cdot \frac{(5)^2}{1} = 25$$

so $\log_{1/5} 25 = -2$

36. $\log 8 = 0.903$ **37.** $\ln 10 = 2.303$

38. $\log \sqrt{2} = 0.151$ **39.** $\log 3.724 = 0.571$

40. $\log 2.54 = 0.405$ **41.** $\log 0.3 = -0.523$

42. $\log 4.05 = 0.607$ **43.** $\log 3.5 = 0.544$

44. $\ln 4.6 = 1.526$ **45.** $\ln 150 = 5.011$

46. $\ln 6.9 = 1.932$ **47.** $\ln 22.5 = 3.114$

48. $5^{\log_5 x} = x$ from the definition

49. $\log_2 2^x = x$ from the definition

50. $9^{\log_9 x} = x$ from the definition

51. $35^{\log_{35} x} = x$ from the defintion

52. $\log_4 16^x = \log_4 (4^2)^x = \log_4 (4)^{2x} = 2x$

53. $7^{\log_7 x} = x$ **54.** $\log 100^x = \log(10^2)^x = \log(10)^{2x} = 2x$

55. $\log_{20} 8000^x = \log_{20}(20^3)^x = \log_{20}(20)^{3x} = 3x$

56. $y = \log_9 x \quad y = 9^x$ **57.** $y = \log_{1/4} x \quad y = \frac{1}{4}^x$

58. $y = \log_5 x \quad y = 5^x$ **59.** $y = \log_{1/2} x \quad y = \frac{1}{2}^x$

60. $y = \log_7 49^x$

$y = \log_7 (7^2)^x$

$y = \log_7 (7)^{2x}$

$x = \log_7 (7)^{2y}$ Switch x and y.

$x = 2y$ Write in exponential form.

$\frac{x}{2} = y$ Solve for y.

$y = \frac{x}{2}$

61. $y = \ln 6x$

$x = \ln 6y$ Switch x and y.

$e^x = 6y$ Write in exponential form.

$y = \frac{e^x}{6}$ Solve for y.

62. $y = \ln(x - 1)$

$x = \ln(y - 1)$ Switch x and y.

$e^x = y - 1$ Write in exponential form.

$y = e^x + 1$ Solve for y.

$y = 1 + e^x$

63. $y = \ln(x + 2)$

$x = \ln(y + 2)$ Switch x and y.

$e^x = y + 2$ Write in exponential form.

$y = e^x - 2$ Solve for y.

$y = -2 + e^x$

64. $y = \ln(x - 2)$

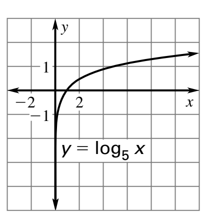
$x = \ln(y - 2)$ Switch x and y.

$e^x = y - 2$ Write in exponential form.

$y = 2 + e^x$ Solve for y.

65. $y = \log_5 x$

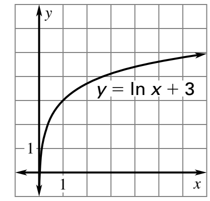
x	y
1	0
$\frac{1}{5}$	-1
25	2



Domain: $x > 0$
Range: all real numbers

66. $y = \ln x + 3$

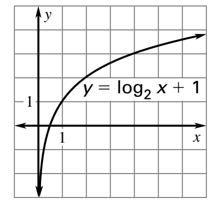
x	y
1	3
2	3.69
0.5	2.31



Domain: $x > 0$
Range: all real numbers

67. $y = \log_2 x + 1$

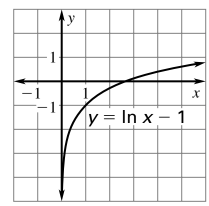
x	y
1	1
2	2
4	3
$\frac{1}{2}$	0



Domain: $x > 0$
Range: all real numbers

68. $y = \ln x - 1$

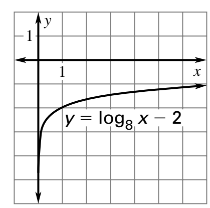
x	y
1	-1
0.5	-1.69
2	-0.31
3	0.10



Domain: $x > 0$
Range: all real numbers

69. $y = \log_8 x - 2$

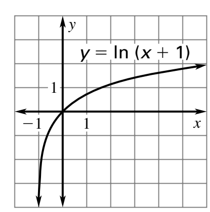
x	y
1	-2
8	-1
$\frac{1}{8}$	-3



Domain: $x > 0$
Range: all real numbers

70. $y = \ln(x + 1)$

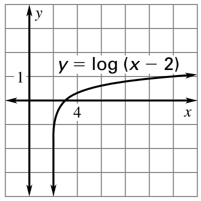
x	y
0	0
1	0.69
2	1.10



Domain: $x > -1$
Range: all real numbers

71. $y = \log(x - 2)$

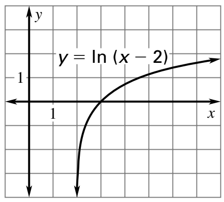
x	y
3	0
2.1	-1
12	1



Domain: $x > 2$
Range: all real numbers

72. $y = \ln(x - 2)$

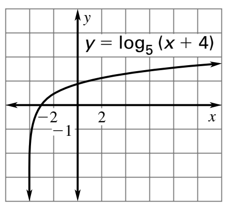
x	y
3	0
2.1	-2.3
4	0.69



Domain: $x > 2$
Range: all real numbers

73. $y = \log_5(x + 4)$

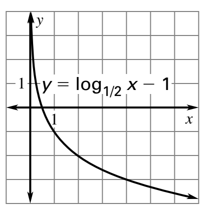
x	y
-3	0
1	1



Domain: $x > -4$
Range: all real numbers

74. $y = \log_{1/2} x - 1$

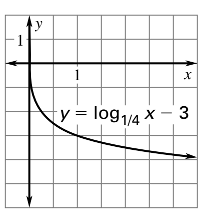
x	y
2	-2
1	-1
$\frac{1}{2}$	0
$\frac{1}{4}$	1



Domain: $x > 0$
Range: all real numbers

75. $y = \log_{1/4} x - 3$

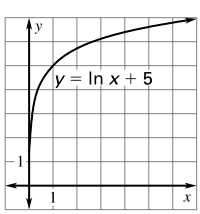
x	y
1	-3
$\frac{1}{4}$	-2
4	-4
$\frac{1}{16}$	-1



Domain: $x > 0$
Range: all real numbers

76. $y = \ln x + 5$

x	y
1	5
0.5	4.3
2	5.69



Domain: $x > 0$
Range: all real numbers

77. a. $\text{pH} = -\log[H^+]$
$= -\log 1 \times 10^{-2.4}$
$= -(10^x = 1 \times 10^{-2.4})$
$= -(x = -2.4)$
$= -(-2.4)$
$= 2.4$

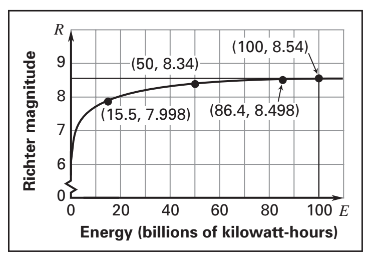
b. $\text{pH} = -\log 1 \times 10^{-3}$
$= -(10^x = 1 \times 10^{-3})$
$= -(x = -3)$
$= -(-3)$
$= 3$

c. $\text{pH} = -\log 1 \times 10^{-3.5}$
$= -(10^x = 1 \times 10^{-3.5})$
$= -(x = -3.5)$
$= -(-3.5)$
$= 3.5$

78. $A = \dfrac{2}{\log e} = \dfrac{2}{0.434294482} = 4.605$

79. $R = 0.67 \log[0.37(15{,}500{,}000{,}000)] + 1.46$
$= 0.67 \log(0.37 \times 1.55 \times 10^{10}) + 1.46$
$= 0.67 \log (5{,}735{,}000{,}000) + 1.46$
$= 0.67(9.758533422) + 1.46$
$= 7.9982$ *or* about 8

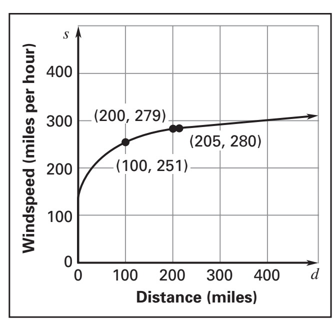
80.



E	R
15.5	7.99
50.0	8.34
75.0	8.46
100.0	8.54
86.4	8.498
86.417	8.5

about 86,000,000,000 kWh

81. $s = 93 \log d + 65$



—CONTINUED—

81 —CONTINUED—

$s = 93 \log d + 65$

$s - 65 = 93 \log d$

$\log d = \dfrac{s - 65}{93}$

$= \dfrac{280 - 65}{93}$

$= \dfrac{215}{93}$

$= 2.312$

$d =$ about 205 miles

$\log 100 = 2$

$s = 93(2) + 65$

$s = 251$ mph

$\log 200 = 2.301$

$s = 93(2.301) + 65$

$s = 278.993$ mph

$\log 205 = 2.312$

$s = 93(2.312) + 65$

$s = 280.02$

	Column A	Column B	
82.	$\log_9 9^{2/3}$	$\log 100$	(B)
83.	$\log_{16} 1$	0	(C)
84.	$\log_4 16$	$\log_8 64$	(C)
85.	$f(8)$ if $f(x) = \log_2 x$	4	(B)
86.	$f(-1)$ if $f(x) = \log_5 5^x$	-1	(C)
87.	$f(\frac{1}{2})$ if $f(x) = \log_3 9^x$	$\log_3 81$	(B)

82. $\log_9 9^{2/3} = 9^x = 9^{2/3} \quad x = \frac{2}{3} = 0.67 \quad \log 100 = 2$ (B)

83. $\log_{16} 1 \quad 16^x = 1 \quad x = 0$ (C)

84. $\log_4 16 \quad 4^x = 16 \quad x = 2 \quad \log_8 64 \quad 8^x = 64 \quad x = 2$ (C)

85. $f(8) = \log_2 (8) \quad 2^x = 8 \quad x = 3$ (B)

86. $f(-1) = \log_5 5^{(-1)} \quad 5^x = 5^{(-1)} \quad x = -1$ (C)

87. $f(\frac{1}{2}) = \log_3 9^{1/2} \quad 3^x = 9^{1/2}$ $\qquad \log_3 81 \quad 3^x = 81$ (B)

$3^x = (3^2)^{1/2}$ $\qquad 3^x = 3^4$

$3^x = 3^1$ $\qquad x = 4$

$x = 1$

88. $\log_{16} 8 \quad 16^x = 8$

$(2^4)^x = 2^3$

$4x = 3$

$x = \frac{3}{4}$

89. $\log_{16} 64 \quad 16^x = 64$

$(2^4)^x = 2^6$

$2^{4x} = 2^6$

$4x = 6$

$x = \frac{3}{2}$

90. $\log_9 27 \quad 9x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

91. $\log_4 512 \quad 4^x = 512$

$(2^2)^x = 2^9$

$2^{2x} = 2^9$

$2x = 9$

$x = \frac{9}{2}$

92. If $b = c^n$ and $x = c^m$, then $\log_b x = \dfrac{m}{n}$.

8.4 Mixed Review (p. 492)

93. $5^2 \cdot 5^3 = 3125$ **94.** $(3^{-4})^2 = 3^{-8} = \dfrac{1}{3^8} = \dfrac{1}{6561}$

95. $7^0 \cdot 7^3 \cdot 7^{-2} = 7^{0+3+(-2)} = 7^1 = 7$

96. $\left(\frac{3}{7}\right)^{-2} = \left(\frac{7}{3}\right)^2 = \frac{49}{9}$ **97.** $\dfrac{6^3}{6^4} = 6^{3-4} = 6^{-1} = \dfrac{1}{6}$

98. $\left(\dfrac{3}{8}\right)^{-3} = \left(\dfrac{8}{3}\right)^3 = \dfrac{512}{27}$ **99.** $(-2^3)^2 = (-8)^2 = 64$

100. $\left(\dfrac{4}{5}\right)^3 = \dfrac{64}{125}$ **101.** $\left(\dfrac{1}{2}\right)^{-4} = \dfrac{1}{\left(\dfrac{1}{2}\right)^4} = \dfrac{1}{\dfrac{1}{16}} = 16$

102. $(-3^2)^{-1} = (-9)^{-1} = -\frac{1}{9}$

103. $\dfrac{2^5}{2^9} = 2^{5-9} = 2^{-4} = \dfrac{1}{16}$ **104.** $\left(\dfrac{7}{9}\right)^{-2} = \left(\dfrac{9}{7}\right)^2 = \dfrac{81}{49}$

105.

$$\begin{array}{r} 2x - 7 \\ x + 4 \overline{\smash{)}\,2x^2 + x - 1} \\ \underline{2x^2 + 8x} \\ -7x - 1 \\ \underline{-7x - 28} \\ 27 \end{array}$$

$2x - 7 + \dfrac{27}{x + 4}$

106.

$$\begin{array}{r} x - 4 \\ x - 1 \overline{\smash{)}\,x^2 - 5x + 4} \\ \underline{x^2 - 1x} \\ -4x + 4 \\ \underline{-4x + 4} \end{array}$$

$x - 4$

107.

$$\begin{array}{r} 4x + 3 \\ x^2 + 2 \overline{\smash{)}\,4x^3 + 3x^2 + 2x - 3} \\ \underline{4x^3 + 0x^2 + 8x} \\ 3x^2 - 6x - 3 \\ \underline{3x^2 - 0x + 6} \\ -6x - 9 \end{array}$$

$4x + 3 - \dfrac{6x + 9}{x^2 + 2}$

108.

$$\begin{array}{r} 6x^2 + 10x + 30 \\ x - 3 \overline{\smash{)}\,6x^3 - 8x^2 + 7} \\ \underline{6x^3 - 18x^2} \\ 10x^2 + 0x + 7 \\ \underline{10x^2 - 30x} \\ 30x + 7 \\ \underline{30x - 90} \\ +97 \end{array}$$

$6x^2 + 10x + 30 + \dfrac{97}{x - 3}$

109. $(2, 0), (-3, 0), (0, 0), (3, -3)$

$y = -\dfrac{1}{6}x(x - 2)(x + 3)$

110. $(3, 0), (2, 0), (-3, 0), (0, -1)$

$y = -\dfrac{1}{18}(x - 3)(x - 2)(x + 3)$

111. (4, 0), (6, 0), (−4, 0), (1, 1)

$$y = \frac{1}{75}(x - 4)(x - 6)(x + 4)$$

112. (−2, 0), (−3, 0), (3, 0), (0, 2)

$$y = -\frac{1}{9}(x + 2)(x + 3)(x - 3)$$

Lesson 8.5

Developing Concepts Activity 8.5 (p. 493)

1.

$\log_b u$	$\log_b v$	$\log_b uv$
$\log 10 = 1$	$\log 100 = 2$	$\log 1000 = 3$
$\log 0.1 = -1$	$\log 0.01 = -2$	$\log 0.001 = -3$
$\log_2 4 = 2$	$\log_2 8 = 3$	$\log_2 32 = 5$

2. $\log_b uv = \log_b u + \log_b v$

8.5 Guided Practice (p. 496)

1. a. product property example:

$\log_5 21 = \log_5 (3 \cdot 7)$
$= \log_5 3 + \log_5 7$
$\approx 0.683 + 1.209 = 1.892$

b. quotient property example:

$\log_5 \frac{3}{7} = \log_5 3 - \log_5 7$
$\approx 0.683 - 1.209 = -0.526$

c. power property example:

$\log_5 49 = \log_5 7^2$
$= 2 \log_5 7 \approx 2(1.209) = 2.418$

2. $\log\left(\frac{7}{9}\right)^2 = 2 \log \frac{7}{9}$
$= 2(\log 7 - \log 9)$ (A)

3. $\log_8(5x^2 + 3)$ (C)

None of the properties of logarithms applies to a sum of terms.

4. $\log_6 11 = \dfrac{\ln 11}{\ln 6}$; $\log_6 11 = \dfrac{\log 11}{\log 6}$

5. $\log_3(3 \cdot 9) = \log_3 3 + \log_3 9 = 1 + 2 = 3$

6. $\log_2 4^5 = 5 \log_2 4 = 5(2) = 10$

7. $\log_3 \dfrac{1}{3} = \dfrac{\log \frac{1}{3}}{\log 3} = -1$

8. $\log_5 \left(\dfrac{1}{5}\right)^3 = 3 \log_5 \dfrac{1}{5} = 3 \dfrac{\log \frac{1}{5}}{\log 5} = 3(-1) = -3$

9. $\log_2 3 = \log_2 \frac{21}{7} = \log_2 21 - \log_2 7$
$\approx 4.39 - 2.81 = 1.58$

10. $\log_2 49 = \log_2 7^2 = 2 \log_2 7 \approx 2(2.81) = 5.62$

11. $\log_2 147 = \log_2 21(7) = \log_2 21 + \log_2 7$
$\approx 4.39 + 2.81 = 7.2$

12. $\log_2 441 = \log_2 21^2 = 2(\log_2 21) = 2(4.39) = 8.78$

13. Difference in loudness $L_2 = L_1$

$= 10 \log \dfrac{1.26 \times 10^{-7}}{10^{-12}} - 10 \log \dfrac{3.16 \times 10^{-10}}{10^{-12}}$

$= 10 \log (1.26 \times 10^5) - 10 \log (3.16 \times 10^2)$

$= 10(\log 1.26 + \log 10^5) - 10(\log 3.16 + \log 10^2)$

$= 10(\log 1.26 + 5 \log 10) - 10(\log 3.16 + 2 \log 10)$

$\approx 10(5.1) - 10(2.5)$

$\approx 51 - 25$

≈ 26 decibels

8.5 Practice and Applications (p. 496)

14. $\log_2(4 \cdot 16) = \log_2 64 = \dfrac{\log 64}{\log 2} = 6$

15. $\ln e^{-2} = -2 \ln e = -2(1) = -2$

16. $\log_2 4^3 = 3 \log_2 4 = 3\left(\dfrac{\log 4}{\log 2}\right) = 3(2) = 6$

17. $\log_5 125 = \dfrac{\log 125}{\log 5} = 3$

18. $\log_3 9^4 = 4 \log_3 9 = 4\left(\dfrac{\log 9}{\log 3}\right) = 4(2) = 8$

19. $\log \dfrac{1}{10} = \log 1 - \log 10 = 0 - 1 = -1$

20. $\ln \dfrac{1}{e^3} = \ln 1 - \ln e^3 = 0 - 3 \ln e = 0 - 3(1) = -3$

21. $\log(0.01)^3 = 3 \log(0.01) = 3(-2) = -6$

22. $\log 3 = \log \frac{15}{5} = \log 15 - \log 5$
$\approx 1.176 - 0.699 = 0.477$

23. $\log 25 = \log 5 \cdot 5 = \log 5 + \log 5$
$= 0.699 + 0.699 = 1.398$

24. $\log 75 = \log 5 \cdot 15 = \log 5 + \log 15$
$= 0.699 + 1.176 = 1.875$

25. $\log 125 = \log 5^3 = 3 \log 5 = 3(0.699) = 2.097$

26. $\log \frac{1}{5} = \log 1 - \log 5 = 0 - 0.699 = -0.699$

27. $\log 225 = \log 15^2 = 2 \log 15 = 2(1.176) = 2.352$

28. $\log \frac{1}{15} = \log 1 - \log 15 = 0 - 1.176 = -1.176$

29. $\log \frac{1}{3} = \log 1 - \log 3 = \log 1 - \log \frac{15}{5}$
$= 0 - (\log 15 - \log 5)$
$= -(1.176 - 0.699)$
$= -0.477$

30. $\log_2 9x = \log_2 9 + \log_2 x$ **31.** $\ln 22x = \ln 22 + \ln x$

32. $\log 4x^5 = \log 4 + \log x^5 = \log 4 + 5\log x$

33. $\log_6 x^6 = 6\log_6 x$ **34.** $\log_4 \frac{4}{3} = 1 - \log_4 3$

35. $\log_3 25 = \log_3 5^2 = 2\log_3 5$

36. $\log_6 \frac{10}{3} = \log_6 10 - \log_6 3$

37. $\ln 3xy^3 = \ln 3 + \ln x + \ln y^3 = \ln 3 + \ln x + 3\ln y$

38. $\log 6x^3yz = \log 6 + \log x^3 + \log y + \log z$

$= \log 6 + 3\log x + \log y + \log z$

39. $\log_8 64x^2 = \log_8 64 + \log_8 x^2 = \log_8 64 + 2\log_8 x$

$= \frac{\log 64}{\log 8} + 2\log_8 x$

$= 2 + 2\log_8 x$

40. $\ln x^{1/2}y^3 = \ln x^{1/2} + \ln y^3 = \frac{1}{2}\ln x + 3\ln y$

41. $\log_3 12^{5/6} x^9 = \log_3 12^{5/6} + \log_3 x^9$

$= \frac{5}{6}\log_3 12 + 9\log_3 x$

42. $\log \sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$

43. $\ln \frac{3y^4}{x^3} = \ln 3y^4 - \ln x^3$

$= \ln 3 + \ln y^4 - 3\ln x$

$= \ln 3 + 4\ln y - 3\ln x$

44. $\log \sqrt[4]{x^3} = \log(x^3)^{1/4} = \log x^{3/4} = \frac{3}{4}\log x$

45. $\log_2 \sqrt{4x} = \log_2(4x)^{1/2} = \log_2 4^{1/2} + \log_2 x^{1/2}$

$= \frac{1}{2}\left(\frac{\log 4}{\log 2}\right) + \frac{1}{2}\log_2 x$

$= 1 + \frac{1}{2}\log_2 x$

46. $\log_5 8 - \log_5 12 = \log_5 \frac{8}{12} = \log_5 \frac{2}{3}$

47. $\ln 16 - \ln 4 = \ln \frac{16}{4} = \ln 4$

48. $2\log x + \log 5 = \log x^2 + \log 5 = \log 5x^2$

49. $4\log_{16} 12 - 4\log_{16} 2 = \log_{16} 12^4 - \log_{16} 2^4$

$= \log_{16} 20736 - \log_{16} 16$

$= \frac{\log_{16} 20736}{\log_{16} 16}$

$= \log_{16} 1296$

50. $3\ln x + 5\ln y = \ln x^3 + \ln y^5 = \ln x^3y^5$

51. $7\log_4 2 + 5\log_4 x + 3\log_4 y = \log_4 2^7 + \log_4 x^5 + \log_4 y^3$

$= \log_4 128x^5y^3$

52. $\ln 20 + 2\ln \frac{1}{2} + \ln x = \ln 20 + \ln\left(\frac{1}{2}\right)^2 + \ln x$

$= \ln 20 + \ln \frac{1}{4} + \ln x$

$= \ln 20\left(\frac{1}{4}\right)x$

$= \ln 5x$

53. $\log_3 2 + \frac{1}{2}\log_3 y = \log_3 2 + \log_3 y^{1/2} = \log_3 2y^{1/2}$

$= \log_3 2\sqrt{y}$

54. $10\log x + 2\log 10 = \log x^{10} + \log 10^2$

$= \log 100x^{10}$

55. $3(\ln 3 - \ln x) + (\ln x - \ln 9) = (\ln 3^3 - \ln x^3) + \left(\frac{\ln x}{\ln 9}\right)$

$= \left(\frac{\ln 3^3}{\ln x^3}\right)\left(\frac{\ln x}{\ln 9}\right)$

$= (\ln 3^3x^{-3})(\ln 3^{-2}x)$

$= \ln 3^{3-2}x^{1-3}$

$= \ln 3x^{-2} = \ln \frac{3}{x^2}$

56. $2(\log_6 15 - \log_6 5) + \frac{1}{2}\log_6 \frac{1}{25}$

$= \left(\frac{\log_6 15^2}{\log_6 5^2}\right) + \log_6\left(\frac{1}{25}\right)^{1/2}$

$= \left(\frac{\log_6 225}{\log_6 25}\right) + \log_6 \frac{1}{5}$

$= \log_6 9 + \log_6 \frac{1}{5}$

$= \log_6 \frac{9}{5}$

57. $\frac{1}{4}\log_5 81 - \left(2\log_5 6 - \frac{1}{2}\log_5 4\right)$

$= \log_5 (81)^{1/4} - (\log_5 6^2 - \log_5 4^{1/2})$

$= \log_5 3 - (\log_5 36 - \log_5 2)$

$= \log_5 3 - \log_5 18$

$= \log_5 \frac{3}{18} = \log_5 \frac{1}{6}$

58. $\log_5 7 = \frac{\log 7}{\log 5} = 1.209$ **59.** $\log_7 12 = \frac{\log 12}{\log 7} = 1.277$

60. $\log_3 16 = \frac{\log 16}{\log 3} = 2.524$

61. $\log_9 25 = \frac{\log 25}{\log 9} = 1.465$ **62.** $\log_2 5 = \frac{\log 5}{\log 2} = 2.322$

63. $\log_6 9 = \frac{\log 9}{\log 6} = 1.226$ **64.** $\log_3 17 = \frac{\log 17}{\log 3} = 2.579$

65. $\log_5 32 = \frac{\log 32}{\log 5} = 2.153$

66. $\log_2 125 = \frac{\log 125}{\log 2} = 6.966$

67. $\log_6 24 = \frac{\log 24}{\log 6} = 1.774$

68. $\log_4 19 = \frac{\log 19}{\log 4} = 2.124$

69. $\log_{16} 81 = \frac{\log 81}{\log 16} = 1.585$

70. $\log_8 \frac{22}{7} = \frac{\log \frac{22}{7}}{\log 8} = 0.551$

71. $\log_9 \frac{5}{16} = \frac{\log \frac{5}{16}}{\log 9} = -0.529$

72. $\log_2 \frac{4}{15} = \frac{\log \frac{4}{15}}{\log 2} = -1.907$

73. $\log_5 \frac{32}{3} = \frac{\log \frac{32}{3}}{\log 5} = 1.471$

74. $s = \log_2 f^2 = 2 \log_2 f^2$

75. $s = 2 \log_2 1.414 = 2\left(\frac{\log 1.414}{\log 2}\right) = 1$

$s = 2 \log_2 2.000 = 2\left(\frac{\log 2.000}{\log 2}\right) = 2$

$s = 2 \log_2 2.828 = 2\left(\frac{\log 2.828}{\log 2}\right) = 3$

$s = 2 \log_2 4.000 = 2\left(\frac{\log 4.000}{\log 2}\right) = 4$

$s = 2 \log_2 5.657 = 2\left(\frac{\log 5.657}{\log 2}\right) = 5$

$s = 2 \log_2 8.000 = 2\left(\frac{\log 8.000}{\log 2}\right) = 6$

$s = 2 \log_2 11.314 = 2\left(\frac{\log 11.314}{\log 2}\right) = 7$

$s = 2 \log_2 16.000 = 2\left(\frac{\log 16.000}{\log 2}\right) = 8$

f	1.414	2.000	2.828	4.000
s	1	2	3	4

f	5.657	8.000	11.314	16.000
s	5	6	7	8

As you change f-steps on the 35 mm camera the s increases by 1.

76. $s = 2 \log_2 f$

$9 = 2 \log_2 f$

$9 = 2 \frac{\log f}{\log 2}$

$\frac{9}{2} = \frac{\log f}{\log 2}$

$\log f = \frac{9}{2} \log 2$

$\log f = 1.35463498$

$10^x = f$

$10^{1.35463498} \approx 22.627 = 16\sqrt{2}$

77. $E = 1.4(\log C_2 - \log C_1)$

$E = 1.4 \log \frac{C_2}{C_1}$

78. $E = 1.4 \log \frac{2\,C_2}{C_1}$

$E = 1.4 \log 2 = 0.421$ kcal/g-molecule

79. $E = 1.4 \log \frac{6\,C_2}{C_1}$

$E = 1.4 \log 6$

$E = 1.089$ kcal/g-molecule

80. $L = 10 \log \frac{0.316}{10^{-12}}$

$L \approx 115$ decibels

between rock concert (110 decibels) and riveting machine (120 decibels).

81. $L = 10 \log \frac{0.003}{10^{-12}}$

$L \approx 95$ decibels

between subway train (90 decibels) and boiler shop (100 decibels)

82. $L = 10 \log \frac{3(1.4 \times 10^{-7})}{10^{-12}}$

$= 10(\log 3 + \log 1.4 \times 10^{-7} - \log 10^{-12})$

$\approx 10[0.477 + (-6.854) + (12)] \approx 56.2$ decibels

83. $L = 10 \log \frac{5(3.16 \times 10^{-4})}{10^{-12}}$

$= 10(\log 5 + \log 3.16 \times 10^{-4} - \log 10^{-12})$

$\approx 10[0.699 + (-3.5) + (12)]$

$\approx 10(9.199) \approx 92$ decibels

84. $L = 10 \log \frac{3 \times I}{10^{-12}} - 10 \log \frac{I}{10^{-12}}$

$= 10\left(\log \frac{3I}{10^{-12}} - \log \frac{I}{10^{-12}}\right)$

$= 10\left(\log 3 + \log \frac{I}{10^{-12}} - \log \frac{I}{10^{-12}}\right)$

$= 10 \log 3$ or about 4.8 decibels

85. $L = 10 \log \frac{I}{10^{-12}} - 10 \log \frac{0.5I}{10^{-12}}$

$= 10\left(\log \frac{I}{10^{-12}} - \log \frac{0.5I}{10^{-12}}\right)$

$= 10\left(\log \frac{I}{10^{-12}} - \log \frac{I}{10^{-12}} + \log 0.5\right)$

$= 10 \log 0.5 \approx 3$ decibels less

86. $\log(u + v) = \log u + \log v$ false

Sample answer: $\log(10 + 10) = \log 20 \approx 1.301$

but $\log 10 + \log 10 = 1 + 1 = 2$

87. $\log 1 = \log \frac{2}{2} = \log 2 - \log 2 = 0$

log 2 and log 3 are given.

$\log 4 = \log 2^2 = 2 \log 2 \approx 0.6020$

$\log 5 = \log 10 - \log 2 \approx 0.6990$

$\log 6 = \log(2 \cdot 3) = \log 2 + \log 3 \approx 0.6781$

$\log 8 = \log 2^3 = 3 \log 2 \approx 0.9030$

$\log 9 = \log 3^2 = 2 \log 3 \approx 0.9542$

$\log 10 = 1$

$\log 12 = \log(3 \cdot 4) = \log 3 + 2 \log 2 \approx 1.0791$

$\log 15 = \log(3 \cdot 5) = \log 3 + \log 5 \approx 1.1761$

$\log 16 = \log 2^4 = 4 \log 2 \approx 1.204$

$\log 18 = \log(2 \cdot 9) = \log 2 + 2 \log 3 \approx 1.2552$

$\log 20 = \log(2 \cdot 10) = \log 2 + 1 \approx 1.3010$

log 7, log 11, log 13, log 14, log 17, and log 19 cannot be found. Those numbers with a prime factorization involving only 2, 3, and 5 can be written in terms of these logs.

Conclusion: for the values of n (1 to 20) that I cannot find log n log 1 = 0, log 5's value is between 0.6020 (log 4) and 0.7781 (log 6), log 7's value is between 0.7781 (log 6) and 0.9030 (log 8), log 10 = 1 and log 11's value is between 1.000 (log 10) and 1.0791 (log 12), log 13's, log 14's, and log 15's values are between 1.0791 (log 12) and 1.204 (log 16), log 17's value is between 1.204 (log 16) and 1.2552 (log 18), and log 19's and log 20's values are slightly higher than 1.2552 (log 18).

88. (C) $\log_2 24 = \log_2 8 + \log_2 16$ incorrect

89. $\log_5 8 =$ (E) both (B) and (C)

(B) $\log_5 8 = \dfrac{\log 8}{\log 5}$

(C) $\log_5 8 = \dfrac{\ln 8}{\ln 5}$

90. $4 \log_3 5 = \log_3 5^4 = \log_3 625$ (B)

91. a. Product Property

$\log_b uv = \log_b u + \log_b v$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b v$ then $v = b^y$

so that $\log_5 uv = \log_b(b^x \cdot b^y) = \log_b(b^{x+y})$

$= x + y = \log_b u + \log_b v$

b. Quotient Property

$\log_b \dfrac{u}{v} = \log_b u - \log_b v$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b u$ then $v = b^y$

so that $\log_b \dfrac{u}{v} = \log_b \dfrac{b^x}{b^y} = \log_b(b^{x-y})$

$= x - y = \log_b u - \log_b v$

c. Power Property

$\log_b u^n = n \log_b u$

let $x = \log_b u$ then $u = b^x$ and $u^n = b^{nx}$

so that $\log_b u^n = \log_b(b^{nx}) = nx = n \log_b u$

d. Change of base formula

$\log_c u = \dfrac{\log_b u}{\log_b c}$

let $x = \log_b u$ then $u = b^x$

let $y = \log_b c$ then $c = b^y$

let $z = \log_c u$ then $u = c^z$

so that $b^x = c^z$

Thus, $x = \log_b u = \log_b b^x = \log_b c^z$

$= z \log_b c = zy$

Thus, $x = yz$, so $z = \dfrac{x}{y}$, or $\log_c u = \dfrac{\log_b u}{\log_b c}$

8.5 Mixed Review (p. 499)

92. $3 \cdot y^2 \cdot y^2 = 3y^{2+2} = 3y^4$ **93.** $(y^4)^3 = y^{4 \cdot 3} = y^{12}$

94. $(x^3y)^4 = x^{3 \cdot 4}y^4 = x^{12}y^4$ **95.** $(-3x^2)^2 = -3^2x^{2 \cdot 2} = 9x^4$

96. $4x^{-1}y = \dfrac{4y}{x}$ **97.** $xy^{-2}x = \dfrac{x^2}{y^2}$ **98.** $\dfrac{x^3}{x^{-1}} = x^{3-(-1)} = x^4$

99. $\dfrac{4x^2y^7}{8xy^{-1}} = \dfrac{1}{2}x^{2-1}y^{7+1} = \dfrac{xy^8}{2}$

100.
$$\sqrt[4]{x+2} + 9 = 14$$
$$\sqrt[4]{x+2} = 5$$
$$\left(\sqrt[4]{x+2}\right)^4 = (5)^4$$
$$x + 2 = 625$$
$$x = 623$$

101.
$$\sqrt[3]{3x-4} = \sqrt[3]{x+10}$$
$$\left(\sqrt[3]{3x-4}\right)^3 = \left(\sqrt[3]{x+10}\right)^3$$
$$3x - 4 = x + 10$$
$$2x = 14$$
$$x = 7$$

102.
$$\sqrt{3x+7} = x + 3$$
$$\left(\sqrt{3x+7}\right)^2 = (x+3)^2$$
$$3x + 7 = x^2 + 6x + 9$$
$$x^2 + 6x - 3x + 9 - 7 = 0$$
$$x^2 + 3x + 2 = 0$$
$$(x+2)(x+1) = 0$$
$$x = -2 \text{ or } x = -1$$

103. $(5x)^{1/2} - 18 = 32$ **104.** $e^9 = 8103.084$

$$(5x)^{1/2} = 50$$
$$\left[(5x)^{1/2}\right]^2 = (50)^2$$
$$5x = 2500$$
$$x = 500$$

105. $e^{-12} = 6.14 \times 10^{-6}$ **106.** $e^{1.7} = 5.474$

107. $e^{-5.632} = 3.581 \times 10^{-3}$ **108.** $\log 15 = 1.176$

109. $\log 1.729 = 0.238$ **110.** $\ln 16 = 2.773$

111. $\ln 5.89 = 1.773$

Math and History (p. 499)

1. approximate log 3 and log 5

$\log 3 \approx 0.5 \quad \log 5 \approx 0.7$

2. $\log 15 = \log(3 \cdot 5) = \log 3 + \log 5$

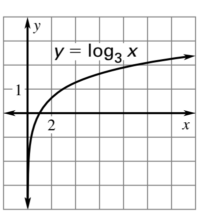
$$= 0.5 + 0.7$$
$$= 1.2$$

8.5 Technology Activity (p. 500)

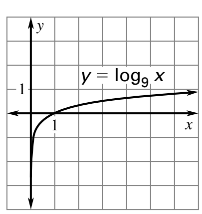
	Point	Vertical asymptote
1. $y = \log_3 x = \dfrac{\log x}{\log 3}$	(1, 0)	$x = 0$
2. $y = \log_9 x = \dfrac{\log x}{\log 9}$	(1, 0)	$x = 0$
3. $y = \log_4 x = \dfrac{\log x}{\log 4}$	(1, 0)	$x = 0$
4. $y = \log_7 x = \dfrac{\log x}{\log 7}$	(1, 0)	$x = 0$
5. $y = \log_5 x = \dfrac{\log x}{\log 5}$	(1, 0)	$x = 0$
6. $y = \log_{11} x = \dfrac{\log x}{\log 11}$	(1, 0)	$x = 0$

Graphs

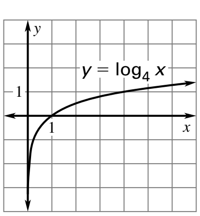
1.



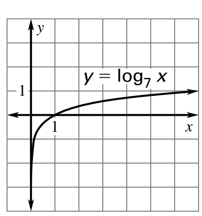
2.



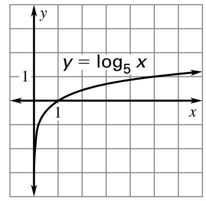
3.



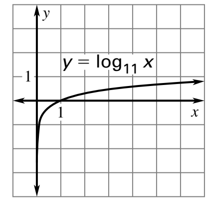
4.



5.



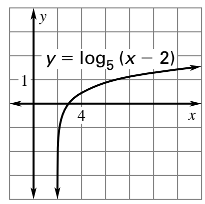
6.



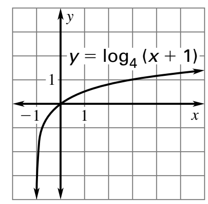
	Point	Vertical asymptote
7. $y = y = \log_5 (x - 2)$	(3, 0)	$x = 2$
8. $y = y = \log_4 (x + 1)$	(0, 0)	$x = -1$

Graphs

7.



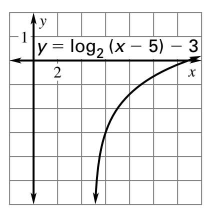
8.



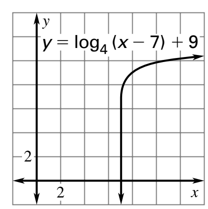
	Point	Vertical asymptote
9. $y = \log_2 (x - 5) - 3$	(6, −3)	$x = 5$
10. $y = \log_4 (x - 7) + 9$	(8, 9)	$x = 7$
11. $y = \log_5 (x + 2) + 6$	(−1, 6)	$x = -2$
12. $y = \log_7 (x - 4) + 4$	(5, 4)	$x = 4$

Graphs

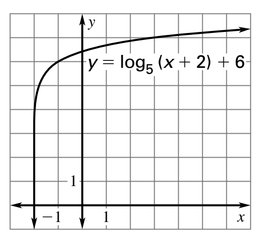
9.



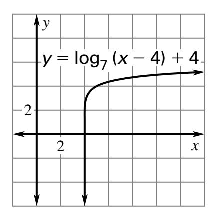
10.



11.



12.



13. The domain of $y = \log x$ is all real numbers greater than 0, while the domain of $y = \log |x|$ is all real numbers except 0. The graph of $y = \log |x|$ is the graph of $y = \log x$ and its reflection is the y-axis.

Lesson 8.6

8.6 Guided Practice (p. 505)

1. $2^{4x} = 8^{x-3}$; $\log (x + 1) + \log (x - 1) = 2.32$

2. Both types of equations can be solved by equating exponents or the expressions whose logarithms you are trying to find; otherwise by using the inverse of the function. In the case of an exponential equation this means taking the logarithm of each side; in the case of a logarithmic equation, raising each side to the same power.

3. Logarithmic equations sometimes have extraneous solutions because the domain of a logarithmic function does not generally include all real numbers.

4. $3^x = 14$

$\log_3 3^x = \log_3 14$

$x = \log_3 14$

$x = \frac{\log 14}{\log 3}$

$x = 2.402$

5. $5^x = 8$

$\log_5 5^x = \log_5 8$

$x = \log_5 8$

$x = \frac{\log 8}{\log 5}$

$x = 1.292$

6. $9^{2x} = 3^{x-6}$

$(3^2)^{2x} = 3^{x-6}$

$3^{4x} = 3^{x-6}$

$4x = x - 6$

$3x = -6$

$x = -2$

7. $10^{3x-4} = 0.1$

$10^{3x-4} = 10^{-1}$

$3x - 4 = -1$

$3x = 3$

$x = 1$

8. $2^{3x} = 4^{x-1}$

$2^{3x} = (2^2)^{x-1}$

$2^{3x} = 2^{2(x-1)}$

$2^{3x} = 2^{2x-2}$

$3x = 2x - 2$

$x = -2$

9. $10^{3x-1} + 4 = 32$

$10^{3x-1} = 28$

$\log 10^{3x-1} = \log 28$

$3x - 1 = \log 28$

$3x = \log 28 + 1$

$x = \frac{\log 28 + 1}{3}$

≈ 0.816

10. $\log x = 2.4$

$10^{2.4} = x$

$x = 251.189$

11. $\log x = 3$

$10^3 = x$

$x = 1000$

12. $\log_3 (2x - 1) = 3$

$3^3 = 2x - 1$

$27 = 2x - 1$

$28 = 2x$

$x = 14$

13. $12 \ln x = 44$

$\ln x = \frac{44}{12}$

$\ln x = 3.67$

$e^{3.67} = x$

$x = 39.121$

14. $\log_2 (x + 2) = \log_2 x^2$

$(x + 2) = x^2$

$x + 2 = x^2$

$x^2 - x - 2 = 0$

$(x + 1)(x - 2) = 0$

$x = -1$ or $x = 2$

15. $\log 3x + \log(x + 2) = 1$

$\log [3x(x + 2)] = 1$

$\log (3x^2 + 6x) = 1$

$3x^2 + 6x = 10^1$

$3x^2 + 6x - 10 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-10)}}{2(3)}$

$x = \frac{-6 \pm \sqrt{36 + 120}}{6}$

$x = \frac{-6 \pm \sqrt{156}}{6}$

$x = -\frac{6}{6} \pm \frac{\sqrt{4}\sqrt{39}}{6}$

$x = -1 \pm \frac{2\sqrt{39}}{6}$

$x = -1 \pm \frac{\sqrt{39}}{3} \approx 1.082$

16. $4^{x+1} = 8^x$

$(2^2)^{x+1} = (2^3)^x$

$2^{2x+2} = 2^{3x}$

$2x + 2 = 3x$

$x = 2$

$\log_4 8 \neq 2$

$4^{x+1} = 8^x$

$\log_4 4^{x+1} = \log_4 8^x$

$x + 1 = x \log + 8$

17. $\log_2 5x = 8$

$e^{\log_2 5x} = e^8$

$e^{\log_2 5x} \neq 5x$, since e^x and $\log_2 x$ are not inverse functions.

18. $M = 0.291 \ln E + 1.17$

$9.2 = 0.291 \ln E + 1.17$

$8.03 = 0.291 \ln E$

$27.595 \approx \ln E$

$e^{27.595} \approx e^{\ln E}$

$9.646 \times 10^{11} \approx E$

$964{,}600{,}000{,}000 \approx E$

about 960 billion ergs

8.6 Practice and Applications (p. 505)

19. $\ln x = 27$, $x = e^{27}$ yes

$e^{27} = x$

20. $5 - \log_4 2x = 3$, $x = 8$ yes

$5 - 3 = \log_4 2x$

$\log_4 2x = 2$

$4^2 = 2x$

$16 = 2x$

$x = 8$

21. $\ln 5x = 4,\ x = \frac{1}{4}e^5$ no

$e^4 = 5x$

$\frac{e^4}{5} = x$

$x = \frac{1}{5}e^4$

22. $\log_5 \frac{1}{2}x = 17,\ x = 2e^{17}$ no

$(5)^{17} = \frac{1}{2}x$

$2(5)^{17} = x$

23. $5e^x = 15,\ x = \ln 3$ yes

$e^x = \frac{15}{5}$

$e^x = 3$

$\ln 3 = x$

24. $e^x + 2 = 18,\ x = \log_2 16$ no

$e^x = 16$

$\ln 16 = x$

25. $10^{x-3} = 100^{4x-5}$

$10^{x-3} = (10^2)^{4x-5}$

$10^{x-3} = 10^{8x-10}$

$x - 3 = 8x - 10$

$7 = 7x$

$x = 1$

26. $25^{x-1} = 125^{4x}$

$(5^2)^{x-1} = (5^3)^{4x}$

$5^{2x-2} = 5^{12x}$

$2x - 2 = 12x$

$-2 = 10x$

$x = -\frac{2}{10} = -\frac{1}{5}$

27. $3^{x-7} = 27^{2x}$

$3^x - 7 = (3^3)^{2x}$

$3^{x-7} = 3^{6x}$

$x - 7 = 6x$

$-7 = 5x$

$x = -\frac{7}{5}$

28. $36^{x-9} = 6^{2x}$

$(6^2)^{x-9} = 6^{2x}$

$6^{2x-18} = 6^{2x}$

$2x - 18 = 2x$

$0 \neq -18$

no solution

29. $8^{5x} = 16^{3x+4}$

$(2^3)^{5x} = (2^4)^{3x+4}$

$2^{15x} = 2^{12x+16}$

$15x = 12x + 16$

$3x = 16$

$x = \frac{16}{3}$

30. $e^{-x} = 6$

$e^x = \frac{1}{6}$

$\ln \frac{1}{6} = x \approx -1.792$

31. $2^x = 15$

$\log_2 2^x = \log_2 15$

$x = \log_2 15$

$x = \frac{\log 15}{\log 2}$

$x = 3.907$

32. $1.2e^{-5x} + 2.6 = 3$

$1.2e^{-5x} = 0.4$

$e^{-5x} = \frac{0.4}{1.2}$

$e^{-5x} = \frac{1}{3}$

$e^{5x} = 3$

$\ln 3 = 5x$

$\frac{\ln 3}{5} = x$

$x = 0.220$

33. $4^x - 5 = 3$

$4^x = 8$

$\log_4 4x = \log_4 8$

$x = \log_4 8$

$x = \frac{\log 8}{\log 4} = 1.5 = \frac{3}{2}$

34. $-5e^{-x} + 9 = 6$

$-5e^{-x} = -3$

$e^{-x} = \frac{3}{5}$

$e^x = \frac{5}{3}$

$\ln \frac{5}{3} = x$

$x = 0.511$

35. $10^{2x} + 3 = 8$

$10^{2x} = 5$

$\log 10^{2x} = \log 5$

$2x = \log 5$

$x = \frac{\log 5}{2}$

$x = 0.349$

36. $0.25^x - 0.5 = 2$

$0.25^x = 2.5$

$\log_{1/4}\left(\frac{1}{4}\right)^x = \log_{1/4} 2.5$

$x = \log_{1/4} 2.5$

$x = \frac{\log 2.5}{\log 0.25}$

$x = -0.661$

37. $\frac{1}{4}(4)^{2x} + 1 = 5$

$\frac{1}{4}(4)^{2x} = 4$

$(4)^{2x} = 16$

$\log_4(4)^{2x} = \log_4 16$

$2x = \log_4 16$

$x = \frac{1}{2}(\log_4 16)$

$x = \frac{1}{2}\left(\frac{\log 16}{\log 4}\right) = \frac{1}{2}(2) = 1$

38. $\frac{2}{3}e^{4x} + \frac{1}{3} = 4$

$\frac{2}{3}e^{4x} = \frac{11}{3}$

$e^{4x} = \frac{11}{3}\left(\frac{3}{2}\right)$

$e^{4x} = \frac{11}{2}$

$\ln e^{4x} = \ln \frac{11}{2}$

$4x = \ln \frac{1}{2}$

$x = \frac{1}{4}\left(\ln \frac{11}{2}\right)$

$x = 0.426$

39. $10^{-12x} + 6 = 100$

$10^{-12x} = 94$

$\log 10^{-12x} = \log 94$

$-12x = \log 94$

$x = -\frac{1}{12}(\log 94)$

$x = -0.164$

40. $4 - 2e^x = -23$

$-2e^x = -23 - 4$

$-2e^x = -27$

$e^x = \frac{27}{2}$

$\ln \frac{27}{2} = x$

$x \approx 2.603$

41. $3^{0.1x} - 4 = 5$

$3^{0.1x} = 9$

$3^{0.1x} = 3^2$

$0.1x = 2$

$x = 20$

42. $$\begin{aligned} -16 + 0.2(10)^x &= 35 \\ 0.2(10)^x &= 51 \\ \tfrac{1}{5}(10)^x &= 51 \\ (10)^x &= 255 \\ \log 255 &= x \\ x &\approx 2.407 \end{aligned}$$

43. $$\begin{aligned} \ln(4x + 1) &= \ln(2x + 5) \\ 4x + 1 &= 2x + 5 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

44. $$\begin{aligned} \log_2 x &= -1 \\ 2^{-1} &= x \\ \tfrac{1}{2} &= x \end{aligned}$$

45. $$\begin{aligned} 4 \log_3 x &= 28 \\ \log_3 x &= \tfrac{28}{4} \\ \log_3 x &= 7 \\ 3^7 &= x \\ x &= 2187 \end{aligned}$$

46. $$\begin{aligned} 16 \ln x &= 30 \\ \ln x &= \tfrac{30}{16} \\ \ln x &= \tfrac{15}{8} \\ e^{15/8} &= x \\ x &\approx 6.521 \end{aligned}$$

47. $$\begin{aligned} \tfrac{1}{2}\log_6 16x &= 3 \\ \log_6 16x &= 6 \\ 6^6 &= 16x \\ \tfrac{46,656}{16} &= x \\ x &= 2916 \end{aligned}$$

48. $$\begin{aligned} 1 - 2 \ln x &= -4 \\ -2 \ln x &= -5 \\ \ln x &= \tfrac{5}{2} \\ e^{5/2} &= x \end{aligned}$$

49. $$\begin{aligned} 2 \ln(-x) + 7 &= 14 \\ 2 \ln(-x) &= 7 \\ \ln(-x) &= \tfrac{7}{2} \\ e^{7/2} &= -x \\ x &= -e^{7/2} \end{aligned}$$

50. $$\begin{aligned} \log_5(2x + 15) &= \log_5 3x \\ 2x + 15 &= 3x \\ 15 &= x \end{aligned}$$

51. $$\begin{aligned} \ln x + \ln(x - 2) &= 1 \\ \ln[x(x - 2)] &= 1 \\ e^{\ln(x^2 - 2x)} &= e^1 \\ x^2 - 2x &= e \\ x^2 - 2x - e &= 0 \end{aligned}$$

$$\begin{aligned} x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-e)}}{2(1)} &= \frac{2 \pm \sqrt{4 + 4e}}{2} \\ &= 1 + \sqrt{1 + e} \\ &\approx 2.928 \end{aligned}$$

52. $$\begin{aligned} \ln x + \ln(x + 3) &= 1 \\ \ln[x(x + 3)] &= 1 \\ e^{\ln(x^2 + 3x)} &= e^1 \\ x^2 + 3x &= e \\ x^2 + 3x - e &= 0 \end{aligned}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-e)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 4e}}{2} \approx 0.729$$

53. $$\begin{aligned} \log_8(11 - 6x) &= \log_8(1 - x) \\ 11 - 6x &= 1 - x \\ 11 - 1 &= 6x - x \\ 10 &= 5x \\ 2 &= x \quad \text{extraneous root} \end{aligned}$$

$$\begin{aligned} \log_8[11 - 6(2)] &= \log_8(1 - 2) \\ \log_8(-1) &\neq \log_8(-1) \end{aligned}$$

no solution

54. $$\begin{aligned} 15 + 2 \log_2 x &= 31 \\ 2 \log_2 x &= 16 \\ \log_2 x &= 8 \\ 2^8 &= x \\ x &= 256 \end{aligned}$$

55. $$\begin{aligned} -5 + 2 \ln 3x &= 5 \\ 2 \ln 3x &= 10 \\ \ln 3x &= 5 \\ e^5 &= 3x \\ \frac{e^5}{3} &= x \\ x &= \frac{1}{3}e^5 \end{aligned}$$

56. $$\begin{aligned} \log(5 - 3x) &= \log(4x - 9) \\ 5 - 3x &= 4x - 9 \\ 14 &= 7x \quad \text{no solution} \\ x &= 2 \quad \text{extraneous root} \end{aligned}$$

57. $$\begin{aligned} 6.5 \log_5 3x &= 20 \\ \log_5 3x &= \frac{20}{6.5} \\ 5^{20/6.5} &= 3x \\ \frac{5^{20/6.5}}{3} &= x \\ x &= 47.158 \end{aligned}$$

58. $$\begin{aligned} \ln(x + 5) &= \ln(x - 1) - \ln(x + 1) \\ x + 5 &= (x - 1) - (x + 1) \\ x + 5 &= x - x - 1 - 1 \\ x + 5 &= -2 \quad \text{no solution} \\ x &= -7 \quad \text{extraneous root} \end{aligned}$$

59. $$\begin{aligned} \ln(5.6 - x) &= \ln(18.4 - 2.6x) \\ 5.6 - x &= 18.4 - 2.6x \\ 1.6x &= 12.8 \\ x &= \frac{12.8}{1.6} \\ x &= 8 \quad \text{extraneous root} \end{aligned}$$

no solution

60. $10 \ln 100x - 3 = 117$

$$10 \ln 100x = 120$$
$$\ln 100x = 12$$
$$e^{12} = 100x$$
$$\frac{e^{12}}{100} = x$$
$$x = 0.01e^{12}$$

61. $4^{3x} = 8^{x+1}$

$$\log 4^{3x} = \log 8^{x+1}$$
$$3x \log 4 = (x + 1)\log 8$$
$$3x \log 4 = x \log 8 + \log 8$$
$$3x \log 4 - x \log 8 = \log 8$$
$$x(3 \log 4 - \log 8) = \log 8$$
$$x = \frac{\log 8}{3 \log 4 - \log 8} = 1$$

Sample answer: This method is needlessly complicated.

62. $T = (T_0 - T_R)e^{-rt} + T_R$

$$95 = (205 - 68)e^{-0.03t} + 68$$
$$27 = 137e^{-0.03t}$$
$$\frac{27}{137} = e^{-0.03t}$$
$$0.197 \approx e^{-0.03t}$$
$$\ln 0.197 \approx \ln e^{-0.03t}$$
$$-1.625 \approx -0.03t$$
$$t \approx \frac{-1.625}{-0.03}$$

$t \approx 54.2$ about 54 minutes

63. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$$2400 = 2000\left(1 + \frac{0.02}{4}\right)^{4t}$$
$$2400 = 2000(1.005)^{4t}$$
$$\frac{2400}{2000} = (1.005)^{4t}$$
$$1.2 = (1.005)^{4t}$$
$$\log 1.2 = \log(1.005)^{4t}$$
$$0.079 = 4t \log (1.005)$$
$$\frac{0.079}{4 \log(1.005)} = t$$
$$t = \frac{0.079}{0.009}$$

$t \approx 8.78$ or about 9 years

64. $y = a(1 - r)^t$

$$10 = 20(1 - 0.05)^t$$
$$10 = 20(0.95)^t$$
$$\frac{1}{2} = (0.95)^t$$
$$\log \frac{1}{2} = \log(0.95)^t$$
$$-0.301 = t \log(0.95)$$
$$\frac{-0.301}{-0.022} \approx t$$

$13.5 \approx t$ about 13.5 days

65. $A = Pe^{rt}$

$$1000 = 500e^{0.025t}$$
$$2 = e^{0.025t}$$
$$\ln 2 = \ln e^{0.025t}$$
$$\ln 2 = 0.025t \ln e$$
$$\frac{\ln 2}{0.025 \ln e} = t$$

27.7 years $\approx t$

66. $P = 8863(1.04)^t$

$$345{,}000 = 8863(1.04)^t$$
$$\frac{345{,}000}{8863} = (1.04)^t$$
$$38.926 = (1.04)^t$$
$$\log 38.926 = t \log(1.04)$$
$$\frac{\log 38.926}{\log(1.04)} = t$$
$$93.36 \approx t$$

about 93.4 years or in 1713

67. a. Subantarctic:

$$d = 1.0245 - e^{0.1266T - 7.828}$$
$$1.0234 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.0234$$
$$e^{0.1266T - 7.828} = 0.0011$$
$$\ln e^{0.1266T - 7.828} = \ln 0.0011$$
$$0.1266T - 7.828 \ln e = \ln 0.0011$$
$$T = \frac{\ln 0.0011 + 7.828 \ln e}{0.1266}$$
$$T \approx 8.022°C$$

—CONTINUED—

67. —CONTINUED—

b. Antarctic intermediate:

$$1.02384 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.02384$$
$$e^{0.1266T - 7.828} = 0.00066$$
$$\ln e^{0.1266T - 7.828} = \ln 0.00066$$
$$0.1266T - 7.828 \ln e = \ln 0.00066$$
$$T = \frac{\ln 0.00066 + 7.828 \ln e}{0.1266}$$
$$T = 3.99°C$$

c. North Atlantic deep:

$$1.02399 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.02399$$
$$e^{0.1266T - 7.828} = 0.00051$$
$$\ln e^{0.1266T - 7.828} = \ln 0.00051$$
$$0.1266T - 7.828 \ln e = \ln 0.00051$$
$$T = \frac{\ln 0.00051 + 7.828 \ln e}{0.1266}$$
$$T = 1.95°C$$

d. Antarctic bottom:

$$1.0241 = 1.0245 - e^{0.1266T - 7.828}$$
$$e^{0.1266T - 7.828} = 1.0245 - 1.0241$$
$$e^{0.1266T - 7.828} = 0.0004$$
$$\ln e^{0.1266T - 7.828} = \ln 0.0004$$
$$0.1266T - 7.828 \ln e = \ln 0.0004$$
$$T = \frac{\ln 0.0004 + 7.828 \ln e}{0.1266}$$
$$T = 0.03°\text{C}$$

68.

$$m = e^{6.331 - 0.403t}$$
$$204 = e^{6.331 - 0.403t}$$
$$\ln 204 = \ln e^{6.331 - 0.403t}$$
$$\ln 204 = 6.331 - 0.403t \ln e$$
$$\ln 204 - 6.331 = -0.403t \ln e$$
$$\frac{\ln 204 - 6.331}{-0.403 \ln e} = t$$
$$2.51\ \mu s \approx t$$

69.

$$m = 5 \log D + 2$$
$$12 = 5 \log D + 2$$
$$12 - 2 = 5 \log D$$
$$10 = 5 \log D$$
$$2 = \log D$$
$$10^2 = D$$
$$100\text{ mm} = D$$

70.

$$h = -8005 \ln \frac{P}{101{,}300}$$
$$4000 = -8005 \ln \frac{P}{101{,}300}$$
$$-\frac{4000}{8005} = \ln \frac{P}{101{,}300}$$
$$-0.499 \approx \ln P - \ln 101{,}300$$
$$\ln 101{,}300 - 0.499 \approx \ln P$$
$$11.027 \approx \ln P$$
$$e^{11.027} \approx P$$
$$P \approx 61{,}461 \text{ pascals}$$

71. a.

$$l = 45 - 25.7e^{-0.09a}$$
$$36 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 9$$
$$e^{-0.09a} = \frac{9}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.350194553$$
$$a(-0.09) \ln e \approx \ln 0.350194553$$
$$a \approx \frac{\ln 0.350194553}{(-0.09) \ln e}$$
$$a \approx 11.66 \text{ years or about 12 years}$$

$$32 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 13$$
$$e^{-0.09a} = \frac{13}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.506$$
$$a(-0.09) \ln e \approx \ln 0.506$$
$$a \approx \frac{\ln 0.506}{(-0.09) \ln e}$$
$$a \approx 7.57 \text{ years or about 8 years}$$

$$28 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 17$$
$$e^{-0.09a} = \frac{17}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.661$$
$$a(-0.09) \ln e \approx \ln 0.661$$
$$a \approx \frac{\ln 0.661}{(-0.09) \ln e}$$
$$a \approx 4.59 \text{ years or about 5 years}$$

—CONTINUED—

71. a. **—CONTINUED—**

$$24 = 45 - 25.7e^{-0.09a}$$
$$25.7e^{-0.09a} = 21$$
$$e^{-0.09a} = \frac{21}{25.7}$$
$$\ln e^{-0.09a} \approx \ln 0.817$$
$$a(-0.09)\ln e \approx \ln 0.817$$
$$a \approx \frac{\ln 0.817}{(-0.09)\ln e}$$
$$a \approx 2.24 \text{ years or about 2 years}$$

b. $I = 45 - 25.7e^{-0.09a}$

$$25.7e^{-0.09a} = 45 - I$$
$$e^{-0.09a} = \frac{45 - I}{25.7}$$
$$\ln e^{-0.09a} = \ln \frac{45 - I}{25.7}$$
$$(-0.09)a(\ln e) = \ln \frac{45 - I}{25.7}$$
$$a = \frac{\ln \frac{45 - I}{25.7}}{-0.09}$$
$$a = \frac{\ln \frac{45 - 36}{25.7}}{-0.09} \approx 11.66 \text{ years or about 12 years}$$
$$a = \frac{\ln \frac{45 - 32}{25.7}}{-0.09} \approx 7.57 \text{ years or about 8 years}$$
$$a = \frac{\ln \frac{45 - 28}{25.7}}{-0.09} \approx 4.59 \text{ years or about 5 years}$$
$$a = \frac{\ln \frac{45 - 24}{25.7}}{-0.09} \approx 2.24 \text{ years or about 2 years}$$

c. It is difficult to solve for a in part 71(b), so I prefer the method in 71(a).

72.

$$2^{x+3} = 5^{3x-1}$$
$$\log_2 2^{x+3} = \log_2 5^{3x-1}$$
$$x + 3 = \log_2 5^{3x-1}$$
$$x + 3 = 3x - 1(\log_2 5)$$
$$x + 3 = 3x(\log_2 5) - (\log_2 5)$$
$$3 + \log_2 5 = 3x(\log_2 5) - x$$
$$3 + \log_2 5 = x(3\log_2 5 - 1)$$
$$\frac{3 + \log_2 5}{3\log_2 5 - 1} = x$$
$$\frac{3 + \frac{\log 5}{\log 2}}{3\left(\frac{\log 5}{\log 2}\right) - 1} \approx 0.892 = x$$

73.

$$10^{5x+2} = 5^{4-x}$$
$$\log 10^{5x+2} = \log 5^{4-x}$$
$$5x + 2 = \log 5^{4-x}$$
$$5x + 2 = (4 - x)\log 5$$
$$5x + 2 = 4\log 5 - x\log 5$$
$$5x + x\log 5 = 4\log 5 - 2$$
$$x(5 + \log 5) = 4\log 5 - 2$$
$$x = \frac{4\log 5 - 2}{5 + \log 5} = \frac{4(0.698970004) - 2}{5 + (0.698970004)}$$
$$= \frac{0.795880017}{5.698970004} \approx 0.14$$

74.

$$\log_3(x - 6) = \log_9 2x$$
$$\log_3(x - 6) = \frac{\log_3 2x}{\log_3 9}$$
$$\log_3(x - 6) = \frac{\log_3 2x}{2}$$
$$2\log_3(x - 6) = \log_3 2x$$
$$\log_3(x - 6)^2 = \log_3 2x$$
$$(x - 6)^2 = 2x$$
$$x^2 - 14x + 36 = 0$$
$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(36)}}{2(1)}$$
$$x = \frac{14 \pm \sqrt{52}}{2}$$
$$x = \frac{14 \pm 2\sqrt{13}}{2}$$
$$x = 7 - \sqrt{13} \text{ extraneous solution}$$
$$x = 7 + \sqrt{13}$$

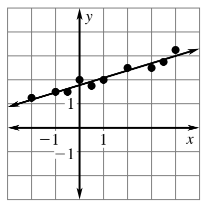
75.

$$\log_4 x = \log_8 4x$$
$$\log_4 x = \frac{\log_4 4x}{\log_4 8}$$
$$\log_4 x = \frac{\log_4 4x}{\frac{3}{2}}$$
$$\frac{3}{2}\log_4 x = \log_4 4x$$
$$\log_4 x^{\frac{3}{2}} = \log_4 4x$$
$$x^{\frac{3}{2}} = 4x$$
$$x^{\frac{3}{2}} - 4x = 0$$
$$x(x^{\frac{1}{2}} - 4) = 0$$
$$x = 0 \qquad x^{\frac{1}{2}} - 4 = 0$$
$$\left(x^{\frac{1}{2}}\right)^2 = (4)^2$$
$$x = 16$$

76. *Sample answer:* For an exponential equation, take the logarithm of both sides to one of the bases, use the change of base formula for the other side of the equation or take the common logarithm or natural logarithm of both sides of the equation. For a logarithmic equation, use the change of base formula to rewrite both sides in terms of logarithms with a single base, then solve normally.

8.6 Mixed Review (p. 508)

77. $m = \dfrac{3.25 - 1.25}{4 + 2} = \dfrac{2}{6} = \dfrac{1}{3}$



$\dfrac{y - 1.25}{x + 2} = \dfrac{1}{3}$

$3(y - 1.25) = x + 2$

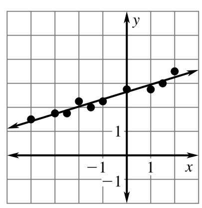
$3y - 3.75 = x + 2$

$3y = x + 5.75$

$y = \dfrac{1}{3}x + \dfrac{5.75}{3}$

$y = \dfrac{1}{3}x + 1.92$

78. $m = \dfrac{3.5 - 1.5}{2 + 4} = \dfrac{2}{6} = \dfrac{1}{3}$



$\dfrac{y - 1.5}{x + 4} = \dfrac{1}{3}$

$3(y - 1.5) = x + 4$

$3y - 4.5 = x + 4$

$3y = x + 8.5$

$y = \dfrac{1}{3}x + \dfrac{8.5}{3}$

$y = \dfrac{1}{3}x + 2.83$

79. $2x - y = 3$ $\quad y = 2x - 3$

$-y = -2x + 3$ $\quad y = 2(4) - 3$

$y = 2x - 3$ $\quad y = 8 - 3$

$3x - 2y = 2$ $\quad y = 5$

$3x - 2(2x - 3) = 2$ $\quad (4, 5)$

$3x - 4x + 6 = 2$

$-x = 2 - 6$

$-x = -4$

$x = 4$

80. $2x + y = 4$ $\quad 2x + (3 - x) = 4$

$x + y = 3$ $\quad 2x + 3 - x = 4$

$y = 3 - x$ $\quad x = 1$

$y = 3 - 1 = 2$ $\quad (1, 2)$

81. $x + 4y = -24$ $\quad x = -24 - 4y$

$x - 4y = 24$ $\quad x - 4y = 24$

$x + 4(-6) = -24$ $\quad (-24 - 4y) - 4y = 24$

$x = 0$ $\quad -24 - 8y = 24$

$(0, -6)$ $\quad -8y = 48$

$y = \dfrac{48}{-8} = -6$

82. $x - 3y = -3$ $\quad x = 3y - 3$

$2x + y = 8$ $\quad 2(3y - 3) + y = 8$

$x - 3(2) = -3$ $\quad 6y - 6 + y = 8$

$x - 6 = -3$ $\quad 7y = 14$

$x = 3$ $\quad y = 2$

$(3, 2)$

83. $2x + y = -1$

$-4x - 2y = -5$

$y = -2x - 1$

$-4x - 2(-2x - 1) = -5$

$-4x + 4x + 2 = -5$

$0 \neq -7$

no solution

84. $-x + 6y = -32$ $\quad -x = -32 - 6y$

$7x - 2y = 24$ $\quad x = 6y + 32$

$7(6y + 32) - 2y = 24$ $\quad x = 6(-5) + 32$

$42y + 224 - 2y = 24$ $\quad x = -30 + 32$

$40y = -200$ $\quad x = 2$

$y = -5$ $\quad (2, -5)$

85. $3x^3 - 6x^2 + 4x - 8 = (3x^2)x - (3x^2)2 + 4(x) - 4(2)$

$= 3x^2(x - 2) + 4(x - 2)$

$= (3x^2 + 4)(x - 2)$

86. $2x^3 - 5x^2 + 16x - 40 = 2x(x^2) - 5(x^2) + 2x(8) - 5(8)$

$= x^2(2x - 5) + 8(2x - 5)$

$= (x^2 + 8)(2x - 5)$

87. $7x^3 + 4x^2 + 35x + 20 = 7x(x^2) + 4(x^2) + 5(7x) + 5(4)$

$= x^2(7x + 4) + 5(7x + 4)$

$= (x^2 + 5)(7x + 4)$

88. $4x^3 - 3x^2 + 8x - 6 = x^2(4x) - x^2(3) + 2(4x) - 2(3)$

$= x^2(4x - 3) + 2(4x - 3)$

$= (x^2 + 2)(4x - 3)$

Quiz 2 (p. 508)

1. $\log_2 8$

$2^x = 8$

$2^3 = 8$ so $x = 3$

2. $\log_5 625$

$5^x = 625$

$5^4 = 625$ so $x = 4$

3. $\log_8 512$

$8^x = 512$

$8^3 = 512$

so $x = 3$

4. $y = \ln(x + 3)$

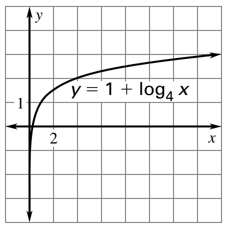
$x = \ln(y + 3)$

$e^x = y + 3$

$y = e^x - 3$

5. $y = 1 + \log_4 x$

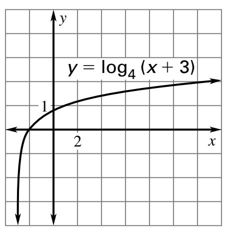
x	y
1	0
$\frac{1}{4}$	-1
4	1



Domain: $x > 0$
Range: all real numbers

6. $y = \log_4 (x + 3)$

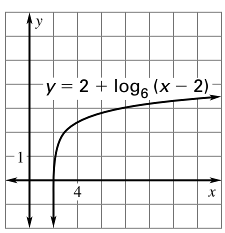
x	y
1	1
-2	0
-2.75	-1



Domain: $x > -3$
Range: all real numbers

7. $y = 2 + \log_6(x - 2)$

x	y
3	2
8	3
$2\frac{1}{16}$	1



Domain: $x > 2$
Range: all real numbers

8. $\log_3(3 \cdot 27) = \log_3 3 + \log_3 27 = 1 + 3 = 4$

9. $\log_2 \frac{1}{2}$ $\quad 2^x = \frac{1}{2}$

$2^{-1} = \frac{1}{2}$ so $x = -1$

10. $\ln e^2 = 2 \ln e = 2(1) = 2$

11. $\log_4 x^{1/2} y^4 = \log_4 x^{1/2} + \log_4 y^4$

$= \frac{1}{2}\log_4 x + 4 \log_4 y$

12. $2 \log_6 14 + 3 \log_6 x - \log_6 7$

$= \log_6(14)^2 + \log_6 x^3 - \log_6 7$

$= \log_6 \dfrac{196x^3}{7}$

$= \log_6 28x^3$

13. $\log_4 22 = \dfrac{\log 22}{\log 4} = 2.230$

14. $3e^x - 1 = 14$

$3e^x = 15$

$e^x = 5$

$\ln e^x = \ln 5$

$x \ln e = \ln 5$

$x = \ln 5$

15. $3 \log_2 x = 28$

$\log_2 x = \frac{28}{3}$

$2^{28/3} = x$

$x = 2^{28/3}$

16. $\ln(2x + 7) = \ln(x - 4)$

$2x + 7 = x - 4$

$x = -11$

no solution

17. $M = 0.291 \ln E + 1.17$

$8.5 = 0.291 \ln E + 1.17$

$7.33 = 0.291 \ln E$

$\dfrac{7.33}{0.291} = \ln E$

$25.189 \approx \ln E$

$e^{25.189} \approx e^{\ln E}$

$8.699 \times 10^{10} \approx E$

$E \approx 86{,}990{,}000{,}000$

$E \approx 87$ billion ergs

Lesson 8.7

8.7 Guided Practice (p. 513)

1. exponential model

2. 2 points: exponential function

2 points: power function

3. No, since 0 is not in the domain of $f(x) = \ln x$.

4. (1, 3), (2, 36)

$3 = ab^1$

$36 = ab^2$

$y = ab^x$

$a = \dfrac{3}{b^1}$

$36 = \left(\dfrac{3}{b^1}\right)b^2$

$36 = 3b$

$12 = b$

$a = \dfrac{3}{12} = \dfrac{1}{4}$

$y = \dfrac{1}{4} \cdot 12^x$

5. (2, 2), (4, 18)

$2 = ab^2$

$18 = ab^4$

$y = ab^x$

$a = \dfrac{2}{b^2}$

$18 = \left(\dfrac{2}{b^2}\right)b^4$

$18 = 2b^2$

$9 = b^2$

$3 = b$

$a = \dfrac{2}{(3)^2} = \dfrac{2}{9}$

$y = \dfrac{2}{9} \cdot 3^x$

6. $(1, 4), (3, 16)$

$4 = ab^1$

$16 = ab^3$

$y = ab^x$

$a = \frac{4}{b^1}$

$16 = \left(\frac{4}{b^1}\right)b^3$

$16 = 4b^2$

$4 = b^2$

$2 = b$

$a = \frac{4}{2}$

$a = 2$

$y = 2 \cdot 2^x$

7. $(2, 3.5), (1, 5.2)$

$3.5 = ab^2$

$5.2 = ab^1$

$y = ab^x$

$a = \frac{3.5}{b^2}$

$5.2 = \left(\frac{3.5}{b^2}\right)b^1$

$5.2 = 3.5b^{-1}$

$\frac{5.2}{3.5} = b^{-1}$

$\frac{35}{52} = b$

$a = \frac{3.5}{\left(\frac{35}{52}\right)^2}$

$a = \left(\frac{3.5}{1}\right)\left(\frac{52}{35}\right)^2$

$a = \frac{2704}{350}$

$y = \frac{2704}{350} \cdot \left(\frac{35}{52}\right)^x$

8. $(5, 8), (3, 32)$

$8 = ab^5$

$32 = ab^3$

$y = ab^x$

$a = \frac{8}{b^5}$

$32 = \left(\frac{8}{b^5}\right)b^3$

$32 = 8b^{-2}$

$\frac{32}{8} = b^{-2}$

$\frac{8}{32} = b^2$

$\frac{1}{4} = b^2$

$\frac{1}{2} = b$

$a = \frac{8}{\left(\frac{1}{2}\right)^5}$

$a = \frac{8}{\frac{1}{32}}$

$a = 32 \cdot 8$

$a = 256$

$y = 256 \cdot \left(\frac{1}{2}\right)^x$

9. $\left(1, \frac{1}{2}\right), \left(1, \frac{3}{8}\right)$

$\frac{1}{2} = ab^1$

$\frac{3}{8} = ab^1$

$y = ab^x$

$a = \frac{\frac{1}{2}}{b^1}$

$\frac{3}{8} = \left(\frac{\frac{1}{2}}{b^1}\right)b^1$

$\frac{3}{8} \neq \frac{1}{2}$ no solution

10. $y = ax^b$ $(3, 27), (9, 243)$

$27 = a(3)^b$

$243 = a(9)^b$

$a = \frac{27}{(3)^b}$

$243 = \left(\frac{27}{(3)^b}\right)(9)^b$

$243 = 27 \cdot 3^b$

$9 = 3^b$

$\log_3 9 = \log_3 3^b$

$\log_3 9 = b$

$\frac{\log 9}{\log 3} = b$

$b = 2$

$a = \frac{27}{(3)^2} = \frac{27}{9} = 3$

$y = 3x^2$

11. $(1, 2), (4, 32)$

$2 = a1^b$

$32 = a4^b$

$y = ax^b$

$a = \frac{2}{1^b}$

$32 = \left(\frac{2}{1^b}\right)4^b$

$32 = 2 \cdot 4^b$

$16 = 4^b$

$\log_4 16 = \log_4 4^b$

$\log_4 16 = b$

$\frac{\log 16}{\log 4} = b$

$2 = b$

$a = \frac{2}{1^2} = 2$

$y = 2x^2$

12. $(4, 48), (2, 6)$

$48 = a4^b$

$6 = a2^b$

$y = ax^b$

$a = \frac{48}{4^b}$

$6 = \left(\frac{48}{4^b}\right)2^b$

$6 = 48 \cdot 2^b \cdot 2^{-2b}$

$6 = 48 \cdot 2^{b-2b}$

$6 = 48 \cdot 2^{-b}$

$\frac{1}{8} = 2^{-b}$

$8 = 2^b$

$\log_2 8 = \log_2 2^b$

$\log_2 8 = b$

$\frac{\log 8}{\log 2} = b$

$3 = b$

$a = \frac{48}{4^3}$

$a = \frac{48}{64} = \frac{6}{8} = \frac{3}{4}$

$y = \frac{3}{4}x^3$

13. $(1, 4), (3, 8)$

$4 = a1^b$

$8 = a3^b$

$y = ax^b$

$a = \frac{4}{1^b}$

$8 = \left(\frac{4}{1^b}\right)3^b$

$8 = 4 \cdot 3^b$

$2 = 3^b$

$\log_3 2 = \log_3 3^b$

$\log_3 2 = b$

$\frac{\log 2}{\log 3} = b$

$0.631 \approx b$

$a = \frac{4}{1^b} = \frac{4}{1^{0.631}} = 4$

$y = 4x^{0.631}$

14. $(4.5, 9.2), (1, 6.4)$

$9.2 = a(4.5)^b$

$6.4 = a1^b$

$\frac{\log 1.4375}{\log 4.5} = b$

$0.241 = b$

$a = \frac{9.2}{4.5^{0.241}}$

$a \approx 6.4$

$y = 6.4x^{0.241}$

$y = ax^b$

$a = \frac{9.2}{(4.5)^b}$

$6.4 = \left[\frac{9.2}{(4.5)^b}\right]1^b$

$6.4 = 9.2 \cdot 4.5^{-b}$

$\frac{6.4}{9.2} = 4.5^{-b}$

$\frac{9.2}{6.4} = 4.5^b$

$1.4375 = 4.5^b$

$\log_{4.5} 1.4375 = \log_{4.5} 4.5^b$

$\log_{4.5} 1.4375 = b$

15. $\left(2, \frac{1}{2}\right), \left(4, \frac{3}{5}\right)$

$\frac{1}{2} = a2^b$

$\frac{3}{5} = a4^b$

$0.6 = \left(\frac{0.5}{2^b}\right)4^b$

$0.6 = 0.5 \cdot 2^b$

$\frac{0.6}{0.5} = 2^b$

$\log_2 1.2 = \log_2 2^b$

$\log_2 1.2 = \text{b}$

$\frac{\log 1.2}{\log_2} = b$

$0.263 = b$

$y = ax^b$

$a = \frac{0.5}{2^b}$

$a = \frac{0.5}{2^{0.263}}$

$a = 0.417$

$y = 0.417x^{0.263}$

16. $y = 1.30(1.46)^{18}$

$y = 1{,}181$ million or about 1.18 billion cell-phone subscribers. The answer tells me about the model that the model probably isn't applicable over such a long period of time.

8.7 Practice and Applications (pp. 513–516)

17. $(1, 4), (2, 12)$

$4 = ab^1$

$12 = ab^2$

$y = ab^x$

$a = \frac{4}{b^1}$

$12 = \left(\frac{4}{b^1}\right)b^2$

$12 = 4b^1$

$3 = b$

$a = \frac{4}{3}$

$y = \frac{4}{3} \cdot 3^x$

18. $(2, 18), (3, 108)$

$18 = ab^2$

$108 = ab^3$

$y = ab^x$

$a = \frac{18}{b^2}$

$108 = \left(\frac{18}{b^2}\right)b^3$

$108 = 18b^1$

$6 = b$

$a = \frac{18}{(6)^2} = \frac{18}{36} = \frac{1}{2}$

$y = \frac{1}{2} \cdot 6^x$

19. $(6, 8), (7, 32)$

$8 = ab^6$

$32 = ab^7$

$y = ab^x$

$a = \frac{8}{b^6}$

$32 = \left(\frac{8}{b^6}\right) \cdot b^7$

$32 = 8b$

$4 = b$

$a = \frac{8}{b^6} = \frac{8}{(4)^6}$

$= \frac{8}{4096} = \frac{1}{512}$

$y = \frac{1}{512} \cdot 4^x$

20. $(1, 7), (3, 63)$

$7 = ab^1$

$63 = ab^3$

$y = ab^x$

$a = \frac{7}{b^1}$

$63 = \left(\frac{7}{b^1}\right)b^3$

$63 = 7b^2$

$9 = b^2$

$3 = b$

$a = \frac{7}{b^1} = \frac{7}{3}$

$y = \frac{7}{3} \cdot 3^x$

21. $(3, 8), (6, 64)$

$8 = ab^3$

$64 = ab^6$

$y = ab^x$

$a = \frac{8}{b^3}$

$64 = \left(\frac{8}{b^3}\right)b^6$

$64 = 8b^3$

$8 = b^3$

$2 = b$

$a = \frac{8}{(2)^3} = \frac{8}{8} = 1$

$y = 1 \cdot 2^x = 2^x$

22. $(-3, 3), (4, 6561)$

$3 = ab^{-3}$

$6561 = ab^4$

$y = ab^x$

$a = \frac{3}{b^{-3}} = 3b^3$

$6561 = (3b^3)b^4$

$6561 = 3b^7$

$2187 = b^7$

$3^7 = 2187$ so $b = 3$

$a = 3b^3 = 3 \cdot (3)^3$

$a = 81$

$y = 81(3)^x$

23. $\left(4, \frac{112}{81}\right), \left(-1, \frac{21}{2}\right)$

$$\frac{112}{81} = ab^4 \qquad \frac{21}{2} = ab^{-1}$$

$$y = ab^x$$

$$a = \frac{\frac{112}{81}}{b^4}$$

$$\frac{21}{2} = \left(\frac{\frac{112}{81}}{b^4}\right)b^{-1}$$

$$b^5 \cdot \frac{21}{2} = \frac{112}{81}$$

$$b^5 = \frac{112}{81} \cdot \frac{2}{21}$$

$$b^5 = \frac{224}{1701}$$

$$b^5 = 0.132$$

$$b = 0.667$$

$$b = \frac{2}{3}$$

$$a = \frac{\frac{112}{81}}{b^4} = \frac{\frac{112}{81}}{\left(\frac{2}{3}\right)^4} = \frac{\frac{112}{81}}{\frac{16}{8}} = \frac{112}{\cancel{81}} \cdot \frac{\cancel{81}}{16} = 7$$

$$y = 7\left(\frac{2}{3}\right)^x$$

24. (3, 13.5), (5, 30.375)

$$13.5 = ab^3 \qquad 30.375 = ab^5$$

$$y = ab^x$$

$$a = \frac{13.5}{b^3}$$

$$30.375 = \left(\frac{13.5}{b^3}\right)b^5$$

$$30.375 = 13.5b^2$$

$$\frac{30.375}{13.5} = b^2$$

$$2.25 = b^2$$

$$1.5 = b$$

$$a = \frac{13.5}{(1.5)^3}$$

$$a = \frac{13.5}{3.375}$$

$$a = 4$$

$$y = 4(1.5)^x$$

25. $\left(2, \frac{25}{4}\right), \left(4, \frac{625}{4}\right)$

$$\frac{25}{4} = ab^2 \qquad \frac{625}{4} = ab^4$$

$$y = ab^x$$

$$a = \frac{25}{4} \cdot b^{-2}$$

$$\frac{625}{4} = \frac{25}{4} \cdot b^{-2} \cdot b^4$$

$$\frac{625}{4} = \frac{25}{4} \cdot b^2$$

$$\frac{625}{\cancel{4}} \cdot \frac{\cancel{4}}{25} = b^2$$

$$25 = b^2$$

$$5 = b$$

$$a = \frac{25}{4} \cdot (5)^{-2}$$

$$a = \frac{\cancel{25}}{4} \cdot \frac{1}{\cancel{25}} = \frac{1}{4}$$

$$y = \frac{1}{4}(5)^x$$

26.

x	1	2	3	4	5	6	7	8
y	14	28	56	112	224	448	896	1792
x	1	2	3	4	5	6	7	8
ln y	2.64	3.33	4.03	4.72	5.41	6.10	6.80	7.49

$$m = \frac{7.49 - 2.64}{8 - 1} = \frac{4.85}{7} = 0.693$$

$$\frac{y - 2.64}{x - 1} = \frac{4.85}{7}$$

$$4.85(x - 1) = 7(y - 2.64)$$

$$4.85x - 4.85 = 7y - 18.48$$

$$7y = 4.85x + 13.63$$

$$y = \frac{4.85}{7}x + \frac{13.63}{7}$$

$$y = 0.693x + 1.947$$

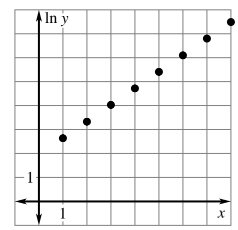
$$\ln y = 0.693x + 1.947$$

$$y = e^{0.693x + 1.947}$$

$$y = (e^{0.693})^x(e^{1.947})$$

$$y = (e^{1.947})(e^{0.693})^x$$

$$y = 7(2)^x$$



27.

x	1	2	3	4	5	6	7	8
y	10.2	30.5	43.4	61.2	89.7	120.6	210.4	302.5
x	1	2	3	4	5	6	7	8
ln y	2.3	3.4	3.8	4.1	4.5	4.8	5.3	5.7

Sample answer:

(1, 2.3), (8, 5.7)

$$m = \frac{5.7 - 2.3}{8 - 1} = \frac{3.4}{7}$$

$$m = 0.486$$

$$\frac{y - 2.3}{x - 1} = \frac{3.4}{7}$$

$$3.4(x - 1) = 7(y - 2.3)$$

$$3.4x - 3.4 = 7y - 16.1$$

$$7y = 3.4x + 12.7$$

$$y = \frac{3.4}{7}x + \frac{12.7}{7}$$

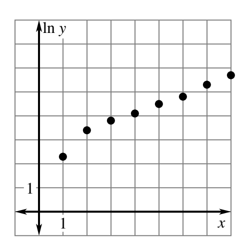
$$y = 0.486x + 1.814$$

$$\ln y = 0.486x + 1.814$$

$$y = e^{0.486x + 1.814}$$

$$y = e^{1.814}(e^{0.486})^x$$

$$y = 6.135(1.626)^x$$



28.

x	1	2	3	4	5	6	7	8
y	12.8	20.48	32.77	52.43	83.89	134.22	214.75	343.6
x	1	2	3	4	5	6	7	8
$\ln y$	2.55	3.02	3.49	3.96	4.43	4.90	5.37	5.84

(2, 2.55), (16, 5.84)

$$m = \frac{5.84 - 2.55}{16 - 2} = \frac{3.29}{14}$$

$$\frac{y - 2.55}{x - 2} = \frac{3.29}{14}$$

$$14(y - 2.55) = 3.29(x - 2)$$

$$14y - 35.7 = 3.29x - 6.58$$

$$14y = 3.29x + 29.12$$

$$y = \frac{3.29}{14}x + \frac{29.12}{14}$$

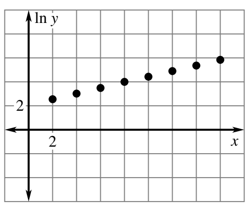
$$y = 0.235x + 2.08$$

$$\ln y = 0.235x + 2.08$$

$$y = e^{0.235x + 2.08}$$

$$y = (e^{2.08})(e^{0.235})^x$$

$$y = 8(1.265)^x$$



29. (2, 1), (6, 5)

$$1 = a \cdot 2^b$$

$$5 = a \cdot 6^b$$

$$y = ax^b$$

$$a = \frac{1}{2^b}$$

$$5 = \left(\frac{1}{2^b}\right)6^b$$

$$5 = 3^b$$

$$\log_3 5 = \log_3 3^b$$

$$\log_3 5 = b$$

$$b = \frac{\log 5}{\log 3} = 1.465$$

$$a = \frac{1}{2^{1.465}} = \frac{1}{2.761}$$

$$a = 0.362$$

$$y = 0.362x^{1.465}$$

30. (6, 8), (12, 36)

$$8 = a \cdot 6^b$$

$$36 = a \cdot 12^b$$

$$y = ax^b$$

$$a = \frac{8}{6^b}$$

$$36 = \left(\frac{8}{6^b}\right)12^b$$

$$36 = 8 \cdot 2^b$$

$$\frac{36}{8} = 2^b$$

$$\log_2 \frac{36}{8} = \log_2 2^b$$

$$\log_2 \frac{36}{8} = b$$

$$\frac{\log \frac{36}{8}}{\log 2} = b$$

$$2.17 = b$$

$$a = \frac{8}{6^b}$$

$$a = \frac{8}{6^{2.17}}$$

$$a = \frac{8}{48.81892515}$$

$$a = 0.164$$

$$y = 0.164x^{2.17}$$

31. (5, 12), (7, 25)

$$12 = a \cdot 5^b$$

$$25 = a \cdot 7^b$$

$$y = ax^b$$

$$a = \frac{12}{5^b}$$

$$25 = \left(\frac{12}{5^b}\right)7^b$$

$$25 = 12 \cdot 1.4^b$$

$$\frac{25}{12} = 1.4^b$$

$$\log_{1.4} 2.08 = \log_{1.4} 1.4^b$$

$$\log_{1.4} 2.08 = b$$

$$b = \frac{\log 2.08}{\log 1.4}$$

$$b = 2.18$$

$$a = \frac{12}{5^{2.18}} = 0.359$$

$$y = 0.359x^{2.18}$$

32. (3, 4), (6, 18)

$$4 = a \cdot 3^b$$

$$18 = a \cdot 6^b$$

$$y = ax^b$$

$$a = \frac{4}{3^b}$$

$$18 = \left(\frac{4}{3^b}\right)6^b$$

$$18 = 4 \cdot 2^b$$

$$\frac{18}{4} = 2^b$$

$$\log_2 \frac{18}{4} = \log_2 2^b$$

$$\log_2 4.5 = b$$

$$b = \frac{\log 4.5}{\log 2} = 2.17$$

$$a = \frac{4}{3^{2.17}} = 0.369$$

$$y = 0.369x^{2.17}$$

33. (2, 10), (8, 25)

$$10 = a \cdot 2^b$$

$$25 = a \cdot 8^b$$

$$y = ax^b$$

$$a = \frac{10}{2^b}$$

$$25 = \left(\frac{10}{2^b}\right)8^b$$

$$25 = 10 \cdot 4^b$$

$$\frac{25}{10} = 4^b$$

$$\log_4 2.5 = \log_4 4^b$$

$$\log_4 2.5 = b$$

$$b = \frac{\log 2.5}{\log 4} = 0.661$$

$$a = \frac{10}{2^{0.661}} = 6.324$$

$$y = 6.324x^{0.661}$$

34. (6, 11), (24, 72)

$$11 = a \cdot 6^b$$

$$72 = a \cdot 24^b$$

$$y = ax^b$$

$$a = \frac{11}{6^b}$$

$$72 = \left(\frac{11}{6^b}\right)24^b$$

$$72 = 11 \cdot 4^b$$

$$\frac{72}{11} = 4^b$$

$$\log_4 6.545 = \log_4 4^b$$

$$\log_4 6.545 = b$$

$$b = \frac{\log 6.545}{\log 4} = 1.355$$

$$a = \frac{11}{6^{1.355}} = 0.971$$

$$y = 0.971x^{1.355}$$

35. (2.2, 10.4), (8.8, 20.3)

$10.4 = a \cdot 2.2^b$

$20.3 = a \cdot 8.8^b$

$\log_4 1.95 = b$

$\dfrac{\log 1.95}{\log 4} = b$

$b = 0.482$

$a = \dfrac{10.4}{2.2^{0.482}} = 7.11$

$y = 7.11x^{0.482}$

$y = ax^b$

$a = \dfrac{10.4}{2.2^b}$

$20.3 = \left(\dfrac{10.4}{2.2^b}\right)8.8^b$

$20.3 = 10.4 \cdot 4^b$

$\dfrac{20.3}{10.4} = 4^b$

$1.95 = 4^b$

$\log_4 1.95 = \log_4 4^b$

36. (2.9, 9.4), (7.3, 12.8)

$9.4 = a \cdot 2.9^b$

$12.8 = a \cdot 7.3^b$

$b = \dfrac{\log 1.36}{\log 2.5} = 0.335$

$a = \dfrac{9.4}{2.9^{0.335}} = 6.579$

$y = 6.579x^{0.335}$

$y = ax^b$

$a = \dfrac{9.4}{2.9^b}$

$12.8 = \left(\dfrac{9.4}{2.9^b}\right)7.3^b$

$12.8 = 9.4 \cdot 2.5^b$

$\dfrac{12.8}{9.4} = (2.5)^b$

$\log_{2.5} 1.36 = \log_{2.5} 2.5^b$

$\log_{2.5} 1.36 = b$

37. (2.71, 6.42), (13.55, 29.79)

$6.42 = a \cdot 2.71^b$

$29.79 = a \cdot 13.55^b$

$b = \dfrac{\log 4.64}{\log 5} = 0.954$

$a = \dfrac{6.42}{(2.71)^{0.954}}$

$a = 2.48$

$y = ax^b$

$a = \dfrac{6.42}{2.71^b}$

$29.79 = \left[\dfrac{6.42}{(2.71)^b}\right]13.55^b$

$29.79 = 6.42 \cdot 5^b$

$\dfrac{29.79}{6.42} = 5^b$

$\log_5 4.64 = \log_5 5^b$

$\log_5 4.64 = b$

$y = 2.48x^{0.954}$

38.

x	1	2	3	4	5	6	7
y	0.78	7.37	27.41	69.63	143.47	259.0	426.79
ln x	0	0.69	1.1	1.39	1.61	1.79	1.95
ln y	−0.25	2.0	3.3	4.2	4.97	5.56	6.06

$0.78 = a \cdot 1^b$

$426.79 = a \cdot 7^b$

$426.79 = \left(\dfrac{0.78}{1^b}\right) \cdot 7^b$

$426.79 = 0.78 \cdot 7^b$

$547.167 = 7^b$

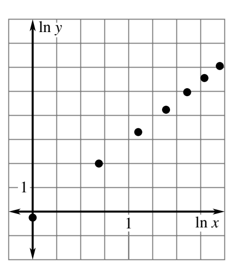
$\log_7 547.167 = \log_7 7^b$

$\dfrac{\log 547.167}{\log 7} = b$

$3.24 = b$

$a = \dfrac{0.78}{1^b} = \dfrac{0.78}{1^{3.24}} = 0.78$

$y = 0.78x^{3.24}$



39.

ln x	0	0.69	1.1	1.39	1.61	1.79	1.95
ln y	0.18	1.69	2.28	2.66	3.24	3.72	4.19

$1.2 = a \cdot 1^b$

$65.8 = a \cdot 7^b$

$65.8 = \left(\dfrac{1.2}{1^b}\right) \cdot 7^b$

$65.8 = 1.2 \cdot 7^b$

$54.83 = 7^b$

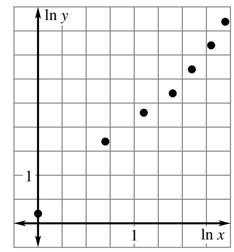
$\log_7 54.83 = \log_7 7^b$

$\dfrac{\log 54.83}{\log 7} = b$

$2.057 = b$

$a = \dfrac{1.2}{1^b} = \dfrac{1.2}{1^{2.057}} = 1.2$

$y = 1.2x^{2.057}$



40.

ln x	0.69	1.39	1.79	2.07	2.3	2.48	2.64
ln y	0.64	0.36	0.20	0.09	0	−0.07	−0.14

$1.89 = a \cdot 2^b$

$0.87 = a \cdot 14^b$

$0.87 = \left(\dfrac{1.89}{2^b}\right) \cdot 14^b$

$0.87 = 1.89 \cdot 7^b$

$0.46 = 7^b$

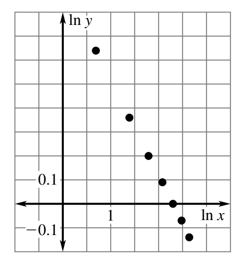
$\log_7 0.46 = \log_7 7^b$

$\dfrac{\log 0.46}{\log 7} = b$

$-0.399 = b$

$a = \dfrac{1.89}{2^b} = \dfrac{1.89}{2^{-0.399}} = 2.492$

$y = 2.492x^{-0.399}$



41. $\log y = 0.24x + 4.5$

$10^{\log y} = 10^{0.24x + 4.5}$

$y = (10^{4.5})(10^{0.24})^x$

$y = 31{,}623(1.738)^x$

42. $\log y = 0.2 \log x + 0.8$

$\log y = \log x^{0.2} + 0.8$

$10^{\log y} = 10^{\log x^{0.2} + 0.8}$

$y = 10^{0.8}(x^{0.2})$

$y = (6.310)x^{0.2}$

43. $\ln y = x + 4$

$y = e^{x+4}$

$y = e^4(e^x)$

$y = 54.598e^x$

44. $\log y = -0.12 + 0.88x$

$10^{\log y} = 10^{-0.12 + 0.88x}$

$y = 10^{-0.12}(10^{0.88})^x$

$y = (0.759)7.586^x$

45. $\log y = -0.48 \log x - 0.548$

$\log y = \log x^{-0.48} - 0.548$

$10^{\log y} = 10^{\log x - 0.48 - 0.548}$

$y = 10^{-0.548}(x^{-0.48})$

$y = 0.283x^{-0.48}$

46. $\ln y = 2.3 \ln x + 4.7$

$\ln y = \ln x^{2.3} + 4.7$

$y = e^{\ln x^{2.3} + 4.7}$

$y = e^{4.7}(x^{2.3})$

$y = 109.947x^{2.3}$

47. $\ln y = -2.38x + 0.98$

$y = e^{-2.38x + 0.98}$

$y = e^{0.98}(e^{-2.38})^x$

$y = 2.664(0.0926)^x$

48. $\log y = -1.48 + 3.751 \log x$

$\log y = -1.48 + \log x^{3.751}$

$10^{\log y} = 10^{-1.48 + \log x^{3.751}}$

$y = 10^{-1.48}(x^{3.751})$

$y = 0.0331x^{3.751}$

49. $\ln y = -1.5x + 2.5$

$y = e^{-1.5x + 2.5}$

$y = (e^{2.5})(e^{-1.5})^x$

$y = 12.182(0.223)^x$

50. $1.2 \log y = 3.4 \log x$

$12 \log y = 34 \log x$

$\log y = \frac{34}{12} \log x$

$\log y = \frac{17}{6} \log x$

$\log y = \log x^{17/6}$

$10^{\log y} = 10^{\log x^{17/6}}$

$y = x^{17/6}$

51. $\frac{1}{2} \log y = \frac{5}{6} \log x$

$\log y = \frac{5}{6}(2) \log x$

$\log y = \frac{5}{3} \log x$

$\log y = \log x^{5/3}$

$10^{\log y} = 10^{\log x^{5/3}}$

$y = x^{5/3}$

52. $2\frac{1}{8} \ln y = 4\frac{1}{4} \ln x + \frac{3}{8}$

$\ln y = \left(\frac{8}{17}\right)\left(\frac{17}{4} \ln x + \frac{3}{8}\right)$

$\ln y = 2 \ln x + \frac{3}{8}\left(\frac{8}{17}\right)$

$y = e^{\ln x^2 + 3/17}$

$y = (e^{3/17})x^2$

$y = e^{0.1765}x^2$

53. (1, 3), (2, 12)

$m = \frac{12 - 3}{2 - 1} = 9$

$\frac{y - 3}{x - 1} = \frac{9}{1}$

$y = 9(x - 1)$

$y = 9x - 9 + 3$

$y = 9x - 6$

$y = ab^x$

$3 = ab^1$

$12 = ab^2$

$a = \frac{3}{b^1}$

$12 = \frac{3}{b}(b^2)$

$12 = 3b$

$b = 4$

$a = \frac{3}{4}$

$y = \frac{3}{4}(4)^x$

$y = ax^b$

$3 = a1^b$

$12 = a2^b$

$a = \frac{3}{1^b}$

$12 = \frac{3}{1^b}(2^b)$

$12 = 3 \cdot 2^b$

$4 = 2^b$

$\log_2 4 = b$

$\frac{\log 4}{\log 2} = b$

$2 = b$

$a = \frac{3}{1^2} = 3$

$y = 3x^2$

$y = 9x - 6$

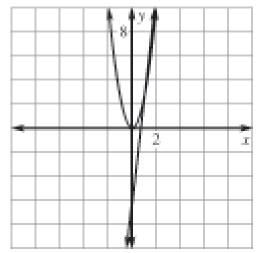
x	y
0	-6
1	3
$\frac{1}{3}$	-3
$\frac{6}{9}$	0

$y = \frac{3}{4} \cdot 4^x$

x	y
0	$\frac{3}{4}$
1	3
$\frac{1}{2}$	$\frac{3}{2}$

$y = 3x^2$

x	y
1	3
2	12
-1	3
0	0



The linear function grows the slowest, the exponential function is in the middle, and the power function grows at the fastest rate.

54. a.

x	1	2	3	4	5
ln *y*	3.091	3.664	4.248	4.836	5.425

x	6	7	8	9	10
ln *y*	6.011	6.600	7.187	7.775	8.363

$\frac{8.363 - 3.091}{10 - 1} = \frac{5.272}{9}$

$\frac{\ln y - 3.091}{x - 1} = \frac{5.272}{9}$

$9(\ln y - 3.091) = 5.272(x - 1)$

$9 \ln y - 27.819 = 5.272x - 5.272$

$9 \ln y = 5.272x + 22.547$

$\ln y = \frac{5.272}{9}x + 2.505$

$\ln y = 0.586x + 2.505$

$y = e^{0.586x + 2.505}$

$y = e^{2.505}(e^{0.586})^x$

$y = 12.244(1.797)^x$

b. $y = 12.244(1.797)^{12}$

$y = 13{,}863$

c. $y = 12.244(1.797)^{34}$

$y = 5.53$ billion

This number is unrealistically large–almost 1 hit for every person in the world.

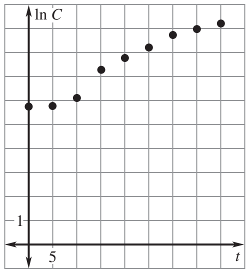
55. a.

t	0	5	10	15	20
ln C	5.680	5.700	6.082	7.361	7.756

t	25	30	35	40
ln C	8.202	8.631	8.937	9.206

—CONTINUED—

55. —CONTINUED—



yes

b. $\frac{9.206 - 5.680}{40 - 0} = \frac{3.526}{40} = 0.088$

$\frac{\ln C - 5.68}{t - 0} = \frac{3.526}{40}$

$40(\ln C - 5.68) = 3.526(t - 0)$

$40 \ln C = 3.526t + 227.2$

$\ln C = \frac{3.526t}{40} + \frac{227.2}{40}$

$\ln C = 0.088t + 5.68$

$C = e^{0.088t + 5.68}$

$C = e^{5.68}(e^{0.088})^t$

$C = 292.95(1.092)^t$

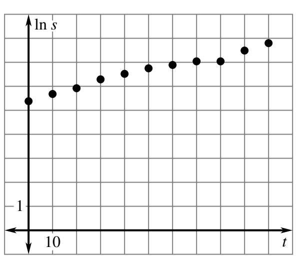
$C = 292.95(1.092)^{50}$

$C \approx 23{,}874$

56. a.

t	0	10	20	30	40	50
ln s	5.384	5.680	5.924	6.293	6.524	6.755

t	60	70	80	90	100
ln s	6.894	7.037	7.037	7.492	7.799



yes

b. $\frac{6.894 - 6.524}{60 - 40} = \frac{0.37}{20} = 0.0185$

$\frac{\ln s - 6.524}{t - 40} = \frac{0.37}{20}$

$20(\ln s - 6.524) = 0.37(t - 40)$

$20 \ln s - 130.48 = 0.37t - 14.8$

$20 \ln s = 0.37t + 115.68$

$\ln s = \frac{0.37}{20}t + \frac{115.68}{20}$

$\ln s = 0.0185t + 5.784$

$s = e^{0.0185t + 5.784}$

$s = e^{5.784}(e^{0.0185})^t$

$s = 325.057(1.019)^t$

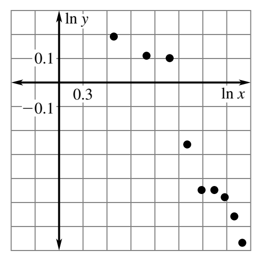
$s = 325.057(1.019)^{111}$

$s = 2626$

57. a.

x	2	3	4	5	6
y	1.21	1.12	1.11	0.77	0.64
ln *x*	0.693	1.099	1.386	1.609	1.792
ln *y*	0.191	0.113	0.104	−0.261	−0.446

x	7	8	9	10
y	0.64	0.64	0.57	0.51
ln *x*	1.946	2.079	2.197	2.303
ln *y*	−0.446	−0.478	−0.562	−0.673



b. $y = ax^b$ *Sample answer:*

$\frac{-0.673 - 0.191}{2.303 - 0.693} = \frac{-0.864}{1.61}$

$\frac{\ln y - 0.191}{\ln x - 0.693} = \frac{-0.864}{1.61}$

$1.61(\ln y - 0.191) = -0.864(\ln x - 0.693)$

$1.61 \ln y - 0.308 = -0.864 \ln x + 0.599$

$1.61 \ln y = -0.864 \ln x + 0.599 + 0.308$

$1.61 \ln y = -0.864 \ln x + 0.907$

$\ln y = \frac{-0.864}{1.61} \ln x + \frac{0.907}{1.61}$

$\ln y = -0.537 \ln x + 0.563$

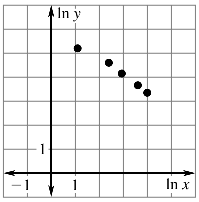
$\ln y = \ln x^{-0.537} + 0.563$

$y = x^{-0.537}(e^{0.563})$

$y = e^{0.563}x^{-0.537}$

$y = 1.756x^{-0.537}$

$y = 1.75(20)^{-0.537} \approx 351{,}000$

58. a. 

yes

b. (3, 180.5), (55, 28.5)

$180.5 = a \cdot 3^b$

$28.5 = a \cdot 55^b$

$28.5 = \left(\frac{180.5}{3^b}\right) \cdot 55^b$

$28.5 = 180.5 \cdot 18.33^b$

$0.1579 = 18.33^b$

$\log_{18.33} 0.1579 = \log_{18.33} 18.33^b$

$\frac{\log 0.1579}{\log 18.33} = b$

$-0.6346 = b$

$a = \frac{180.5}{3^b}$

$= \frac{180.5}{3^{-0.6346}}$

$= 362.46$

$y = 362.46x^{-0.6346}$

c. $y = 362.46(87)^{-0.6346}$

$y \approx 21.303°C$

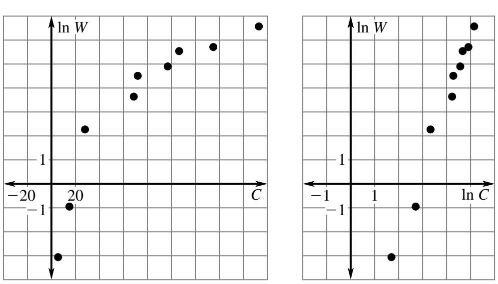
59. a.

C	5.5	15	28	68.7	72
ln W	−3.058	−0.955	2.27	3.638	4.505

C	97	106.5	135	173
ln W	4.902	5.545	5.707	6.565

C	1.704	2.708	3.332	4.230	4.277
ln W	−3.058	−0.955	2.27	3.638	4.505

C	4.575	4.668	4.905	5.153
ln W	4.902	5.545	5.707	6.565



b. *Sample answer:* a power model; the scatter plot of ln *W* vs. *C* looks curved, like the graph of a log function, while the scatter plot of ln *W* vs. ln *C* looks more like a straight line.

c. (5.5, 0.047), (173, 710)

$0.047 = a \cdot 5.5^b$

$710 = a \cdot 173^b$

$710 = \left(\frac{0.047}{5.5^b}\right) \cdot 173^b$

$710 = 0.047 \cdot \left(\frac{173}{5.5}\right)^b$

$\frac{710}{0.047} = \left(\frac{173}{5.5}\right)^b$

$15{,}106.4 = 31.5^b$

$\log_{31.5} 15{,}106.4 = \log_{31.5} 31.5^b$

$\frac{\log 15{,}106.4}{\log 31.5} = b$

$2.789 = b$

$y = ax^b$

$a = \frac{0.047}{5.5^b}$

$a = \frac{0.047}{5.5^{2.789}} = 0.000404$

$y = (0.000404)x^{2.789}$

Note: Using a graphing calculator with power regression gives $y = (0.000349)x^{2.834}$.

d.

Using the power model from part (c):	Using a regression model from a calculator:
racoon: 4.4 kg	racoon: 4.4 kg
cougar: 37.2 kg	cougar: 38.7 kg
bison: 644.4 kg	bison: 700.9 kg
hippo: 1179 kg	hippo: 1295 kg

60. If $y = ab^x$, then:

$\ln y = \ln (ab^x)$

$\ln y = \ln a + \ln b^x$

$\ln y = \ln a + x \cdot \ln b$

$\ln y = (\ln b)x + \ln a$

$m = \ln b$; y-int $= \ln a$

If $y = ax^b$, then:

$\ln y = \ln (ax^b)$

$\ln y = \ln a + \ln x^b$

$\ln y = \ln a + b \cdot \ln x$

$\ln y = b \ln x + \ln a$

$m = b$; y-int $= \ln a$

61. $f(x) = -x^3 + x^2 - x + 4$

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

62. $f(x) = x^4 - 7x^2 + 2$

$f(x) \to +\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

63. $f(x) = -x^4 + 3x - 3$

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to -\infty$ as $x \to +\infty$

64. $f(x) = 3x^5 - x^4 - x^2 + 1$

$f(x) \to -\infty$ as $x \to -\infty$

$f(x) \to +\infty$ as $x \to +\infty$

65. $f(x) = x^6 - 2x - 1$

$f(x) \to +\infty$ as $x \to -\infty$

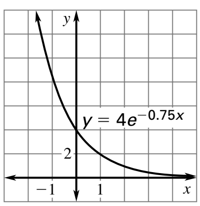
$f(x) \to +\infty$ as $x \to +\infty$

66. $f(x) = -2x^5 + 3x^4 - 2x^3 + x^2 + 5$

$f(x) \to +\infty$ as $x \to -\infty$

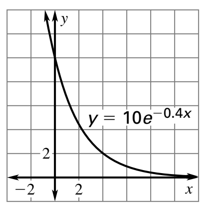
$f(x) \to -\infty$ as $x \to +\infty$

67. $y = 4e^{-0.75x}$



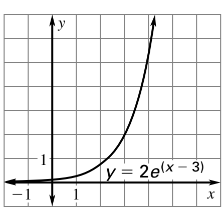
x	y
0	4
1	1.890
2	0.223

68. $y = 10e^{-0.4x}$



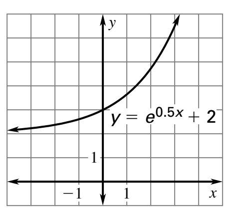
x	y
0	10
1	6.7
2	4.49

69. $y = 2e^{x-3}$



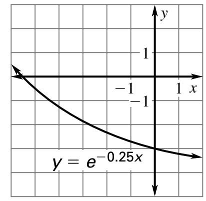
x	y
3	2
0	0.99
4	5.43
−1	0.037

70. $y = e^{0.5x} + 2$



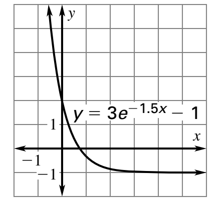
x	y
0	3
1	3.65
−1	2.61
−2	2.37

71. $y = e^{-0.25x} - 4$



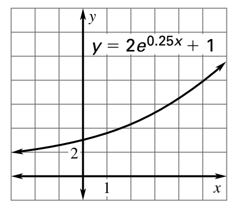
x	y
0	−3
1	−3.22
−1	−2.72
2	−3.39

72. $y = 3e^{-1.5x} - 1$



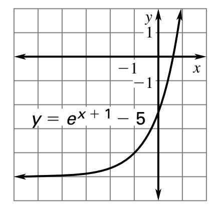
x	y
0	2
1	−0.78
−1	3.48
2	−0.95

73. $y = 2e^{0.25x} + 1$



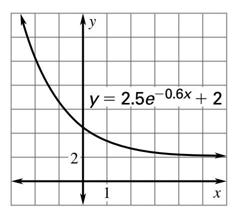
x	y
0	3
−1	2.56
−2	2.21
−4	1.74

74. $y = e^{x+1} - 5$



x	y
0	−2.3
−1	−4
−2	−4.6

75. $y = 2.5e^{-0.6x} + 2$



x	y
0	4.5
−2	6.3
2	2.76

76. $5 \log 2 - \log 8 = \log 2^5 - \log 8$

$= \log 32 - \log 8$

$= \log \frac{32}{8} = \log 4$

77. $2 \log 9 - \log 3 = \log 9^2 - \log 3 = \log \frac{81}{3} = \log 27$

78. $\ln x + 5 \ln 3 = \ln x + \ln 3^5 = \ln x(3^5) = \ln 243x$

79. $2 \ln x - \ln 4 = \ln x^2 - \ln 4 = \ln \frac{x^2}{4}$

80. $\log_2 8 + 3\log_2 3 - \log_2 6$

$= \log_2 8 + \log_2 3^3 - \log_2 6$

$= \log_2 8 + \log_2 27 - \log_2 6$

$= \log_2 (8 \cdot 27) - \log_2 6$

$= \log_2 \dfrac{216}{6}$

$= \log_2 36$

81. $\log_7 12 + 3\log_7 4 + \log_7 5$

$= \log_7 12 + \log_7 4^3 + \log_7 5$

$= \log_7 (12 \cdot 64) + \log_7 5$

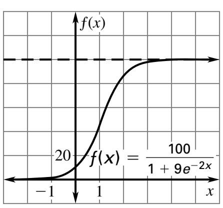
$= \log_7 (12 \cdot 64) \cdot 5$

$= \log_7 3840$

Lesson 8.8

Developing Concepts Activity 8.8 (p. 517)

1.



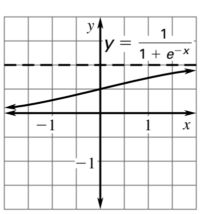
2. The graph has two horizontal asymptotes, the x-axis and the line $y = c$. The graph is continuously increasing, symmetric about the point where it crosses the line $y = \frac{c}{2}$. The graph grows steeply in the center, and is flat at each end.

8.8 Guided Practice (p. 520)

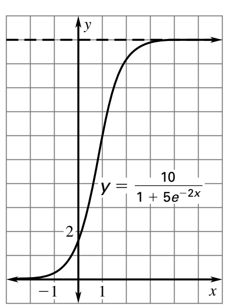
1. a logistic growth function

2. An exponential function increases without bound, while a logistic growth function approaches a finite limiting value.

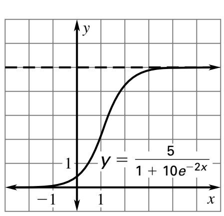
a.



b.



c.



3. This is the point of maximum growth for the function.

4. $f(x) = \dfrac{12}{1 + 5e^{-2x}}$

$f(0) = \dfrac{12}{1 + 5e^{-2(0)}}$

$f(0) = \dfrac{12}{6} = 2$

5. $f(-2) = \dfrac{12}{1 + 5e^{-2(2)}}$

$= \dfrac{12}{1 + 5e^{4}}$

$f(-2) = \dfrac{12}{273.99} \approx 0.0438$

6. $f(5) = \dfrac{12}{1 + 5e^{-2(5)}}$

$f(5) = \dfrac{12}{1 + 5e^{-10}}$

$f(5) = \dfrac{12}{1.000227} \approx 12.00$

7. $f\left(-\dfrac{1}{2}\right) = \dfrac{12}{1 + 5e^{-2(-1/2)}}$

$f\left(-\dfrac{1}{2}\right) = \dfrac{12}{1 + 5e}$

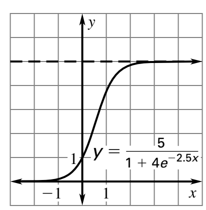
$f\left(-\dfrac{1}{2}\right) = \dfrac{12}{14.591} \approx 0.822$

8. $f(10) = \dfrac{12}{1 + 5e^{-2(10)}}$

$f(10) = \dfrac{12}{1 + 5e^{-20}}$

$f(10) = \dfrac{12}{1.00000001} \approx 11.99$ almost 12

9. $f(x) = \dfrac{5}{1 + 4e^{-2.5x}}$



x	y
0	1
0.555	2.5
1	3.76
−1	0.1

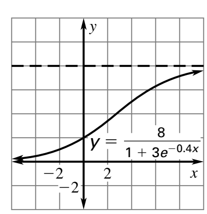
asymptotes: x-axis, $y = 5$

y-intercept $= 1$

point of maximum growth $= \left(\dfrac{\ln a}{r}, \dfrac{c}{2}\right) = \left(\dfrac{\ln 4}{2.5}, \dfrac{5}{2}\right)$

$= (0.555, 2.5)$

10. $f(x) = \dfrac{8}{1 + 3e^{-0.4x}}$



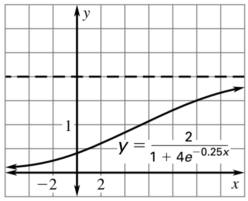
x	y
0	2
2.747	2.5
−4	0.5

asymptotes: x-axis, $y = 8$

y-intercept $= 2$

point of maximum growth $= \left(\dfrac{\ln 3}{0.4}, \dfrac{8}{2}\right) = (2.747, 4)$

11. $f(x) = \dfrac{2}{1 + 4e^{-0.25x}}$



x	y
0	$\frac{2}{5}$
5.545	1
−1	0.33

asymptotes: x-axis, $y = 2$

y-intercept $= \dfrac{2}{5}$

point of maximum growth $= \left(\dfrac{\ln 4}{0.25}, \dfrac{2}{2}\right) = (5.45, 1)$

12. $\dfrac{18}{1 + 2e^{-2x}} = 10$

$$18 = (1 + 2e^{-2x})(10)$$
$$18 = 10 + 20e^{-2x}$$
$$8 = 20e^{-2x}$$
$$0.4 = e^{-2x}$$
$$\ln 0.4 = -2x$$
$$-\tfrac{1}{2}\ln 0.4 = x$$
$$0.458 \approx x$$

13. $\dfrac{30}{1 + 4e^{-x}} = 10$

$$30 = (1 + 4e^{-x})(10)$$
$$30 = 10 + 40e^{-x}$$
$$20 = 40e^{-x}$$
$$0.5 = e^{-x}$$
$$\ln 0.5 = -x$$
$$-(\ln 0.5) = x$$
$$0.693 \approx x$$

14. $\dfrac{12.5}{1 + 7e^{-0.2x}} = 9$

$$12.5 = (1 + 7e^{-0.2x})(9)$$
$$12.5 = 9 + 63e^{-0.2x}$$
$$3.5 = 63e^{-0.2x}$$
$$0.056 = e^{-0.2x}$$
$$\ln 0.056 = -0.2x$$
$$\ln 0.056 = -\frac{1}{5}x$$
$$-5(\ln 0.056) = x$$
$$14.452 \approx x$$

15. $h = \dfrac{117}{1 + 18e^{-0.73t}}$

8.8 Practice and Applications (pp. 520–522)

16. $f(1) = \dfrac{7}{1 + 3e^{-1}}$

$f(1) = \dfrac{7}{2.103638324}$

$f(1) \approx 3.328$

17. $f(3) = \dfrac{7}{1 + 3e^{-3}}$

$f(3) = \dfrac{7}{1.149361205}$

$f(3) \approx 6.090$

18. $f(-1) = \dfrac{7}{1 + 3e^{-(-1)}}$

$f(-1) = \dfrac{7}{9.154845485}$

$f(-1) = 0.765$

19. $f(-6) = \dfrac{7}{1 + 3e^{-(-6)}}$

$f(-6) = \dfrac{7}{1211.28638}$

$f(-6) = 0.00578$

20. $f(0) = \dfrac{7}{1 + 3e^{-0}}$

$f(0) = \dfrac{7}{1 + 3(1)}$

$f(0) = \dfrac{7}{4}$

21. $f\left(\dfrac{3}{4}\right) = \dfrac{7}{1 + 3e^{-(3/4)}}$

$f\left(\dfrac{3}{4}\right) = \dfrac{7}{2.417099658}$

$f\left(\dfrac{3}{4}\right) \approx 2.896$

22. $f(2.2) = \dfrac{7}{1 + 3e^{-(2.2)}}$

$f(2.2) = \dfrac{7}{1.332409475}$

$f(2.2) \approx 5.254$

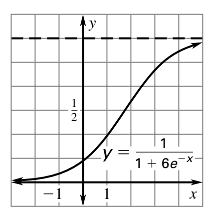
23. $f(-0.9) = \dfrac{7}{1 + 3e^{-(-0.9)}}$

$f(-0.9) = \dfrac{7}{8.378809333}$

$f(-0.9) \approx 0.835$

24. C **25.** A **26.** B

27. $y = \dfrac{1}{1 + 6e^{-x}}$



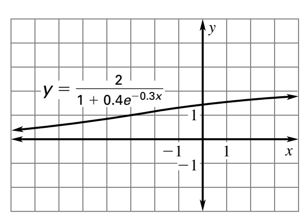
x	y
0	$\frac{1}{7}$
1.79	$\frac{1}{2}$
−1	0.06

asymptotes: x-axis, $y = 1$

y-intercept $= \dfrac{1}{7}$

point of maxumum growth $= \left(\dfrac{\ln 6}{1}, \dfrac{1}{2}\right) = \left(1.79, \dfrac{1}{2}\right)$

28. $y = \dfrac{2}{1 + 0.4e^{-0.3x}}$



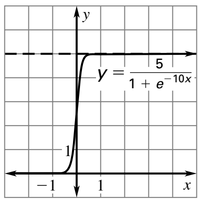
x	y
0	1.429
−3.054	1
−6	0.585

asymptotes: x-axis, $y = 2$

y-intercept $= 1.429$

point of maxumum growth $= \left(\dfrac{\ln 0.4}{0.3}, \dfrac{2}{2}\right) = (-3.054, 1)$

29. $y = \dfrac{5}{1 + e^{-10x}}$



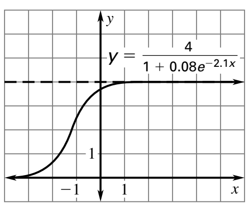
x	y
0	$\frac{5}{2}$
1	4.99
−1	0.0002

asymptotes: x-axis, $y = 5$

y-intercept $= \dfrac{5}{2}$

point of maxumum growth $= \left(\dfrac{\ln 1}{10}, \dfrac{5}{2}\right) = \left(0, \dfrac{5}{2}\right)$

30. $y = \dfrac{4}{1 + 0.08e^{-2.1x}}$



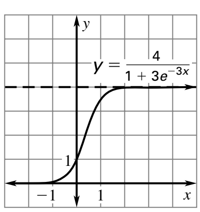
x	y
0	3.704
1	3.961
−1	2.420
−3	0.09
−1.203	2

asymptotes: x-axis, $y = 4$

y-intercept $= 3.704$

point of maxumum growth $= \left(\dfrac{\ln 0.08}{2.1}, \dfrac{4}{2}\right) = (-1.203, 2)$

31. $y = \dfrac{4}{1 + 3e^{-3x}}$



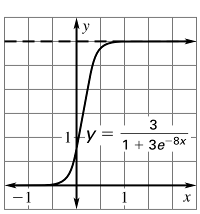
x	y
0	1
0.366	2
−2	0.003
2	3.972

asymptotes: x-axis, $y = 4$

y-intercept $= 1$

point of maxumum growth $= \left(\dfrac{\ln 3}{3}, \dfrac{4}{2}\right) = (0.366, 2)$

32. $y = \dfrac{3}{1 + 3e^{-8x}}$



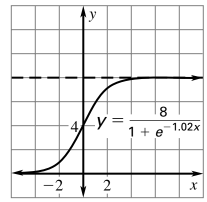
x	y
0	$\frac{3}{4}$
0.137	$\frac{3}{2}$
1	2.99
−1	0.0003

asymptotes: x-axis, $y = 3$

y-intercept $= \dfrac{3}{4}$

point of maxumum growth $= \left(\dfrac{\ln 3}{8}, \dfrac{3}{2}\right) = \left(0.137, \dfrac{3}{2}\right)$

33. $y = \dfrac{8}{1 + e^{-1.02x}}$



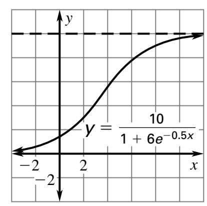
x	y
0	4
1	5.878
−4	0.133

asymptotes: x-axis, $y = 8$

y-intercept $= 4$

point of maxumum growth $= \left(\dfrac{\ln 1}{1.02}, \dfrac{8}{2}\right) = (0, 4)$

34. $y = \dfrac{10}{1 + 6e^{-0.5x}}$



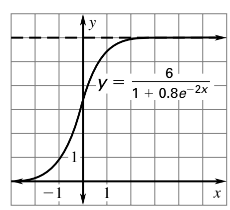
x	y
0	$\frac{10}{7}$
3.584	5
−2	0.577

asymptotes: x-axis, $y = 10$

y-intercept $= \dfrac{10}{7}$

point of maxumum growth $= \left(\dfrac{\ln 6}{0.5}, \dfrac{10}{2}\right) = (3.584, 5)$

35. $y = \dfrac{6}{1 + 0.8e^{-2x}}$



x	y
0	3.333
−0.112	3
−0.5	1.89

asymptotes: x-axis, $y = 6$

y-intercept $= 3.333$

point of maxumum growth $= \left(\dfrac{\ln 0.8}{2}, \dfrac{6}{2}\right) = (-0.112, 3)$

36. $\dfrac{8}{1 + 3e^{-x}} = 5$

$$8 = (1 + 3e^{-x})5$$
$$8 = 5 + 15e^{-x}$$
$$3 = 15e^{-x}$$
$$0.2 = e^{-x}$$
$$\ln 0.2 = -x$$
$$-1(\ln 0.2) = x$$
$$1.609 \approx x$$
$$x \approx \ln 5$$

37. $\dfrac{10}{1 + 2e^{-4x}} = 9$

$$10 = (1 + 2e^{-4x})9$$
$$10 = 9 + 18e^{-4x}$$
$$1 = 18e^{-4x}$$
$$\frac{1}{18} = e^{-4x}$$
$$\ln \frac{1}{18} = -4x$$
$$-\frac{1}{4}\left(\ln \frac{1}{18}\right) = x$$
$$0.723 \approx x$$

38. $\dfrac{3}{1 + 18e^{-x}} = 1$

$$3 = (1 + 18e^{-x})1$$
$$3 = 1 + 18e^{-x}$$
$$2 = 18e^{-x}$$
$$\frac{1}{9} = e^{-x}$$
$$\ln \frac{1}{9} = -x$$
$$-1\left(\ln \frac{1}{9}\right) = x$$
$$2.197 \approx x$$
$$x = \ln 9$$

39. $\dfrac{28}{1 + 13e^{-2x}} = 20$

$$28 = (1 + 13e^{-2x})20$$
$$28 = 20 + 260e^{-2x}$$
$$8 = 260e^{-2x}$$
$$\frac{8}{260} = e^{-2x}$$
$$\ln \frac{8}{260} = -2x$$
$$-\frac{1}{2}\left(\ln \frac{8}{260}\right) = x$$
$$1.741 \approx x$$

40. $\dfrac{82}{1 + 50e^{-x}} = 68$

$$82 = (1 + 50e^{-x})68$$
$$82 = 68 + 3400e^{-x}$$
$$14 = 3400e^{-x}$$
$$\frac{14}{3400} = e^{-x}$$
$$\ln \frac{14}{3400} = -x$$
$$-1\left(\ln \frac{14}{3400}\right) = x$$
$$5.492 \approx x$$

41. $\dfrac{36}{1 + 7e^{-10x}} = 30$

$$36 = (1 + 7e^{-10x})30$$
$$36 = 30 + 210e^{-10x}$$
$$6 = 210e^{-10x}$$
$$\frac{6}{210} = e^{-10x}$$
$$\ln \frac{6}{210} = -10x$$
$$-\frac{1}{10}\left(\ln \frac{6}{210}\right) = x$$
$$0.356 \approx x$$

42. $\dfrac{41}{1 + 14.9e^{-6x}} = 7$

$$41 = (1 + 14.9e^{-6x})7$$
$$41 = 7 + 104.3e^{-6x}$$
$$34 = 104.3e^{-6x}$$
$$\frac{34}{104.3} = e^{-6x}$$
$$\ln\left(\frac{34}{104.3}\right) = -6x$$
$$-\frac{1}{6}\left(\ln \frac{34}{104.3}\right) = x$$
$$0.187 \approx x$$

43. $\dfrac{9}{1 + 5e^{-0.2x}} = \dfrac{3}{4}$

$$9 = (1 + 5e^{-0.2x})\frac{3}{4}$$

$$9 = \frac{3}{4} + \frac{15}{4}e^{-0.2x}$$

$$\frac{33}{4} = \frac{15}{4}e^{-0.2x}$$

$$\left(\frac{4}{15}\right)\frac{33}{4} = e^{-0.2x}$$

$$\frac{33}{15} = e^{-0.2x}$$

$$\ln\frac{33}{15} = -0.2x$$

$$-5\left(\ln\frac{33}{15}\right) = x$$

$$x \approx -3.942$$

44. $\dfrac{40}{1 + 2.5e^{-0.4x}} = 6.4$

$$40 = (1 + 2.5e^{-0.4x})6.4$$

$$40 = 6.4 + 16e^{-0.4x}$$

$$33.6 = 16e^{-0.4x}$$

$$\frac{33.6}{16} = e^{-0.4x}$$

$$\ln\frac{33.6}{16} = -0.4x$$

$$-\frac{10}{4}\left(\ln\frac{33.16}{16}\right) = x$$

$$x \approx -1.855$$

45. $86 = \dfrac{91.86}{1 + 22.96e^{-0.4t}}$

$$91.86 = (1 + 22.96e^{-0.4t})86$$

$$91.86 = 86 + 1974.56e^{-0.4t}$$

$$5.86 = 1974.56e^{-0.4t}$$

$$\frac{5.86}{1974.56} = e^{-0.4t}$$

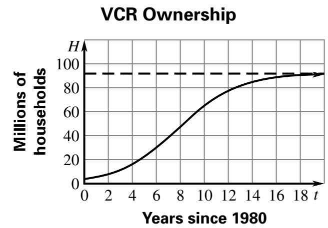
$$\ln\frac{5.86}{1974.56} = -0.4t$$

$$-\frac{10}{4}\left(\ln\frac{5.86}{1974.56}\right) = t$$

$$t \approx 14.550$$

46.

t	H
0	3.83
7	38.3
10	64.64
14.55	86



47. to approach 91.86 million households

48. $5000 = \dfrac{9200}{1 + 8.03e^{-0.121t}}$

$$9200 = (1 + 8.03e^{-0.121t})5000$$

$$9200 = 5000 + 40{,}150e^{-0.121t}$$

$$4200 = 40{,}150e^{-0.121t}$$

$$\frac{4200}{40{,}150} = e^{-0.121t}$$

$$\ln\frac{4200}{40{,}150} = -0.121t$$

$$-\frac{1}{0.121}\left(\ln\frac{4200}{40{,}150}\right) = t$$

$$t \approx 18.657$$

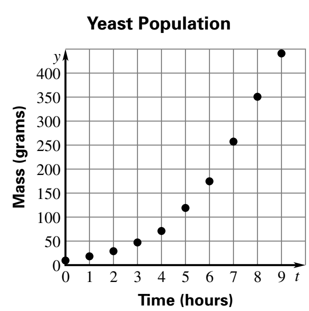
49.

t	H
0	1019
18.657	5000
17	4539

$$\text{point of maximum growth} = \left(\frac{\ln 8.03}{0.121}, \frac{9200}{2}\right)$$

$$= (17.216, 4600)$$

50.



51. $y = \dfrac{721}{1 + 72e^{-0.526t}}$

52. exponential model: $P = 5.36(1.03)^t$

a. (0, 5.3), (10, 7.2)

$5.3 = ab^0$

$7.2 = ab^{10}$

$a = \frac{5.3}{b^0} = 5.3$

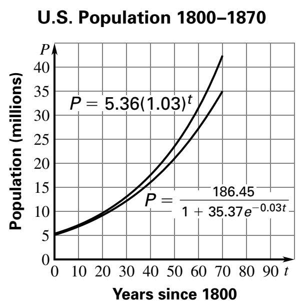
$7.2 = 5.3b^{10}$

$\frac{7.2}{5.3} = b^{10}$

$b = 1.03$

$a = 5.3$

$P = 5.3(1.03)^t$



t	P
0	5.12
118.862	93.225

logistic growth model

$P = \frac{186.45}{1 + 35.37e^{-0.03t}}$

$$\text{point of maximum growth} = \left(\frac{\ln 35.37}{0.03}, \frac{186.45}{2}\right) = (118.862, 93.225)$$

b.

$$P = 5.36(1.03)^t$$
$$92 = 5.36(1.03)^t$$
$$\frac{92}{5.36} = 1.03^t$$
$$17.164 = 1.03^t$$
$$\log_{1.03} 17.164 = t$$
$$\frac{\log 17.164}{\log 1.03} = t$$
$$t \approx 96.175 \text{ years}$$

$$P = \frac{186.45}{1 + 35.37e^{-0.03t}}$$
$$92 = \frac{186.45}{1 + 35.37e^{-0.03t}}$$
$$186.45 = (1 + 35.37e^{-0.03t})92$$
$$186.45 = 92 + 3254.04e^{-0.03t}$$
$$94.45 = 3254.04e^{-0.03t}$$
$$\frac{94.45}{3254.04} = e^{-0.03t}$$
$$\ln \frac{94.45}{3254.04} = -0.03t$$
$$\left(-\frac{1}{0.03}\right)\left(\ln \frac{94.45}{3254.04}\right) = t$$
$$t \approx 117.986 \text{ years}$$

Exponential model gives 1896; logistic model gives 1918 which is closer. The data more closely follows a logistic pattern.

c. $P = 5.36(1.03)^{210}$

$P = 2660.615$

$P \approx 2.7$ billion

$P = \frac{816.45}{1 + 35.37e^{-0.03(210)}}$

$P = \frac{186.45}{1.0649501}$

$P \approx 175.1$ million

Exponential model gives a value of 2.7 billion, which is far too large. Logistic model gives a closer value of 175.1 million.

53. $y = \frac{c}{1 + ae^{-rx}} = \frac{c}{2}$, then $1 + ae^{-rx} = 2$, so $ae^{-rx} = 1$,

$e^{rx} = a$, $rx = \ln a$, $x = \frac{\ln a}{r}$.

8.8 Mixed Review (p. 522)

54. $x = 4, y = 36$

$y = ax$

$36 = a \cdot 4$

$a = 9$

$y = 9x$

55. $x = -5, y = 10$

$y = ax$

$10 = a(-5)$

$a = -2$

$y = -2x$

56. $x = 2, y = 13$

$y = ax$

$13 = a \cdot 2$

$a = \frac{13}{2}$

$y = \frac{13}{2}x$

57. $x = 40, y = 5$

$y = ax$

$5 = a \cdot 40$

$a = \frac{1}{8}$

$y = \frac{1}{8}x$

58. $x = 0.1, y = 0.9$

$y = ax$

$0.9 = a(0.1)$

$a = 9$

$y = 9x$

59. $x = 1, y = 0.2$

$y = ax$

$0.2 = a \cdot 1$

$a = 0.2$

$y = 0.2x$

60. $\log y = 0.9 \log x + 2.11$

$\log y = \log x^{0.9} + 2.11$

$10^{\log y} = 10^{\log x^{0.9} + 2.11}$

$y = (10^{2.11})(x^{0.9})$

$y \approx 128.8x^{0.9}$

61. $\ln y = 0.94 - 2.44x$

$y = e^{0.94 - 2.44x}$

$y = e^{0.94}(e^{-2.44})^x$

$y = 2.560(0.0872)^x$

62. $\log y = -1.82 + 0.4x$

$10^{\log y} = 10^{-1.82 + 0.4x}$

$y = (10^{-1.82})(10^{0.4})^x$

$y = 0.0151(2.512)^x$

63. $\log y = -0.75 \log x - 1.76$

$\log y = \log x^{-1.75} - 1.76$

$10^{\log y} = 10^{\log x^{-0.75} - 1.76}$

$y = (10^{-1.76})x^{-0.75}$

$y = 0.0174x^{-0.75}$

Quiz 3 (p. 522)

1. (2, 3), (5, 12)

$y = ab^x$

$3 = ab^2$

$12 = ab^5$

$a = \frac{3}{b^2}$

$12 = \left(\frac{3}{b^2}\right)b^5$

$12 = 3b^3$

$4 = b^3$

$b = 1.587$

$a = \frac{3}{1.587^2} = 1.191$

$y = 1.191(1.587)^x$

2. (1, 16), (3, 45)

$16 = ab^1$

$45 = ab^3$

$a = \frac{16}{b^1}$

$45 = \left(\frac{16}{b^1}\right)b^3$

$45 = 16b^2$

$\frac{45}{16} = b^2$

$b = 1.677$

$a = \frac{16}{1.677^1} = 9.541$

$y = 9.541(1.677)^x$

3. (5, 9), (8, 35)

$9 = ab^5$

$35 = ab^8$

$a = \frac{9}{b^5}$

$35 = \left(\frac{9}{b^5}\right)b^8$

$35 = 9b^3$

$\frac{35}{9} = b^3$

$b = 1.573$

$a = \frac{9}{(1.573)^5} = \frac{9}{9.630}$

$= 0.936$

$y = 0.936(1.573)^x$

4. (2, 28), (8, 192)

$y = ax^b$

$28 = a \cdot 2^b$

$192 = a \cdot 8^b$

$a = \frac{28}{2^b}$

$192 = \frac{28}{2^b}(8)^b$

$192 = 28 \cdot 4^b$

$6.857 = 4^b$

$\log_4 6.857 = b$

$\frac{\log 6.857}{\log 4} = b$

$b = 1.389$

$a = \frac{28}{(2)^{1.389}} = \frac{28}{2.619}$

$= 10.693$

$y = 10.693x^{1.389}$

5. (1, 0.5), (6, 48)

$0.5 = a \cdot 1^b$

$48 = a \cdot 6^b$

$a = \frac{0.5}{1^b}$

$48 = \left(\frac{0.5}{1^b}\right)6^b$

$48 = 0.5 \cdot 6^b$

$96 = 6^b$

$\log_6 96 = b$

$\frac{\log 96}{\log 6} = b$

$b = 2.547$

$a = \frac{0.5}{(1)^{2.547}}$

$a = 0.5$

$y = \frac{1}{2}x^{2.547}$

6. (5, 40), (2, 6)

$40 = a \cdot 5^b$

$6 = a \cdot 2^b$

$a = \frac{40}{5^b}$

$6 = \left(\frac{40}{5^b}\right)2^b$

$6 = 40(0.4)^b$

$0.15 = (0.4)^b$

$\log_{0.4} 0.15 = b$

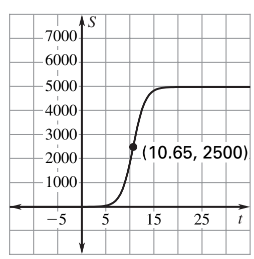
$b = \frac{\log 0.15}{\log 0.4}$

$b = 2.070$

$a = \frac{40}{(5)^{2.070}} = \frac{40}{27.981} = 1.429$

$y = 1.429x^{2.070}$

7. $S = \frac{5000}{1 + 4999e^{-0.8t}}$



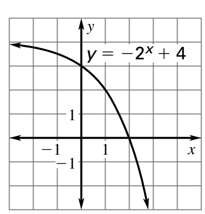
t	*S*
0	1
10.65	2500
−10	0.0003

point of maximum growth $= \left(\frac{\ln 4999}{0.8}, \frac{5000}{2}\right)$

$= (10.65, 2500)$

Chapter 8 Review *(pp. 524–526)*

1. $y = -2^x + 4$

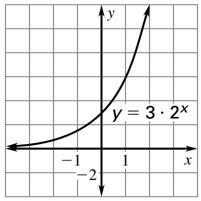


x	*y*
0	3
1	2
2	0

Domain: all real numbers

Range: $y < 4$

2. $y = 3 \cdot 2^x$

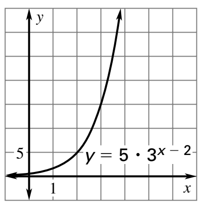


x	y
0	3
-2	$\frac{3}{4}$
1	6

Domain: all real numbers

Range: $y > 0$

3. $y = 5 \cdot 3^{x-2}$

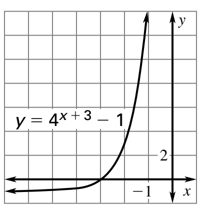


x	y
0	$\frac{5}{9}$
1	$\frac{5}{3}$
2	5
-1	$\frac{5}{27}$

Domain: all real numbers

Range: $y > 0$

4. $y = 4^{x+3} - 1$



x	y
0	63
-3	0
-2	3
-4	$-\frac{3}{4}$

Domain: all real numbers

Range: $y > -1$

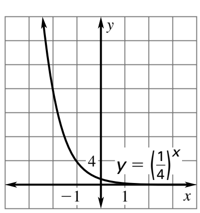
5. $f(x) = 5\left(\frac{3}{4}\right)^x$ exponential decay

6. $f(x) = 2\left(\frac{5}{4}\right)^x$ exponential growth

7. $f(x) = 3(6)^{-x}$ exponential decay

8. $f(x) = 4(3)^x$ exponential growth

9. $y = \left(\frac{1}{4}\right)^x$

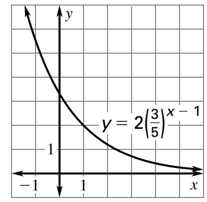


x	y
0	1
-1	4
1	$\frac{1}{4}$

Domain: all real numbers

Range: $y > 0$

10. $y = 2\left(\frac{3}{5}\right)^{x-1}$

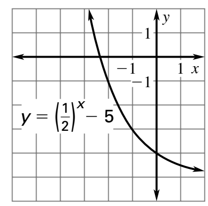


x	y
0	$\frac{10}{3}$
1	2
2	$\frac{6}{5}$
3	$\frac{18}{25}$

Domain: all real numbers

Range: $y > 0$

11. $y = \left(\frac{1}{2}\right)^x - 5$

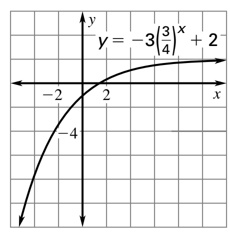


x	y
0	-4
1	-4.5
-1	-3
-2	-1
-3	3

Domain: all real numbers

Range: $y > -5$

12. $y = -3\left(\frac{3}{4}\right)^x + 2$

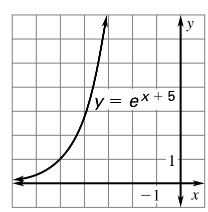


x	y
0	-1
-1	-2
1	$-\frac{1}{4}$
2	$\frac{5}{16}$
4	$\frac{269}{256}$

Domain: all real numbers

Range: $y < 2$

13. $y = e^{x+5}$

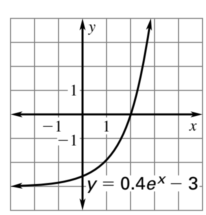


x	y
0	148.413
-5	1
-6	0.368

Domain: all real numbers

Range: $y > 0$

14. $y = 0.4e^x - 3$

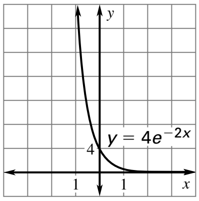


x	y
0	-2.6
-1	-2.853
3	5.034
2	-0.044

Domain: all real numbers

Range: $y > -3$

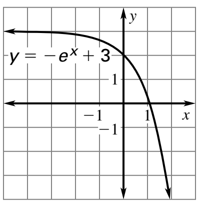
15. $y = 4e^{-2x}$



x	y
0	4
$\frac{1}{2}$	1.47
$-\frac{1}{2}$	10.873

Domain: all real numbers
Range: $y > 0$

16. $y = -e^x + 3$



x	y
0	2
1	0.282
−1	2.632
−2	2.865

Domain: all real numbers
Range: $y < 3$

17. $\log_4 64 = 4^x = 64$, so $x = 3$

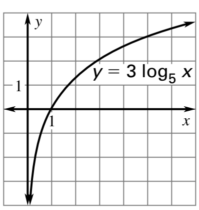
18. $\log_2 \frac{1}{8} = 2^x = \frac{1}{8}$, so $x = -3$

19. $\log_3 \frac{1}{9} = 3^x = \frac{1}{9}$, so $x = -2$

20. $\log_6 1 = 6^x = 1$, so $x = 0$

21. $y = 3 \log_5 x$

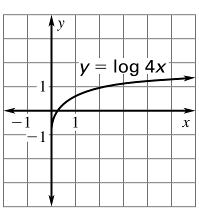
$y = \log_5 x^3$



x	y
1	0
5	3
$\frac{1}{5}$	−3

Domain: $x > 0$
Range: all real numbers

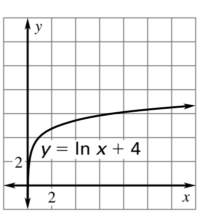
22. $y = \log 4x$



x	y
25	2
$\frac{1}{400}$	−2

Domain: $x > 0$
Range: all real numbers

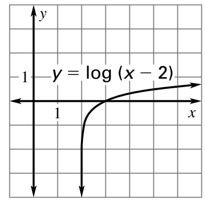
23. $y = \ln x + 4$



x	y
1	4
5	5.609
0.5	3.307
0.1	1.697
0.025	0.311

Domain: $x > 0$
Range: all real numbers

24. $y = \log(x - 2)$



x	y
3	0
4	0.301
12	1

Domain: $x > 2$
Range: all real numbers

25. $\log_3 6xy = \log_3 6 + \log_3 x + \log_3 y$

26. $\ln \frac{7x}{3} = \ln 7x - \ln 3 = \ln 7 + \ln x - \ln 3$

27. $\log 5x^3 = \log 5 + \log x^3 = \log 5 + 3 \log x$

28. $\log \frac{x^5 y^{-2}}{2y} = \log \frac{x^5}{2y^3} = \log x^5 - \log 2y^3$

$= 5 \log x - (\log 2 + \log y^3)$

$= 5 \log x - \log 2 - \log y^3$

$= 5 \log x - \log 2 - 3 \log y$

29. $2 \ln 3 - \ln 5 = \ln(3)^2 - \ln 5 = \ln 9 - \ln 5 = \ln \frac{9}{5}$

30. $\log_4 3 + 3 \log_4 2 = \log_4 3 + \log_4 (2)^3$

$= \log_4 3 + \log_4 8$

$= \log_4 3 \cdot 8$

$= \log_4 24$

31. $0.5 \log 4 + 2(\log 6 - \log 2) = \log(4)^{1/2} + 2\left(\log \frac{6}{2}\right)$

$= \log 2 + 2(\log 3)$

$= \log 2 + \log(3)^2$

$= \log 2 + \log 9$

$= \log 2 \cdot 9$

$= \log 18$

32. $2(3)^{2x} = 5$

$(3)^{2x} = 5\left(\frac{1}{2}\right)$

$\log_3 3^{2x} = \log_3 2.5$

$2x = \log_3 2.5$

$x = \frac{1}{2}(\log_3 2.5)$

$x = \frac{1}{2}\left(\frac{\log 2.5}{\log 3}\right)$

$x = 0.417$

33. $3e^{-x} - 4 = 9$

$3e^{-x} = 13$

$e^{-x} = \frac{13}{3}$

$\ln e^{-x} = \ln \frac{13}{3}$

$-x = \ln \frac{13}{3}$

$x = -\left(\ln \frac{13}{3}\right)$

$x = -1.466$

34. $3 + \ln x = 8$

$\ln x = 5$

$e^5 = x$

$x = 148.41$

35. $5 \log(x - 2) = 11$

$\log(x - 2) = \frac{11}{5}$

$10^{11/5} = x - 2$

$10^{2.2} + 2 = x$

$158.49 + 2 = x$

$x = 160.49$

36. (2, 6), (3, 8)

$y = ab^x$

$6 = ab^2$

$8 = ab^3$

$a = \frac{6}{b^2}$

$8 = \frac{6}{b^2}(b^3)$

$8 = 6b$

$\frac{4}{3} = b$

$a = \frac{6}{\left(\frac{4}{3}\right)^2}$

$a = \frac{6}{16/9}$

$a = \frac{6}{1} \cdot \frac{9}{16} = \frac{54}{16} = \frac{27}{8}$

$y = \frac{27}{8}\left(\frac{4}{3}\right)^x$

37. (2, 8.9), (4, 20)

$8.9 = ab^2$

$20 = ab^4$

$a = \frac{8.9}{b^2}$

$20 = \frac{8.9}{b^2}(b^4)$

$20 = 8.9b^2$

$\frac{20}{8.9} = b^2$

$1.499 = b$

$a = \frac{8.9}{(1.499)^2} = 3.9605$

$y = 3.9605(1.499)^x$

38. (2, 4.2), (4, 3.6)

$4.2 = ab^2$

$3.6 = ab^4$

$a = \frac{4.2}{b^2}$

$3.6 = \frac{4.2}{b^2}(b^4)$

$3.6 = 4.2b^2$

$\frac{3.6}{4.2} = b^2$

$b^2 = 0.857$

$b = 0.926$

$a = \frac{4.2}{b^2} = \frac{4.2}{0.857}$

$= 4.900$

$y = 4.9(0.926)^x$

39. (2, 3.4), (6, 7.3)

$y = ax^b$

$3.4 = a \cdot 2^b$

$7.3 = a \cdot 6^b$

$a = \frac{3.4}{2^b}$

$7.3 = \left(\frac{3.4}{2^b}\right)6^b$

$7.3 = 3.4 \cdot 3^b$

$\frac{7.3}{3.4} = 3^b$

$\log_3 \frac{7.3}{3.4} = b$

$\log_3 2.147 = b$

$\frac{\log 2.147}{\log 3} = b$

$b = 0.696$

$a = \frac{3.4}{(2)^{0.696}} = 2.099$

$y = 2.099x^{0.696}$

40. (2, 12.5), (4, 33.2)

$12.5 = a \cdot 2^b$

$33.2 = a \cdot 4^b$

$a = \frac{12.5}{2^b}$

$33.2 = \frac{12.5}{2^b}(4^b)$

$33.2 = 12.5 \cdot 2^b$

$\frac{33.2}{12.5} = 2^b$

$\log_2 2.656 = b$

$\frac{\log 2.656}{\log 2} = b$

$b = 1.409$

$a = \frac{12.5}{(2)^{1.409}} = 4.706$

$y = 4.706x^{1.409}$

41. (0.5, 1), (10, 150)

$1 = a \cdot (0.5)^b$

$150 = a \cdot 10^b$

$a = \frac{1}{(0.5)^b}$

$150 = \frac{1}{(0.5)^b}10^b$

$150 = 1 \cdot 20^b$

$150 = 20^b$

$\log_{20} 150 = b$

$\frac{\log 150}{\log 20} = b$

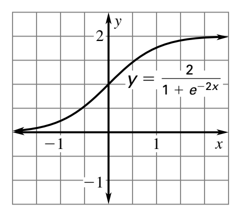
$b = 1.673$

$a = \frac{1}{0.5^{1.673}} = 3.188$

$y = 3.188x^{1.673}$

42. $y = \frac{2}{1 + e^{-2x}}$

x	y
0	1
−1	0.238
1	1.762



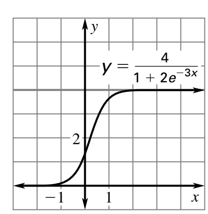
asymptotes: x-axis, $y = 2$

y-intercept $= 1$

point of maximum growth $= \left(\frac{\ln 1}{2}, \frac{2}{2}\right) = (0, 1)$

43. $y = \frac{4}{1 + 2e^{-3x}}$

x	y
0	$\frac{4}{3}$
0.231	2
−1	0.097

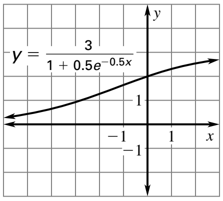


asymptotes: x-axis, $y = 4$

y-intercept $= \frac{4}{3}$

point of maximum growth $= \left(\frac{\ln 2}{3}, \frac{4}{2}\right) = (0.231, 2)$

44. $y = \dfrac{3}{1 + 0.5e^{-0.5x}}$



x	y
0	2
-1.386	$\frac{3}{2}$
1	2.302

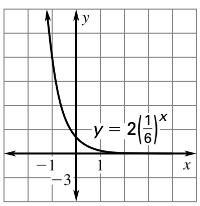
asymptotes: x-axis, $y = 3$

y-intercept $= 2$

point of maximum growth $= \left(\dfrac{\ln 0.5}{0.5}, \dfrac{3}{2}\right) = \left(-1.386, \dfrac{3}{2}\right)$

Chapter 8 Test *(p. 527)*

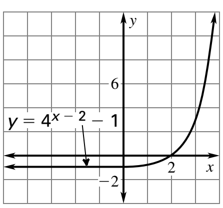
1. $y = 2\left(\frac{1}{6}\right)^x$



x	y
0	2
-1	12
1	$\frac{1}{3}$

Domain: all real numbers
Range: $y > 0$

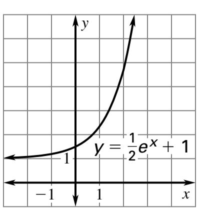
2. $y = 4^{x-2} - 1$



x	y
2	0
3	3
0	$-\frac{15}{16}$

Domain: all real numbers
Range: $y > -1$

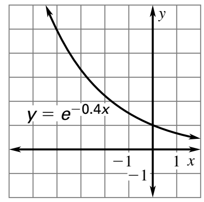
3. $y = \frac{1}{2}e^x + 1$



x	y
0	1.5
1	2.359
-1	1.184

Domain: all real numbers
Range: $y > 1$

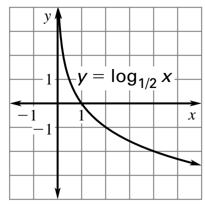
4. $y = e^{-0.4x}$



x	y
0	1
1	0.67
-1	1.49

Domain: all real numbers
Range: $y > 0$

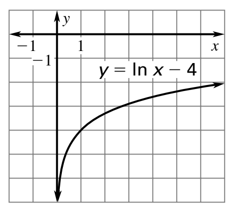
5. $y = \log_{1/2} x$



x	y
1	0
2	-1
4	-2
$\frac{1}{2}$	1
$\frac{1}{4}$	2

Domain: $x > 0$
Range: all real numbers

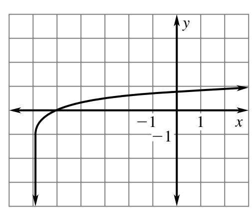
6. $y = \ln x - 4$



x	y
1	-4
10	-1.7

Domain: $x > 0$
Range: all real numbers

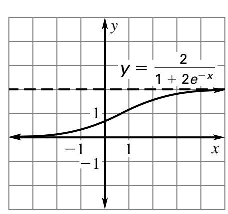
7. $y = \log(x + 6)$



x	y
-5	0
-1	0.69
-5.5	-0.3
0	0.78

Domain: $x > -6$
Range: all real numbers

8. $y = \dfrac{2}{1 + 2e^{-x}}$



x	y
0	$\frac{2}{3}$
-1	0.311
1	1.152

Domain: all real numbers
Range: $0 < y < 2$

9. $(2e^{-1})(3e^2) = 2 \cdot 3e^{-1+2} = 6e$

10. $\frac{-4e}{2e^{5x}} = -2e^{x-5x} = -2e^{-4x} = \frac{-2}{e^{4x}}$

11. $e^6 \cdot e^x \cdot e^{-3x} = e^6(e^{x-3x}) = e^6(e^{-2x}) = e^{-2x+6}$

12. $\log 1000^2 = \log(10^3)^2 = \log 10^6$

$10^x = 10^6$, so $x = 6$

13. $8^{\log_8 x} = x$ **14.** $\log_4 0.25 \quad 4^x = \frac{1}{4}$ so $x = -1$

15. $\log_{1/3} 27 \quad \frac{1}{3}^x = 27$ so $x = -3$

16. $\log 1 \quad 10^x = 1$ so $x = 0$

17. $\ln e^{-2} \quad e^x = e^{-2}$ so $x = -2$

18. $\log_3 243^2$

$3^x = 243^2$

$3^x = (3^5)^2$

$3^x = 3^{10}$ so $x = 10$

19.
$$12 = 10^{x+5} - 7$$
$$12 + 7 = 10^{x+5}$$
$$19 = 10^{x+5}$$
$$10^{x+5} = 19$$
$$\log 10^{x+5} = \log 19$$
$$x + 5 = \log 19$$
$$x = \log 19 - 5 \approx -3.721$$

20.
$$5 - \ln x = 7$$
$$-\ln x = 7 - 5$$
$$-\ln x = 2$$
$$\ln x = -2$$
$$x = e^{-2}$$
$$x \approx 0.135$$

21.
$$\log_2 4x = \log_2 (x + 15)$$
$$4x = (x + 15)$$
$$4x - (x + 15) = 0$$
$$4x - x - 15 = 0$$
$$3x - 15 = 0$$
$$3x = 15$$
$$x = 5$$

22.
$$\frac{4}{1 + 2.5e^{-4x}} = 3.3$$
$$4 = (1 + 2.5e^{-4x})3.3$$
$$4 = 3.3 + 8.25e^{-4x}$$
$$0.7 = 8.25e^{-4x}$$
$$\frac{0.7}{8.25} = e^{-4x}$$
$$0.085 = e^{-4x}$$
$$\ln 0.085 = -4x$$
$$-\frac{1}{4}(\ln 0.085) = x$$
$$x \approx 0.617$$

23. $f(x) = 10(0.87)^x$ exponential decay

24. $y = \log_6 x$

$6^y = x$

$x = 6^y$

$y = 6^x$

25. $\log_2 5 \approx 2.322$

$$\log_2 50 = \log_2 5 \cdot 10 = \log_2 5 + \log_2 10$$
$$= \log_2 5 + \log_2 5 \cdot 2$$
$$= \log_2 5 + \log_2 5 + \log_2 2$$
$$= \log_2 5 + \log_2 5 + \frac{\log 2}{\log 2}$$
$$= 2.322 + 2.322 + 1.000$$
$$= 5.644$$

$$\log_2 0.4 = \log_2 \frac{2}{5} = \log_2 2 - \log_2 5$$
$$= \frac{\log 2}{\log 2} - \log_2 5$$
$$= 1 - 2.322$$
$$= -1.322$$

26.
$$3 \log_4 14 - 3 \log_4 42 = \log_4 (14)^3 - \log_4 (42)^3$$
$$= \log_4 \frac{(14)^3}{(42)^3}$$
$$= \log_4 \left(\frac{14}{42}\right)^3$$
$$= \log_4 \left(\frac{1}{3}\right)^3 = \log_4 \frac{1}{27}$$

27.
$$\ln 2y^2x = \ln 2 + \ln y^2 + \ln x$$
$$= \ln 2 + 2 \ln y + \ln x$$

28. $\log_7 15 = \frac{\log 15}{\log 7} = 1.392$

29. (4, 6), (7, 10)

$y = ab^x$

$6 = ab^4$

$10 = ab^7$

$a = \frac{6}{b^4}$

$10 = \frac{6}{b^4}(b^7) = 6b^3$

$\frac{10}{6} = b^3$

$\frac{5}{3} = b^3$

$b^3 = 1.667$

$b = 1.186$

$a = \frac{6}{(1.186)^4} = 3.036$

$y = 3.036(1.186)^x$

30. (2, 3), (10, 21)

$y = ax^b$

$3 = a \cdot 2^b$

$21 = a \cdot 10^b$

$a = \frac{3}{2^b}$

$21 = \frac{3}{2^b}(10^b)$

$21 = 3 \cdot 5^b$

$7 = 5^b$

$\log_5 7 = b$

$\frac{\log 7}{\log 5} = b$

$b \approx 1.209$

$a = \frac{3}{2^{1.209}}$

$a = 1.298$

$y = 1.298x^{1.209}$

31. $V = a(1 - r)^t$

$= 24{,}900(1 - 0.10)^t$

$= 24{,}900(0.90)^t$

t	*V*
0	24,900
4	16,337
6	13,233
6.58	12,448

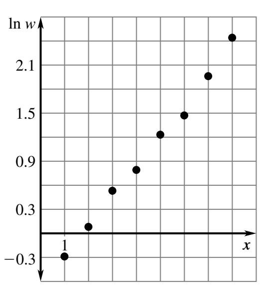
half purchase price about 6.58 years

32. $A = Pe^{rt}$

$= 4000e^{0.07(5)}$

$= \$5676.27$

33. a.



yes

x	1	2	3	4
ln *w*	−0.286	0.076	0.532	0.788

x	5	6	7	8
ln *w*	1.235	1.47	1.96	2.4

33. b. *Sample answer*: $w = ab^x$

(1, 0.751), (8, 11.518)

$0.751 = ab^1$

$11.518 = ab^8$

$11.518 = \left(\frac{0.751}{b^1}\right)b^8$

$11.518 = 0.751b^7$

$\frac{11.518}{0.751} = b^7$

$1.477 = b$

$a = \frac{0.751}{b^1}$

$a = \frac{0.751}{1.477}$

$a = 0.508$

$w = 0.508(1.477)^x$

$w = 0.508(1.477)^9 \approx 17$ kg

Chapter 8 Standardized Test *(pp. 528–529)*

1. E. $f(x) = 4(0.8)^x - 3$

2. $A = Pe^{rt}$

Initial investment \$4000.00

Double investment \$8000.00

$8000 = 4000e^{0.07(t)}$

$\frac{8000}{4000} = e^{0.07t}$

$2 = e^{0.07t}$

$\ln 2 = 0.07t$

$\frac{1}{0.07}(\ln 2) = t$

$9.9 = t$

$t \approx 10$

D

3. $y = \ln(x - 2)$

$x = \ln(y - 2)$

$e^x = y - 2$

$e^x + 2 = y$

A

4. $f(x) = \log(x + 4)$ D

5. $\log_2 7 = \frac{\log 7}{\log 2}$ D

6. $\log \frac{xy^2}{z} = \log x + \log y^2 - \log z$

$= \log x + 2 \log y - \log z$

B

7. $2^{x+14} = 16^{2x}$

$2^{x+14} = (2^4)^{2x}$

$2^{x+14} = 2^{8x}$

$x + 14 = 8x$

$14 = 7x$

$x = 2$

A

8. $0.5 \log_3 x = 2$

$\log_3 x = 2(2)$

$\log_3 x = 4$

$3^4 = x$

$x = 81$

C

9. Asymptote: $y = 0$

$f(x) = \log 6x$

C

x	y
$\frac{1}{6}$	0
$\frac{10}{6}$	2
$\frac{1}{60}$	-2

10. $f(x) = 4e^{0.5x}$ exponential growth function B

11. $\log 10{,}000 = 4$

$\ln e^4 = 4$

C

12. $\log_2 4 = \frac{\log 4}{\log 2} = 2$

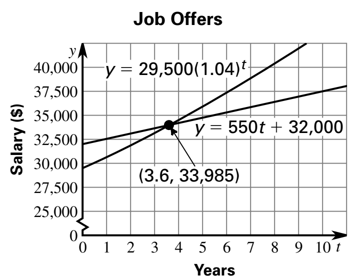
$\log_4 2 = \frac{\log 2}{\log 4} = 0.5$

A

13. a. $y = 550t + 32{,}000$

b. $y = 29{,}500(1.04)^t$

c.



The intersection point $(3.6, 33{,}984)$ represents when the salaries are equal.

d. After about 4 years, the salary from the second offer becomes greater than the first.

y	t
32,000	0
32,550	1
33,100	2
33,650	3
34,200	4
34,750	5
35,300	6
35,850	7
36,400	8
33,980	3.6

y	t
0	0
30,680	1
31,907	2
33,183	3
34,511	4
35,891	5
37,327	6
38,820	7
40,373	8
33,974	3.6

14. a.

t	2	4	6	8	10
$\ln y$	2.351	2.603	2.773	2.918	3.000

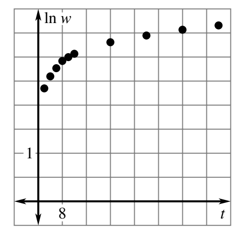
t	12	24	36	48	60
$\ln y$	3.068	3.314	3.45	3.570	3.664

b.

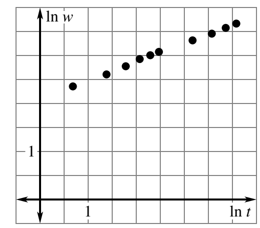
$\ln t$	0.693	1.386	1.792	2.079	2.303
$\ln w$	2.351	2.603	2.773	2.918	3.0

$\ln t$	2.485	3.178	3.584	3.871	4.094
$\ln w$	3.068	3.314	3.45	3.57	3.664

a.



b.



c. A power model is a better fit for the original data. The points in the graph of $(t, \ln w)$ lie on a curve, while those in the graph of $(\ln t, \ln w)$ lie close to a straight line.

d.

$$\frac{3.664 - 2.351}{4.094 - 0.693} = \frac{1.313}{3.401} = 0.386$$

$$\frac{\ln w - 2.351}{\ln t - 0.693} = \frac{1.313}{3.401}$$

$$3.401(\ln w - 2.351) = 1.313(\ln t - 0.693)$$

$$3.401 \ln w - 7.996 = 1.313 \ln t - 0.910$$

$$3.401 \ln w = 1.313 \ln t + 7.086$$

$$\ln w = \frac{1.313}{3.401} \ln t + \frac{7.086}{3.401}$$

$$\ln w = 0.386 \ln t + 2.084$$

$$\ln w = \ln t^{0.386} + 2.084$$

$$w = e^{2.084}(t^{0.386})$$

$$w = 8.037t^{0.386}$$

e. $1\frac{1}{2}$ years old:

$w = 8.037t^{0.386}$

$= 8.037(18)^{0.386}$

$= 24.5$ lb

$2\frac{1}{2}$ years old:

$w = 8.037t^{0.386}$

$= 8.037(30)^{0.386}$

$= 29.9$ lb

$3\frac{1}{2}$ years old:

$w = 8.037t^{0.386}$

$= 8.037(42)^{0.386}$

$= 34.0$ lb

$4\frac{1}{2}$ years old:

$w = 8.037t^{0.386}$

$= 8.037(54)^{0.386}$

$= 37.5$ lb

15. $f(x) = \dfrac{5}{1 + 9e^{-x}}$

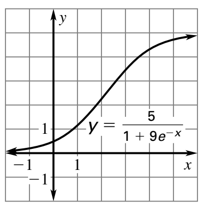
a. $f(-1) = \dfrac{5}{1 + 9e^{-(-1)}}$ $\quad f(0) = \dfrac{5}{1 + 9e^{-0}}$

$f(-1) = \dfrac{5}{25.465}$ $\quad f(0) = \dfrac{5}{10} = \dfrac{1}{2} = 0.5$

$f(-1) = 0.196$

$f(2) = \dfrac{5}{1 + 9e^{-2}} = \dfrac{5}{2.218} = 2.254$

b.



c. Asymptotes: x-axis, $y = 5$

y-intercept $= 0.5$

point of maximum growth $= \left(\dfrac{\ln 9}{1}, \dfrac{5}{2}\right) = (2.197, 2.5)$

d. $f(4) = \dfrac{5}{1 + 9e^{-4}} = \dfrac{5}{1.165} \approx 4.292$

$\left(\dfrac{\ln 9(4)}{1}, \dfrac{5}{2}\right)$

$\left(\dfrac{\ln 36}{1}, \dfrac{5}{2}\right)$

$\left(3.584, \dfrac{5}{2}\right)$

e. At first the function grows very slowly, then it grows faster and faster until the growth rate reaches its peak at the point (ln 9, 2.5). After this, the growth rate drops off until the curve again becomes almost flat, approaching $y = 5$ asymptotically.