CHAPTER 12

Think & Discuss *(p. 699)*

1. Beethoven & Mozart
Beethoven & Dvorak
Beethoven & Brahms
Beethoven & Srauss
Beethoven & Tchaikovsky

Mozart & Dvorak
Mozart & Brahms
Mozart & Strauss
Mozart & Tchaikovsky

Dvorak & Brahms
Dvorak & Strauss
Dvorak & Tchaikovsky

Brahms & Strauss
Brahms & Tchaikovsky

Strauss & Tchaikovsky

15 choices

2. $15 + 6 = 21$

Skill Review *(p. 700)*

1. 0.5, 50%

2. 0.2, 20%

3. 0.15, 15%

4. 0.48, 48%

5. 0.194, 19.4%

6. 0.469, 46.9%

7. $A = 4^2\pi$
$= 16\pi$
$= 50.27$

8. $A = 5^2$
$= 25$

9. $A = \frac{1}{2}(12)(8)$
$= 48$

10. $10^x = 0.5$
$\log 10^x = \log 0.5$
$x \log 10 = \log 0.5$
$x = \dfrac{\log 0.5}{\log 10}$
$x = -0.301$

11. $(0.5)^x + 3 = 3.75$
$(0.5)^x = 0.75$
$x \log 0.5 = \log 0.75$
$x = 0.415$

12. $1 - 9^x = 0.25$
$0.75 = 9^x$
$\log 0.75 = x \log 9$
$x = -0.131$

Lesson 12.1

12.1 Guided Practice (p. 705)

1. A permutation of n objects is an ordering of those objects.

2. The Fundamental Counting Principle states that if one event can occur in n ways, another in m ways, and a third in p ways, the number of ways all can occur is $n \cdot m \cdot p$. The number of permutations of n objects is also a product, $n(n-1)(n-2)\cdot\,.\,.\,.\cdot 3\cdot 2\cdot 1$.

3. Yes; $1! = 1$ so $\dfrac{4!}{2!} = \dfrac{4!}{(2!)(1!)(1!)}$

4. Because we are interested in all numbers with three even digits, those digits can be repeated. Therefore the correct answer is $5 \times 5 \times 5 = 125$.

5. $2! = 2$

6. $6! = 720$

7. $1! = 1$

8. $4! = 24$

9. $\dfrac{6!}{(6-3)!} = 6\cdot 5\cdot 4 = 120$

10. $\dfrac{5!}{(5-1)!} = 5$

11. $\dfrac{3!}{(3-3)!} = 6$

12. $\dfrac{10!}{(10-2)!} = 90$

13. $\dfrac{7!}{4!} = 210$

14. $\dfrac{5!}{3!\cdot 2!} = 10$

12.1 Practice and Applications (pp. 705–707)

15. $3\cdot 1 = 3$ ways

16. $3\cdot 5 = 15$ ways

17. $2\cdot 4\cdot 5 = 40$ ways

18. $4\cdot 6\cdot 9\cdot 7 = 1512$ ways

19. **(a)** $26\cdot 26\cdot 26\cdot 10\cdot 10\cdot 10 = 17{,}576{,}000$
(b) $26\cdot 25\cdot 24\cdot 10\cdot 9\cdot 8 = 11{,}232{,}000$

20. **(a)** $10\cdot 10\cdot 26\cdot 26\cdot 26\cdot 26 = 45{,}697{,}600$
(b) $10\cdot 9\cdot 26\cdot 25\cdot 24\cdot 23 = 32{,}292{,}000$

21. **(a)** $10\cdot 10\cdot 10\cdot 10\cdot 26\cdot 26 = 6{,}760{,}000$
(b) $10\cdot 9\cdot 8\cdot 7\cdot 26\cdot 25 = 3{,}276{,}000$

22. **(a)** $26\cdot 26\cdot 26\cdot 26\cdot 26\cdot 10 = 118{,}813{,}760$
(b) $26\cdot 25\cdot 24\cdot 23\cdot 22\cdot 10 = 78{,}936{,}000$

23. $8! = 40{,}320$

24. $5! = 120$

25. $10! = 3{,}628{,}800$

26. $9! = 362{,}880$

27. $0! = 1$

28. $7! = 5040$

29. $3! = 6$

30. $12! = 479{,}001{,}600$

31. $\dfrac{3!}{(3-3)!} = 6$

32. $\dfrac{5!}{(5-2)!} = 20$

33. $\dfrac{2!}{(2-1)!} = 2$

34. $\dfrac{7!}{(7-6)!} = 5040$

35. $\dfrac{8!}{(8-5)!} = 6720$

36. $\dfrac{9!}{(9-4)!} = 3024$

37. $\dfrac{12!}{(12-3)!} = 1320$

38. $\dfrac{16!}{(16-0)!} = 1$

39. $2! = 2$

40. $3! = 6$

41. $4! = 24$

42. $5! = 120$

43. $6! = 720$

44. $7! = 5040$

45. $8! = 40{,}320$

46. $9! = 362{,}880$

47. $\dfrac{3!}{2!} = 3$

48. $\dfrac{5!}{3!} = 20$

49. $\frac{6!}{2!} = 360$

50. $\frac{6!}{2!2!} = 180$

51. $\frac{7!}{2!} = 2520$

52. $\frac{7!}{4!} = 210$

53. $\frac{8!}{2!2!} = 10{,}080$

54. $\frac{11!}{3!2!2!} = 1{,}663{,}200$

55. $5 \cdot 8 \cdot 12 = 480$

56. $7 \cdot 11 \cdot 6 \cdot 3 \cdot 2 = 2772$

57. **(a)** $36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 \cdot 36 = 2{,}176{,}782{,}336$

(b) $36 \cdot 35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 = 1{,}402{,}410{,}240$

58. ${}_nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

This makes sense because there is only one way to choose zero objects from any set: take none.

59. $24! = 6.20 \times 10^{23}$

60. $10! = 3{,}628{,}800$

61. **(a)** $6! = 720$

(b) $\frac{9!}{(9-6)!} = 60{,}480$

62. $\frac{7!}{3!2!2!} = 210$

63. $\frac{15!}{3!5!4!3!} = 12{,}612{,}600$

64. ${}_nP_{n-1} = \frac{n!}{(n-(n-1))!} = \frac{n!}{1} = n!$ is the same as the number of permutations of all n objects. This makes sense because it can be thought of as choosing one object to leave out (n ways) then doing permutations of the $n - 1$ objects remaining $[(n-1)!$ ways$]$. Now we have $n \times (n-1)! = n!$.

65. B

66. C

67. $\frac{n!}{n} = (n-1)!$; You can arrange n objects in a circle n different ways in the same order so divide $n!$ by n to find the number of unique circular permutations.

12.1 Mixed Review (p. 707)

68. $(x + 9)(x - 9) = x^2 - 81$

69. $(x^2 + 2)^2 = x^4 + 4x^2 + 4$

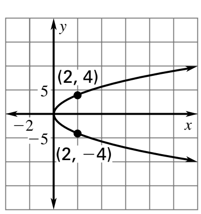
70. $(2x - 1)^3 = 8x^3 - 12x^2 + 6x - 1$

71. $(4x + 5)(4x - 5) = 16x^2 - 25$

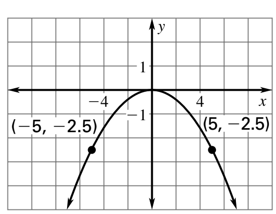
72. $(2y + 3x)^2 = 4y^2 + 12xy + 9x^2$

73. $(8y - x)^2 = 64y^2 - 16xy + x^2$

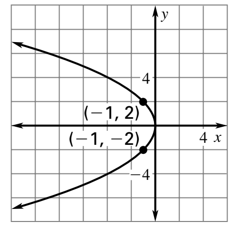
74.



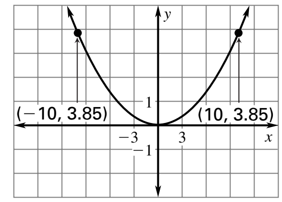
75.



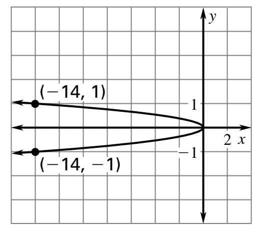
76.



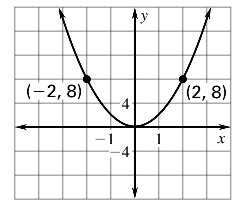
77.



78.



79.



80. $\sum_{n=0}^{\infty} 3\left(\frac{1}{2}\right)^n = 6$

81. $\sum_{n=0}^{\infty} -4\left(\frac{1}{4}\right)^n = -\frac{16}{3}$

82. $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 6$

83. $\sum_{n=1}^{\infty} 2\left(\frac{7}{5}\right)^{n-1}$ no sum

84. $\sum_{n=0}^{\infty} -5\left(\frac{1}{8}\right)^n = -\frac{40}{7}$

85. $\sum_{n=1}^{\infty} \frac{1}{2}(0.3)^{n-1} = 0.714$

86. $R = \frac{V}{I}$

$= \frac{120}{0.80}$

$= 150$ ohms

Lesson 12.2

Activity (p. 710)

1. **(a)** $a^2 + 2ab + b^2$

(b) $(a + b)(a^2 + 2ab + b^2) = a^3 + 3a^2b + 3ab^2 + b^3$

(c) $(a + b)(a^3 + 3a^2b + 3ab^2 + b^3)$

$= a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2$

$+ 3ab^3 + b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

2. The coefficients for $(a + b)^n$ are the numbers in Pascal's triangle for ${}_nC_r$.

3. The exponents of a and b sum to n (where n is the exponent in $(a + b)^n$). The exponents of a decrease from n to zero and the exponents of b increase from zero to n.

12.2 Guided Practice (p. 712)

1. In a permutation, the order of the objects is important. In a combination, order does not matter.

2. **a.** Add combinations when finding the number of ways that event A or event B can happen.

b. Multiply combinations when finding the number of ways that event A and event B can happen.

3. $(x + y)^4 = {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3xy^3 + {}_4C_4x^0y^4$
$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = {}_4C_0x^4(-y)^0 + {}_4C_1x^3(-y)^1 + {}_4C_2x^2(-y)^2 + {}_4C_3x(-y)^3 + {}_4C_4x^0(-y)^4$
$= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

The two expansions have the same variables raised to the same powers and the same coefficients. The signs of the two terms are different.

4. The problem should be $\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210$. The six and five will cancel also.

5. ${}_8C_2 = \frac{8!}{6! \cdot 2!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$

6. ${}_6C_5 = \frac{6!}{1! \cdot 5!} = \frac{6}{1} = 6$

7. ${}_5C_1 = \frac{5!}{4! \cdot 1!} = \frac{5}{1} = 5$

8. ${}_9C_9 = \frac{9!}{0! \cdot 9!} = 1$

9. $(x + y)^3 = {}_3C_0x^3y^0 + {}_3C_1x^2y^1 + {}_3C_2xy^2 + {}_3C_3x^0y^3$
$= x^3 + 3x^2y + 3xy^2 + y^3$

10. $(x + 1)^4 = {}_4C_0x^4(1)^0 + {}_4C_1x^3(1)^1 + {}_4C_2x^2(1)^2 + {}_4C_3x(1)^3 + {}_4C_4x^0(1)^4$
$= x^4 + 4x^3 + 6x^2 + 4x + 1$

11. $(2x + 4)^3 = {}_3C_0(2x)^3(4)^0 + {}_3C_1(2x)^2(4)^1 + {}_3C_2(2x)^1(4)^2 + {}_3C_3(2x)^0(4)^3$
$= 8x^3 + (3)(4x^2)(4) + (3)(2x)(16) + (1)(1)(64)$
$= 8x^3 + 48x^2 + 96x + 64$

12. $(2x + 3y)^5$
$= {}_5C_0(2x)^5(3y)^0 + {}_5C_1(2x)^4(3y)^1 + {}_5C_2(2x)^3(3y)^2 + {}_5C_3(2x)^2(3y)^3 + {}_5C_4(2x)^1(3y)^4 + {}_5C_5(2x)^0(3y)^5$
$= (1)(32x^5)(1) + (5)(16x^4)(3y) + 10(8x^3)(9y^2) + (10)(4x^2)(27y^3) + 5(2x)(81y^4) + (1)(1)(243y^5)$
$= 32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

13. $(x - y)^5 = {}_5C_0x^5(-y)^0 + {}_5C_1x^4(-y)^1 + {}_5C_2x^3(-y)^2 + {}_5C_3x^2(-y)^3 + {}_5C_4x^1(-y)^4 + {}_5C_5x^0(-y)^5$
$= x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

14. $(x - 2)^3 = {}_3C_0x^3(-2)^0 + {}_3C_1x^2(-2)^1 + {}_3C_2x^1(-2)^2 + {}_3C_3x^0(-2)^3$
$= x^3 - 6x^2 + 12x - 8$

15. $(3x - 1)^4 = {}_4C_0(3x)^4(-1)^0 + {}_4C_1(3x)^3(-1)^1 + {}_4C_2(3x)^2(-1)^2 + {}_4C_3(3x)^1(-1)^3 + {}_4C_4(3x)^0(-1)^4$
$= 81x^4 + 4(27x^3)(-1) + 6(9x^2)(1) + 4(3x)(-1) + 1$
$= 81x^4 - 108x^3 + 54x^2 - 12x + 1$

16. $(4x - 4y)^3 = {}_3C_0(4x)^3(-4y)^0 + {}_3C_1(4x)^2(-4y)^1 + {}_3C_2(4x)^1(-4y)^2 + {}_3C_3(4x)^0(-4y)^3$
$= 64x^3 + (3)(16x^2)(-4y) + 3(4x)(16y^2) + (-64y^3)$
$= 64x^3 - 192x^2y + 192xy^2 - 64y^3$

17. ${}_5C_3(27) = 10(27) = 270$
${}_5C_5(3)^5 = 243$

12.2 Practice and Applications (pp. 712–714)

18. ${}_{10}C_2 = \frac{10!}{8! \cdot 2!} = \frac{10 \cdot 9}{2 \cdot 1} = 45$

19. ${}_8C_5 = \frac{8!}{3! \cdot 5!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

20. ${}_5C_2 = \frac{5!}{3! \cdot 2!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$

21. ${}_8C_6 = \frac{8!}{2! \cdot 6!} = \frac{8 \cdot 7}{2 \cdot 1} = 28$

22. ${}_{12}C_4 = \frac{12!}{8! \cdot 4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} = 495$

23. ${}_{12}C_{12} = \frac{12!}{0! \cdot 12!} = 1$

24. ${}_{14}C_6 = \frac{14!}{8! \cdot 6!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003$

25. ${}_{11}C_3 = \frac{11!}{8! \cdot 3!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$

26. ${}_{12}C_5 = \frac{12!}{7! \cdot 5!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792$

27. ${}_4C_4 \cdot {}_{48}C_1 = 1(48)$
$= 48$

28. ${}_4C_1 \cdot {}_{48}C_4 = \frac{4!}{3! \cdot 1!} \cdot \frac{48!}{44! \cdot 4!}$
$= 4 \cdot 194{,}580$
$= 778{,}320$

29. ${}_4C_2 \cdot {}_4C_3 = 6 \cdot 4$
$= 24$

30. ${}_{13}C_1 \cdot {}_{48}C_1 = 13 \cdot 48$
$= 624$

31.

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

32. $(x + 4)^6 = x^6 + 6(4)^1x^5 + 15(4)^2x^4 + 20(4)^3x^3 + 15(4)^4x^2 + 6(4)^5x^1 + 4^6$
$= x^6 + 24x^5 + 240x^4 + 1280x^3 + 3840x^2 + 6144x + 4096$

33. $(x - 3y)^6 = x^6 + 6(-3y)^1x^5 + 15(-3y)^2x^4 + 20(-3y)^3x^3 + 15(-3y)^4x^2 + 6(-3y)^5x^2 + (-3y)^6$
$= x^6 - 18x^5y + 135x^4y^2 - 540x^3y^3 + 1215x^2y^4 - 1458xy^5 + 729y^6$

34. $(x^2 + y)^7 = (x^2)^7 + 7(x^2)^6y^1 + 21(x^2)^5y^2 + 35(x^2)^4y^3 + 35(x^2)^3y^4 + 21(x^2)^2y^5 + 7(x^2)^1y^6 + y^7$
$= x^{14} + 7x^{12}y + 21x^{10}y^2 + 35x^8y^3 + 35x^6y^4 + 21x^4y^5 + 7x^2y^6 + y^7$

35. $(2x - y^3)^7 = (2x)^7 + 7(2x)^6(-y^3)^1 + 21(2x)^5(-y^3)^2 + 35(2x)^4(-y^3)^3 + 35(2x)^3(-y^3)^4 + 21(2x)^2(-y^3)^5 + 7(2x)^1(-y^3)^6 + (-y^3)^7$
$= 128x^7 - 448x^6y^3 + 672x^5y^6 - 560x^4y^9 + 280x^3y^{12} - 84x^2y^{15} + 14xy^{18} - y^{21}$

36. $(x - 2)^3 = {}_3C_0x^3(-2)^0 + {}_3C_1x^2(-2)^1 + {}_3C_2x^1(-2)^2 + {}_3C_3x^0(-2)^3$
$= x^3 - 6x^2 + 12x - 8$

37. $(x + 4)^5 = {}_5C_0x^5(4)^0 + {}_5C_1x^4(4)^1 + {}_5C_2x^3(4)^2 + {}_5C_3x^2(4)^3 + {}_5C_4x^1(4)^4 + {}_5C_5x^0(4)^5$
$= x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$

38. $(x + 3y)^4 = {}_4C_0x^4(3y)^0 + {}_4C_1x^3(3y)^1 + {}_4C_2x^2(3y)^2 + {}_4C_3x^1(3y)^3 + {}_4C_4x^0(3y)^4$
$= x^4 + 12x^3y + 54x^2y^2 + 108xy^3 + 81y^4$

39. $(2x - y)^6 = {}_6C_0(2x)^6(-y)^0 + {}_6C_1(2x)^5(-y)^1 + {}_6C_2(2x)^4(-y)^2 + {}_6C_3(2x)^3(-y)^3 + {}_6C_4(2x)^2(-y)^4 + {}_6C_5(2x)^1(-y)^5 + {}_6C_6(2x)^0(-y)^6$
$= 64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^4 - 12xy^5 + y^6$

40. $(x^2 + 3)^5 = {}_5C_0(x^3)^5(3)^0 + {}_5C_1(x^3)^4(3)^1 + {}_5C_2(x^3)^3(3)^2 + {}_5C_3(x^3)^2(3)^3 + {}_5C_4(x^3)^1(3)^4 + {}_5C_5(x^3)^0(3)^5$
$= x^{15} + 15x^{12} + 90x^9 + 270x^6 + 405x^3 + 243$

41. $(3x^2 - 3)^4 = {}_4C_0(3x^2)^4(-3)^0 + {}_4C_1(3x^2)^3(-3)^1 + {}_4C_2(3x^2)^2(-3)^2 + {}_4C_3(3x^2)^1(-3)^3 + {}_4C_4(3x^2)^0(-3)^4$
$= 81x^8 - 324x^6 + 486x^4 - 324x^2 + 81$

42. $(2x - y^2)^7 = {}_7C_0(2x)^7 + {}_7C_1(2x)^6(-y^2)^1 + {}_7C_2(2x)^5(-y^2)^2 + {}_7C_3(2x)^4(-y^2)^3 + {}_7C_4(2x)^3(-y^2)^4 + {}_7C_5(2x)^2(-y^2)^5 + {}_7C_6(2x)^1(-y^2)^6 + {}_7C_7(-y^2)^7$
$= 128x^7 - 448x^6y^2 + 672x^5y^4 - 560x^4y^6 + 280x^3y^8 - 84x^2y^{10} + 14xy^{12} - y^{14}$

43. $(x^3 + y^2)^3 = {}_3C_0(x^3)^3(y^2)^0 + {}_3C_1(x^3)^2(y^2)^1 + {}_3C_2(x^3)^1(y^2)^2 + {}_3C_3(x^3)^0(y^2)^3$
$= x^9 + 3x^6y^2 + 3x^3y^4 + y^6$

44. $(x - 3)^7; {}_7C_2x^5(-3)^2$
$189x^5$
coefficient $= 189$

45. $(x + 2)^8; {}_8C_4x^4(2)^4$
$1120x^4$
coefficient $= 1120$

46. $(x^2 + 4)^{10}; {}_{10}C_7(x^2)^3(4)^7$
$(120)(16{,}384)x^6$
$1{,}966{,}080x^6$
coefficient $= 1{,}966{,}080$

47. ${}_{10}C_3 = \frac{10!}{7! \cdot 3!}$
$= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$
$= 120$

48. $_{15}C_4 = \dfrac{15!}{11! \cdot 4!}$

$= \dfrac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1}$

$= 1365$

49. $_7C_1 \cdot {_{10}C_2} = \dfrac{7!}{6! \cdot 1!} \cdot \dfrac{10!}{8! \cdot 2!}$

$= 7 \cdot \dfrac{10 \cdot 9}{2 \cdot 1}$

$= 315$

50. $_6C_1 \cdot {_5C_4} = \dfrac{6!}{5! \cdot 1!} \cdot \dfrac{5!}{1! \cdot 4!}$

$= 6 \cdot 5$

$= 30$

51. $_{10}C_0 + {_{10}C_1} + {_{10}C_2} + {_{10}C_3} + {_{10}C_4} + {_{10}C_5} + {_{10}C_6} + {_{10}C_7}$

$= 1 + 10 + 45 + 120 + 210 + 252 + 210 + 120$

$= 968$

52. $_{20}C_{15} + {_{20}C_{16}} + {_{20}C_{17}} + {_{20}C_{18}} + {_{20}C_{19}} + {_{20}C_{20}}$

$= 15{,}504 + 4845 + 1140 + 190 + 20 + 1$

$= 21{,}700$

53. $_{10}C_3 + {_{10}C_4} + {_{10}C_5} + {_{10}C_6} + {_{10}C_7} + {_{10}C_8} + {_{10}C_9} +$

$_{10}C_{10} = 120 + 210 + 252 + 210 + 120 + 45 + 10 +$

$1 = 968$

54. $2^{12} - ({_{12}C_0} + {_{12}C_1} + {_{12}C_2} + {_{12}C_3})$

$= 4096 - (1 + 12 + 66 + 220)$

$= 3797$

55. combinations

$_8C_3 = \dfrac{8!}{5! \cdot 3!}$

$= 56$

56. permutations

$_{15}P_2 = \dfrac{15!}{13!}$

$= 210$

57. permutations

$_{16}P_4 = \dfrac{16!}{12!}$

$= 43{,}680$

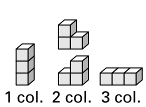
58. combinations

$_{12}C_6 = \dfrac{12!}{6! \cdot 6!}$

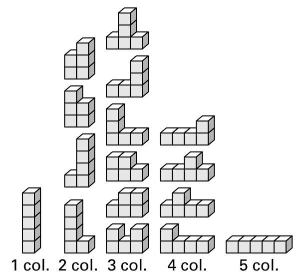
$= 924$

59. $_nP_r = r! \times {_nC_r}$

60.



61.



62. The numbers in row $(n - 1)$ of Pascal's triangle correspond to the number of ways to stack n blocks in r columns. The total number of ways is the sum of the numbers in row $(n - 1)$ of Pascal's triangle.

63. $_9C_0 + {_9C_1} + {_9C_2} + {_9C_3} + {_9C_4} + {_9C_5} + {_9C_6} + {_9C_7} + {_9C_8} + {_9C_9}$

$= 1 + 9 + 36 + 84 + 126 + 126 + 84 + 36 + 9 + 1$

$= 512$

64. The sum of the numbers in row n of Pascal's triangle is 2^n. Each number is the sum of the two numbers above it, so each internal number is added twice to the next row. For the ones on the ends, the additional ones on each end of the new row represents the second use of those values (this can be thought of as adding the one to an imaginary zero on the outside of the previous row).

65. $\displaystyle\sum_{n=0}^{20} 2^n = 2{,}097{,}151$

66. $S_n = S_{n-1} + S_{n-2} = 1, 1, 2, 3, 5, 8, \ldots$, which are the Fibonacci numbers.

67. **a.** $\dfrac{20!}{8! \cdot 7! \cdot 5!} = 99{,}768{,}240$

b. $\dfrac{20!}{8! \times 12!} \times \dfrac{12!}{7! \times 5!} \times \dfrac{5!}{5! \times 0!} = 125970 \times 792 \times 1$

$= 99{,}768{,}240$

c. They are the same. They are two different ways to count the same thing. The additional factorials in the numerators and denominators of part (b) simplify to become the expression in part (a).

68. $_nC_0 = \dfrac{n!}{n! \times 0!} = 1$

69. $_nC_n = \dfrac{n!}{0! \times n!} = 1$

70. $_nC_1 = \dfrac{n!}{(n-1)! \times 1!} = \dfrac{n!}{(n-1)!} = {_nP_1}$

71. $_nC_r = \dfrac{n!}{(n-r)! \times r!}$

$_nC_{n-r} = \dfrac{n!}{(n-(n-r))! \times (n-r)!}$

$= \dfrac{n!}{r! \times (n-r)!}$

Therefore $_nC_r = \dfrac{n!}{(n-r)! \times r!} = {_nC_{n-r}}$

72. ${}_nC_r \cdot {}_rC_m = \dfrac{n!}{(n-r)! \times r!} \cdot \dfrac{r!}{(r-m)! \times m!}$

$$= \frac{n!}{(n-r)! \times (r-m)! \times m!}$$

${}_nC_m \cdot {}_{n-m}C_{r-m}$

$$= \frac{n!}{(n-m)! \times m!} \times \frac{(n-m)!}{((n-m)-(r-m))! \times (r-m)!}$$

$$= \frac{n!}{m! \times (n-r)! \times (r-m)!}$$

Therefore ${}_nC_r \cdot {}_rC_m = {}_nC_m \cdot {}_{n-m}C_{r-m}$

73. ${}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}$

$${}_{n+1}C_r = \frac{(n+1)!}{((n+1)-r)! \cdot r!} = \frac{(n+1)!}{(n+1-r)!r!}$$

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

$${}_nC_{r-1} = \frac{n!}{(n-(r-1))!(r-1)!} = \frac{n!}{(n+1-r)!(r-1)!}$$

${}_nC_r + {}_nC_{r-1}$

$$= \frac{n!}{(n-r)!r!} + \frac{n!}{(n+1-r)!(r-1)!}$$

$$= \frac{n! \cdot (n+1-r)}{(n-r)! \cdot r! \cdot (n+1-r)} + \frac{n! \cdot r}{(n+1-r)! \cdot (r-1)! \cdot r}$$

$$= \frac{n! \cdot (n+1-r) + n! \cdot r}{(n+1-r)! \cdot r!}$$

$$= \frac{n! \cdot (n+1-r+r)}{(n+1-r)!\, r!}$$

$$= \frac{n! \cdot (n+1)}{(n+1-r)!\, r!}$$

$$= \frac{(n+1)!}{r!(n+1-r)!} = {}_{n+1}C_r$$

12.2 Mixed Review (p. 715)

74. $A = \pi(18)^2$
$= 1017.88 \text{ cm}^2$

75. $A = (9.5) \times (11.3)$
$= 107.35 \text{ in.}^2$

76. $A = \frac{1}{2}(13)(9)$
$= 58.5 \text{ ft}^2$

77. $A = \frac{1}{2}(10 + 13)(27)$
$= 310.5 \text{ m}^2$

78. $\dfrac{x^2}{25} - \dfrac{y^2}{144} = 1$

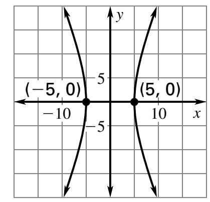
center: (0, 0)

asymptotes: $y = 0 + \frac{12}{5}(x - 0)$

$y = \frac{12}{5}x$

$y = 0 - \frac{12}{5}(x - 0)$

$y = -\frac{12}{5}x$



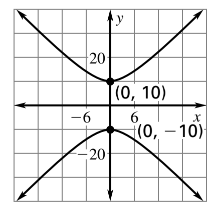
79. $\dfrac{y^2}{100} - \dfrac{x^2}{36} = 1$

center: (0, 0)

asymptotes: $y = 0 + \frac{10}{6}(x - 0)$

$y = \frac{5}{3}x$

$y = -\frac{5}{3}x$



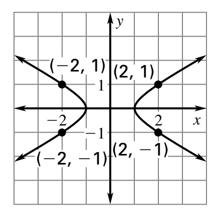
80. $x^2 - \dfrac{49y^2}{16} = 1$

center: (0, 0)

asymptotes: $y = 0 + \frac{7}{4}(x - 0)$

$y = \frac{7}{4}x$

$y = -\frac{7}{4}x$



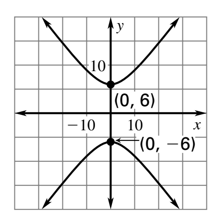
81. $\dfrac{y^2}{4} - \dfrac{x^2}{9} = 9$

$\dfrac{y^2}{36} - \dfrac{x^2}{81} = 1$

center: (0, 0)

asymptotes: $y = \frac{2}{3}x$

$y = -\frac{2}{3}x$



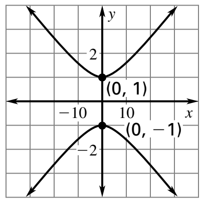
82. $64y^2 - x^2 = 64$

$\frac{y^2}{1} - \frac{x^2}{64} = 1$

center: $(0, 0)$

asymptotes: $y = \frac{1}{8}x$

$y = -\frac{1}{8}x$



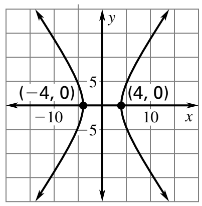
83. $9x^2 - 4y^2 = 144$

$\frac{x^2}{16} - \frac{y^2}{36} = 1$

center: $(0, 0)$

asymptotes: $y = \frac{3}{2}x$

$y = -\frac{3}{2}x$



84. geometric

3^n

85. arithmetic

$-4 + 7n$

86. geometric

$2(5)^{n-1}$

87. geometric

$(-2)^{n-1}$

88. arithmetic

$10 - 2n$

89. arithmetic

$-15 + 5n$

90. $3x + 4y = 70$

$3x + y = 40$

$y = -3x + 40$

$3x + 4(-3x + 40) = 70$

$3x - 12x + 160 = 70$

$90 = 9x$

$10 = x$

$3(10) + y = 40$

$y = 10$

10 plates, 10 bowls

Quiz 1 (p. 715)

1. $\frac{3!}{2!} = 3$

2. $4! = 24$

3. $5! = 120$

4. $\frac{6!}{2! \times 2!} = 180$

5. $7! = 5040$

6. $8! = 40{,}320$

7. $\frac{9!}{3!} = 60{,}480$

8. $\frac{10!}{2! \times 2!} = 907{,}200$

9. $(x + y)^6 = {}_6C_0x^6y^0 + {}_6C_1x^5y^1 + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5x^1y^5 + {}_6C_6y^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

10. $(x + 2)^4 = {}_4C_0x^4 + {}_4C_1x^3(2)^1 + {}_4C_2x^2(2)^2 + {}_4C_3x^1(2)^3 + {}_4C_4x^0(2)^4$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

11. $(x - 2y)^5 = {}_5C_0x^5 + {}_5C_1x^4(-2y)^1 + {}_5C_2x^3(-2y)^2 + {}_5C_3x^2(-2y)^3 + {}_5C_4x^1(-2y)^4 + {}_5C_5(-2y)^5$

$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

12. $(3x - 4y)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(-4y)^1 + {}_3C_2(3x)^1(-4y)^2 + {}_3C_3(-4y)^3$

$= 27x^3 - 108x^2y + 144xy^2 - 64y^3$

13. $(x^2 + 3y)^4 = {}_4C_0(x^2)^4 + {}_4C_1(x^2)^3(3y)^1 + {}_4C_2(x^2)^2(3y)^2 + {}_4C_3(x^2)^1(3y)^3 + {}_4C_4(3y)^4$

$= x^8 + 12x^6y + 54x^4y^2 + 108x^2y^3 + 81y^4$

14. $(4x^2 - 2)^6 = {}_6C_0(4x^2)^6 + {}_6C_1(4x^2)^5(-2)^1 + {}_6C_2(4x^2)^4(-2)^2 + {}_6C_3(4x^2)^3(-2)^3 + {}_6C_4(4x^2)^2(-2)^4 + {}_6C_5(4x^2)^1(-2)^5 + {}_6C_6(-2)^6$

$= 4096x^{12} - 12{,}288x^{10} + 15{,}360x^8 - 10{,}240x^6 + 3840x^4 - 768x^2 + 64$

15. $(x^3 - y^3)^3 = {}_3C_0(x^3)^3 + {}_3C_1(x^3)^2(-y^3)^1 + {}_3C_2(x^3)^1(-y^3)^2 + {}_3C_3(-y^3)^3$

$= x^9 - 3x^6y^3 + 3x^3y^6 - y^9$

16. $(2x^4 + 5y^2)^5 = {}_5C_0(2x^4)^5 + {}_5C_1(2x^4)^4(5y^2)^1 + {}_5C_2(2x^4)^3(5y^2)^2 + {}_5C_3(2x^4)^2(5y^2)^3 + {}_5C_4(2x^4)^1(5y^2)^4 + {}_5C_5(5y^2)^5$

$= 32x^{20} + 400x^{16}y^2 + 2000x^{12}y^4 + 5000x^8y^6 + 6250x^4y^8 + 3125y^{10}$

17. $(x + 3)^5$; ${}_5C_2x^3(3)^2$

$10(9)x^3$

coefficient $= 90$

18. $(5 - y^2)^3 = {}_3C_2(5)(-y^2)^2$

$(3)(5)y^4$

coefficient $= 15$

19. $6 \times 12 \times 6 \times 8 = 3456$

20. ${}_{12}C_3 \cdot {}_6C_1 = \frac{12!}{9! \times 3!} \times \frac{6!}{5! \times 1!}$

$= 220 \times 6$

$= 1320$

Lesson 12.3

12.3 Guided Practice (p. 719)

1. geometric
2. B; The event with the higher probability is more likely to occur.
3. A theoretical probability is based on the number of outcomes of the event and the total number of possible outcomes. An experimental probability is determined through an experiment, survey, or historical data about an event. The theoretical probability of rolling a 5 using a 6-sided die is $\frac{1}{6}$. If you actually rolled a 6-sided die 100 times, the experimental probability of rolling a 5 would be the number of fives you rolled divided by 100.

4. $\frac{2}{6} = \frac{1}{3}$

5. $\frac{1}{6}$

6. $\frac{4}{6} = \frac{2}{3}$

7. $\frac{5}{6}$

8. $$\text{P(hitting shaded region)} = \frac{9\pi}{36} = \frac{\pi}{4} \approx 0.785$$

9. $$\text{P(hitting shaded region)} = \frac{(3\sqrt{2})^2}{3^2(\pi)} = \frac{18}{9\pi} \approx 0.637$$

10. $$\text{P(hitting shaded region)} = \frac{\frac{1}{2}(6)(6)}{6^2} = \frac{\frac{1}{2}(6)(6)}{6^2} = \frac{1}{2}$$

11. **a.** $$\text{P(under 25)} = \frac{94{,}507}{267{,}637} \approx 0.353$$

b. $$\text{P(at least 45)} = \frac{89{,}522}{267{,}637} \approx 0.334$$

12.3 Practice and Applications (pp. 719–722)

12. $\frac{10}{20} = 0.5$

13. $\frac{6}{20} = 0.3$

14. $\frac{4}{20} = 0.2$

15. $\frac{8}{20} = 0.4$

16. $\frac{6}{20} = 0.3$

17. $\frac{12}{20} = 0.6$

18. $\frac{1}{52} \approx 0.0192$

19. $\frac{4}{52} \approx 0.0769$

20. $\frac{13}{52} = 0.25$

21. $\frac{26}{52} = 0.5$

22. $\frac{48}{52} \approx 0.9231$

23. $\frac{12}{52} \approx 0.23$

24. experimental probability: $\frac{26}{120} \approx 0.217$

theoretical probability: $\frac{1}{6} \approx 0.17$

25. experimental probability: $\frac{37}{120} \approx 0.308$

theoretical probability: $\frac{2}{6} \approx 0.3$

26. experimental probability: $\frac{59}{120} \approx 0.4917$

theoretical probability: $\frac{3}{6} = 0.5$

27. experimental probability: $\frac{61}{120} \approx 0.5083$

theoretical probability: $\frac{3}{6} = 0.5$

28. experimental probability: $\frac{87}{120} = 0.725$

theoretical probability: $\frac{4}{6} \approx 0.67$

29. experimental probability: $\frac{105}{120} = 0.875$

theoretical probability: $\frac{5}{6} \approx 0.83$

30. $$P(\text{red center}) = \frac{2^2\pi}{24^2} \approx 0.0218$$

31. $$P(\text{white border}) = \frac{24^2 - (10^2 \cdot \pi)}{24^2} \approx 0.455$$

32. $$P(\text{red center or white border}) = \frac{2^2 \cdot \pi + (24^2 - 10^2 \cdot \pi)}{24^2} \approx 0.477$$

33. $$P(\text{four rings or red center}) = \frac{10^2 \cdot \pi}{24^2} = 0.545$$

34. $$P(\text{the yellow or green ring}) = \frac{8^2 \cdot \pi}{24^2} - \frac{4^2 \cdot \pi}{24^2} = 0.262$$

35. $\frac{1}{26} \approx 0.0385$

36. $\frac{4}{26} \approx 0.154$

37. $$\frac{{}_{44}C_3 \cdot {}_{54}C_4}{{}_{100}C_7} = \frac{13244 \cdot 316251}{1.60075 \times 10^{10}} = 0.2617$$

38. $$\frac{9}{100} \cdot \frac{2}{99} \cdot \frac{2}{98} \cdot \frac{4}{97} \cdot \frac{12}{96} \cdot \frac{2}{95} \cdot \frac{3}{94} = 1.285 \times 10^{-10};$$

39. $\frac{1}{{}_{51}C_6} = 5.6 \times 10^{-8}$

40. $\frac{1}{10 \cdot 9 \cdot 8} = 0.001$

41. a. $\frac{672}{1211} \approx 0.555$

b. $\frac{46}{1211} \approx 0.038$

42. a. $\frac{4}{17} \approx 0.235$

b. $\frac{11}{17} \approx 0.647$

43. a. $\frac{6{,}069{,}589}{115{,}070{,}274} \approx 0.0527$

b. $\frac{99{,}830{,}336}{115{,}070{,}274} \approx 0.868$

44. $\frac{3 - 1}{5.5 - 0} = \frac{2}{5.5} \approx 0.364$

45. $\frac{0.25}{4} = 0.0625$

46. $\frac{3.6^2\pi}{18^2\pi} = 0.04$

47. $\frac{\pi}{26(50)} \approx 0.00242$

48. C

49. $\frac{25\pi - \frac{1}{2}(10)(5)}{25\pi} = 0.682$

E

50. 4 of the 6 graphs intersect the x-axis. $\frac{4}{6} = \frac{2}{3} = 0.667$

12.3 Mixed Review (p. 722)

51. $18 - 35 = -17$

52. $-18 - 0 = -18$

53. $3 - (-16) = 19$

54. $5 - 2(1) + 3(-7), -18$

55. $0 - 1(-2) + 5(-11), -53$

56. $-1(34) - (-2)(32) + 2(10), 50$

57. $\frac{6xy^2}{5x^3y} \cdot \frac{10y^4}{9xy} = \frac{4y^4}{3x^3}$

58. $\frac{\cancel{(x+1)}\cancel{(x+2)}}{\cancel{(x-3)}\cancel{(x+2)}} \cdot \frac{x\cancel{(x-3)}}{(x-2)\cancel{(x+1)}} = \frac{x}{x-2}$

59. $\frac{\cancel{(5x-4)}\cancel{(5x+4)}}{\cancel{(5x-4)}} \cdot \frac{\cancel{(x-7)}(x+3)}{x\cancel{(5x+4)}\cancel{(x-7)}} = \frac{x+3}{x}$

60. $\frac{4x\cancel{(x-3)}}{-\cancel{(x-3)}\cancel{(x^2+3x+9)}} \cdot \cancel{(x^2+3x+9)} = -4x$

61. 3, 10, 17, 24, 31

62. $-1, -3, -9, -27, -81$

63. 2; 8; 512; 134,217,728; 2.418×10^{24}

64. 1, 1, 2, 3, 5

65. $-2, 0, 2, 2, 0$

66. $1, -2, -2, 4, -8$

67. $2^{20} - ({}_{20}C_0 + {}_{20}C_1 + {}_{20}C_2 + {}_{20}C_3 + {}_{20}C_4)$

$= 1048576 - (1 + 20 + 190 + 1140 + 4845)$

$= 1{,}042{,}380$

12.3 Activity (p. 723)

1. Experimental results may vary.

Theoretical probabilities: $\frac{1}{6}$ for each number

Answers may vary but experimental probabilities should be close to theoretical probabilities.

2. Experimental results may vary.

Theoretical probabilities: $\frac{1}{13}$ for each card.

Answers may vary but experimental probabilites should be close to theoretical probabilities.

3. Experimental results may vary.

As the number of trials increases, the experimental results should more closely resemble the theoretical results.

Lesson 12.4

12.4 Guided Practice (p. 727)

1. Two events are mutually exclusive if there are no outcomes shared by both of them.

2. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. When A and B are mutually exclusive, $P(A \text{ and } B) = 0$ so the formula becomes $P(A \text{ or } B) = P(A) + P(B)$.

3. Yes, A' is defined as all outcomes not in A, so there is no intersection between events in A and events not in A.

4. $P(A \text{ or } B) = 0.2 + 0.3$

$= 0.5$

5. $P(A \text{ or } B) = 0.5 + 0.5$

$= 1$

6. $P(A \text{ or } B) = \frac{3}{8} + \frac{1}{8}$

$= \frac{1}{2}$

7. $P(A \text{ or } B) = \frac{1}{3} + \frac{1}{4}$

$= \frac{4}{12} + \frac{3}{12}$

$= \frac{7}{12}$

8. $P(A \text{ or } B) = 0.5 + 0.4 - 0.3$

$= 0.6$

9. $P(A \text{ or } B) = \frac{2}{5} + \frac{3}{5} - \frac{1}{5}$

$= \frac{4}{5}$

10. $0.8 = 0.7 + 0.2 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.1$

11. $\frac{9}{16} = \frac{5}{16} + \frac{7}{16} - P(A \text{ and } B)$

$P(A \text{ and } B) = \frac{3}{16}$

12. $P(A') = 1 - 0.5$

$= 0.5$

13. $P(A') = 1 - 0.75$

$= 0.25$

14. $P(A') = 1 - \frac{1}{3}$

$= \frac{2}{3}$

15. $P(A') = 1 - \frac{4}{7}$

$= \frac{3}{7}$

12.4 Practice and Applications (pp. 727–729)

16. $0.5 = 0.4 + 0.35 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.25$

no

17. $P(A \text{ or } B) = 0.6 + 0.2 - 0.1$

$= 0.7$

no

18. $0.70 = 0.25 + P(B) - 0$

$P(B) = 0.45$

yes

19. $\frac{14}{17} = \frac{13}{17} + P(B) - \frac{6}{17}$

$\frac{7}{17} = P(B)$

no

20. $\frac{7}{12} = \frac{1}{3} + \frac{1}{4} - P(A \text{ and } B)$

$P(A \textit{ and } B) = 0$

yes

21. $P(A \text{ or } B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

$= \frac{9}{12} + \frac{4}{12} - \frac{3}{12}$

$= \frac{10}{12}$

$= \frac{5}{6}$

no

22. $P(A \text{ or } B) = 5 + 29 - 0$

$= 34\%$

yes

23. $10 = 30 + P(B) - 50$

$30\% = P(B)$

no

24. $32 = 16 + 24 - P(A \text{ and } B)$

$P(A \text{ and } B) = 8\%$

no

25. $P(A') = 1 - 0.34$

$= 0.66$

26. $P(A') = 1 - 0$

$= 1$

27. $P(A') = 1 - \frac{3}{4}$

$= \frac{1}{4}$

28. $P(A') = 1 - 1$

$= 0$

29. $P = \frac{1}{52}$

30. $P = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

31. $P = \frac{13}{52} + \frac{13}{52}$

$= \frac{26}{52}$

$= \frac{1}{2}$

32. $P = \frac{4}{52} + \frac{4}{52}$

$= \frac{8}{52}$

$= \frac{2}{13}$

33. $P = 0$

34. $P = \frac{4}{52} + \frac{12}{52}$

$= \frac{16}{52}$

$= \frac{4}{13}$

35. $P(\text{sum is not } 3) = 1 - \frac{2}{36}$

$= \frac{34}{36}$

$= \frac{17}{18} \approx 0.944$

36. $P = 1 - \frac{6}{36}$

$= \frac{30}{36}$

$= \frac{5}{6} \approx 0.833$

37. $P = 1 - \frac{8}{36}$

$= \frac{28}{36}$

$= \frac{7}{9} \approx 0.778$

38. $P = 1 - \frac{3}{36}$

$= \frac{33}{36}$

$= \frac{11}{12} \approx 0.917$

39. $P = 1 - \frac{1}{36}$

$= \frac{35}{36} \approx 0.972$

40. $P = 1 - \frac{14}{36}$

$= \frac{22}{36}$

$= \frac{11}{18} \approx 0.611$

41. *Sample answer:* Not 3: 0.933; ≥ 5: 0.825; not 3 or 7: 0.783; ≤ 10: 0.942; > 2: 0.983; < 8 or > 11: 0.600; the experimental results are very similar to the theoretical results.

42. $P = 0.33 + 0.50$

$= 0.83$

43. $P = 0.40 + 0.35$

$= 0.75$

44. $P = \frac{42}{79} + \frac{44}{79} - \frac{28}{79}$

$= \frac{58}{79} \approx 0.734$

45. $P = \frac{48}{162} + \frac{78}{162} - \frac{14}{162}$

$= \frac{112}{162} \approx 0.691$

46. $0.85 = 0.45 + 0.50 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.10$

47. $0.50 = 0.20 + 0.40 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.10$

48. $P(\text{at least 2 share a dorm}) = 1 - \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{8^5}$

$= 0.795$

49. $P(\text{at least 2 share a dorm}) = 1 - \frac{14 \cdot 13 \cdot 12 \cdot 11}{14^4}$

$= 0.375$

50. A; A' is all outcomes not in A, so A has all outcomes not in A'.

51. **a.** $P = 1 - \dfrac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361}{365^5}$

$= 0.0271$

b. $P = 1 - \dfrac{365 \cdot 364 \cdot 363 \cdot 362 \cdot 361 \cdot 360 \cdot 359 \cdot 358 \cdot 357 \cdot 356}{365^{10}}$

$= 0.117$

c. $1 - {}_{365}P_n\left(\dfrac{1}{365^n}\right)$

d. 23 (Probability ≈ 0.507)

52. $\dfrac{4}{1} = \dfrac{x}{5}$

$x = 20$

total marbles $= 5 + 20 = 25$

53. $\frac{4}{7}; \frac{7}{4}$

54. $P(E) = \dfrac{\text{odds in favor}}{1 + \text{odds in favor}}$

55. Odds in favor of Event $E = \dfrac{P(E)}{P(E')} = \dfrac{P(E)}{1 - P(E)}$

12.4 Mixed Review (p. 729)

56. $7^{2x} = 49^{16}$

$2x \log 7 = 16 \log 49$

$x = 8\dfrac{\log 49}{\log 7}$

$x = 16$

57. $9^x = 3^{x+1}$

$x \log 9 = (x + 1) \log 3$

$x = (x + 1)\dfrac{\log 3}{\log 9}$

$x = \dfrac{\log 3}{\log 9}$

$\dfrac{x}{x + 1} = 0.5$

$x = 0.5x + 0.5$

$0.5x = 0.5$

$x = 1$

58. $2^{4x+8} = 32^{19}$

$(4x + 8) \log 2 = 19 \log 32$

$4x + 8 = 19\dfrac{\log 32}{\log 2}$

$4x = 19(5) - 8$

$x = 21.75$

59. $5^x = 21$

$x \log 5 = \log 21$

$x = \dfrac{\log 21}{\log 5}$

$x = 1.892$

60. $10^{3x-1} - 13 = 8$

$10^{3x-1} = 21$

$(3x - 1) \log 10 = \log 21$

$3x - 1 = \dfrac{\log 21}{\log 10}$

$x = 0.774$

61. $72 = 91e^{-0.023x} + 50$

$\dfrac{22}{91} = e^{-0.023x}$

$\ln\left(\dfrac{22}{91}\right) = -0.023x$

$x = 61.73$

62. $r = \sqrt{(0 - 0)^2 + (7 - 0)^2}$

$= 7$

$x^2 + y^2 = 49$

63. $r = \sqrt{(3 - 0)^2 + (4 - 0)^2}$

$= 5$

$x^2 + y^2 = 25$

64. $r = \sqrt{(-1 - 0)^2 + (6 - 0)^2}$

$= \sqrt{37}$

$x^2 + y^2 = 37$

65. $r = \sqrt{(8 - 0)^2 + (-2 - 0)^2}$

$= \sqrt{68}$

$x^2 + y^2 = 68$

66. $r = \sqrt{(4 - 0)^2 + (4 - 0)^2}$

$= \sqrt{32}$

$x^2 + y^2 = 32$

67. $r = \sqrt{(3 - 0)^2 + (10 - 0)^2}$

$= \sqrt{109}$

$x^2 + y^2 = 109$

68. $r = \sqrt{(-4 - 0)^2 + (-2 - 0)^2}$

$= \sqrt{20}$

$x^2 + y^2 = 20$

69. $r = \sqrt{(16 - 0)^2 + (0)}$

$= 16$

$x^2 + y^2 = 256$

70. **a.** $26^3 \cdot 10^4 = 175{,}760{,}000$

b. $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78{,}624{,}000$

71. **a.** $26^4 \cdot 10^3 = 456{,}976{,}000$

b. $26 \cdot 25 \cdot 24 \cdot 23 \cdot 10 \cdot 9 \cdot 8 = 258{,}336{,}000$

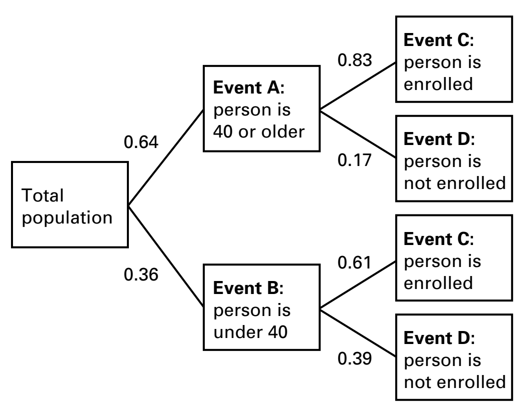
Lesson 12.5

12.5 Guided Practice (p. 734)

1. Events are independent if the occurrence of one has no effect on the occurrence of the other. Events are dependent if the occurrence of one affects the occurrence of the other. For example, rolls of a die are independent events because the number you get on the first roll does not affect the number on the second roll. The sum of the numbers on two rolls of a die is dependent. For example, if the first roll is not a six, there is a 0 probability that the sum will be twelve.
2. dependent
3. independent
4. $P(A \text{ and } B) = 0.3(0.9)$
   $= 0.27$
5. $0.06 = P(A)(0.3)$
   $P(A) = 0.2$
6. $0.15 = (0.75)P(B)$
   $P(B) = 0.2$
7. $P(A \text{ and } B) = (0.1)(0.8)$
   $P(A \text{ and } B) = 0.08$
8. $0.25 = P(A)(0.5)$
   $P(A) = 0.5$
9. $0.54 = (0.9)P(B/A)$
   $P(B/A) = 0.6$
10. $P = \frac{3}{18} \cdot \frac{10}{18} \cdot \frac{5}{18}$
    $= 0.026$
11. $P = \frac{10}{18} \cdot \frac{9}{18} \cdot \frac{8}{18}$
    $= 0.123$

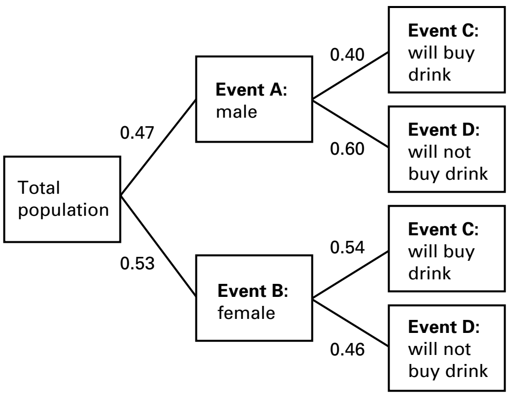
12.5 Practice and Applications (pp. 734–736)

12. $\frac{3}{16} \cdot \frac{4}{16} = 0.0468$
13. $\frac{3}{16} \cdot \frac{4}{16} = 0.0468$
14. $\frac{5}{16} \cdot \frac{3}{16} = 0.0586$
15. $\frac{4}{16} \cdot \frac{5}{16} = 0.0781$
16. $\frac{4}{16} \cdot \frac{5}{16} \cdot \frac{4}{16} = 0.0195$
17. $\frac{4}{16} \cdot \frac{3}{16} \cdot \frac{4}{16} = 0.0117$
18. **a.** $P = \frac{13}{52} \cdot \frac{13}{52}$
    $= 0.0625$
    b. $P = \frac{13}{52} \cdot \frac{13}{51}$
    $= 0.0637$
19. **a.** $P = \frac{4}{52} \cdot \frac{4}{52}$
    $= 0.0059$
    b. $P = \frac{4}{52} \cdot \frac{4}{51}$
    $= 0.0060$
20. **a.** $P = \frac{4}{52} \cdot \frac{12}{52}$
    $= 0.0178$
    b. $P = \frac{4}{52} \cdot \frac{12}{51}$
    $= 0.0181$
21. **a.** $P = \frac{12}{52} \cdot \frac{4}{52}$
    $= 0.0178$
    b. $P = \frac{12}{52} \cdot \frac{4}{51}$
    $= 0.0181$
22. **a.** $P = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$
    $= 0.00046$
    b. $P = \frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50}$
    $= 0.00048$
23. **a.** $P = \frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52}$
    $= 0.0156$
    b. $P = \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50}$
    $= 0.0153$
24. $P = 1 - \left(\frac{17}{25}\right)^3$
    $= 0.0328$
25. $P = (0.12)(0.06)(0.20)$
    $= 0.00144$
26. $P = \frac{{}_{89}C_8}{{}_{90}C_8}$
    $= 1 - \left(\frac{{}_{89}C_8}{{}_{90}C_8}\right)^{10}$
    $= 0.606$
27. $P = 1 - (939)^{10}$
    $= 0.467$
28. $P(\text{chip is not defective}) = 0.9999$
    $1 - (0.9999)^n = 0.5$
    $(0.9999)^n = 0.5$
    $n = \frac{\log 0.5}{\log 0.9999}$
    $n = 6931$
29. $1 - \left(1 - \frac{1}{{}_{42}C_6}\right)^n = 0.01$
    $n = \frac{\log 0.99}{\log\left(1 - \frac{1}{{}_{42}C_6}\right)} \approx 52{,}722$
30. $P = \frac{47{,}403}{86{,}602}$
    ≈ 0.547
31. $P = \frac{31{,}378}{33{,}470}$
    $= 0.937$
32. $P = \frac{0.261(1{,}867{,}000)}{2{,}561{,}000}$
    $= 0.19$
33. $P = \frac{20}{20} \cdot \frac{19}{20} \cdot \frac{18}{20} \cdot \frac{17}{20} \cdot \frac{16}{20}$
    $= 0.581$
34. $P = \left(\frac{1}{2}\right)^6$
    $= 0.0156$
35.



$P(\text{person is enrolled}) = (0.64)(0.83) + (0.36)(0.61)$
$= 0.5312 + 0.2196$
≈ 0.751

36.

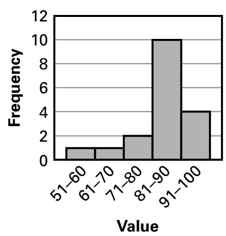


$$P(\text{will buy drink}) = (0.47)(0.4) + (0.53)(0.54) = 0.188 + 0.2862 = 0.474$$

37. B **38.** B **39.** $P = 0.0245$

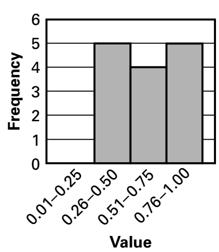
40.

51–60	1
61–70	1
71–80	2
81–90	10
91–100	4



41.

0.01–0.25	0
0.26–0.50	5
0.51–0.75	4
0.76–1.0	5



42. $\dfrac{5}{1 + 2e^{-x}} = 4$

$\dfrac{5}{4} - 1 = 2e^{-x}$

$\dfrac{1}{8} = e^{-x}$

$\ln\left(\dfrac{1}{8}\right) = -x$

$x = 2.08$

43. $\dfrac{10}{1 + 6e^{-x}} = 8$

$\dfrac{10}{8} - 1 = 6e^{-x}$

$\ln\left(\dfrac{1}{24}\right) = -x$

$x = 3.178$

44. $\dfrac{9}{1 + 3e^{-3x}} = 6$

$\dfrac{9}{6} - 1 = 3e^{-3x}$

$\ln\left(\dfrac{1}{6}\right) = -3x$

$x = 0.597$

45. $\dfrac{12}{1 + e^{-2x}} = 4$

$2 = e^{-2x}$

$\ln 2 = -2x$

$x = -0.347$

46. $\dfrac{1}{1 + 3e^{-5x}} = \dfrac{1}{2}$

$1 = 3e^{-5x}$

$\ln\left(\dfrac{1}{3}\right) = -5x$

$x = 0.220$

47. $\dfrac{70}{1 + 12e^{-10x}} = \dfrac{2}{9}$

$630 = 2 + 24e^{-10x}$

$\dfrac{628}{24} = e^{-10x}$

$\ln\left(\dfrac{157}{6}\right) = -10x$

$x = -0.326$

48. $(x + 1)^5 = {}_5C_0x^5 + {}_5C_1x^4 + {}_5C_2x^3 + {}_5C_3x^2 + {}_5C_4x + {}_5C_5$
$= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

49. $(2x - 1)^7 = {}_7C_0(2x)^7 + {}_7C_1(2x)^6(-1) + {}_7C_2(2x)^5(-1)^2$
$+ {}_7C_3(2x)^4(-1)^3 + {}_7C_4(2x)^3(-1)^4 + {}_7C_5(2x)^2(-1)^5$
$+ {}_7C_6(2x)(-1)^6 + {}_7C_7(-1)^7$
$= 128x^7 - 448x^6 + 672x^5 - 560x^4 + 280x^3$
$- 84x^2 + 14x - 1$

50. $(x - 3y)^4 = {}_4C_0x^4 + {}_4C_1x^3(-3y) + {}_4C_2x^2(-3y)^2$
$+ {}_4C_3x(-3y)^3 + {}_4C_4(-3y)^4$
$= x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$

51. $(x - 1)^6 = {}_6C_0x^6 + {}_6C_1x^5(-1) + {}_6C_2x^4(-1)^2$
$+ {}_6C_3x^3(-1)^3 + {}_6C_4x^2(-1)^4 + {}_6C_5x(-1)^5 + {}_6C_6(-1)^6$
$= x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

Quiz 2 (p. 737)

1. $P = \dfrac{12}{25} = 0.48$

2. $P = \dfrac{6}{25} = 0.24$

3. $P = \dfrac{18}{25} = 0.72$

4. $P = \dfrac{4\pi}{36\pi} = \dfrac{1}{9} \approx 0.111$

5. $P = \dfrac{36\pi - 36}{36\pi} = 0.682$

6. $P = \dfrac{36 - 4\pi}{36\pi} = 0.207$

7. $P(A \text{ or } B) = 0.7 + 0.2 - 0.1$

$P(A \text{ or } B) = 0.8$

8. $0.9 = 0.5 + 0.4 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0$

9. $P(A') = 1 - 0.25$

$P(A') = 0.75$

10. $P = \dfrac{5}{5} \cdot \dfrac{4}{5} \cdot \dfrac{3}{5} \cdot \dfrac{2}{5} \cdot \dfrac{1}{5}$

$P = 0.0384$

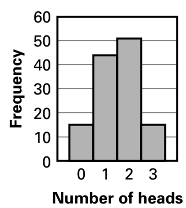
Math & History (p. 737)

1. Since A is closer to winning, A should receive all of the stakes and B should receive none of the stakes.
2. Since A has double the number of points, A should receive $\frac{2}{3}$ and B should receive $\frac{1}{3}$.
3. There are 10 ways in which the game can end: AA, ABA, BAA, ABBA, BABA, BBAA, BBB, BBAB, BABB, ABBB. However, the outcomes are not equally likely. For the outcome AA (with two more points scored), the probability is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. For each outcome with three more points scored, the probability is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. For each outcome with four more points scored, the probability is $(\frac{1}{2})^4 = \frac{1}{16}$. $P(\text{B wins}) = \frac{1}{8} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{5}{16}$, so $P(\text{A wins}) = 1 - \frac{5}{16} = \frac{11}{16}$. A should get $\frac{11}{16}$ of winnings and B should get $\frac{5}{16}$.

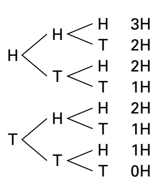
Lesson 12.6

12.6 Developing Concepts Activity (p. 738)
Exploring the Concept

3. Using the results from p. 738:

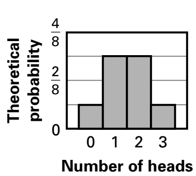


4.



5. $P(k = 0) = {}_3C_0(0.5)^0(0.5)^3 = 0.125$
   $P(k = 1) = {}_3C_1(0.5)^1(0.5)^2 = 0.375$
   $P(k = 2) = {}_3C_2(0.5)^2(0.5)^1 = 0.375$
   $P(k = 3) = {}_3C_3(0.5)^3(0.5)^0 = 0.125$

6.

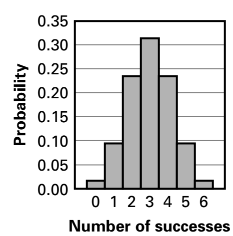


Drawing Conclusions

1. The shapes of the histograms of the experimental results and the theoretical probabilities look very similar. The theoretical probability histogram is perfectly symmetric and the experimental results are nearly, but not quite symmetric.
2. With more trials, the experimental results histogram would tend to look even more like the theoretical probability histogram. This is because with larger values of n you are likely to get results that are less skewed and closer to the theoretical distribution.
3. A histogram of results using weighed coins would differ from those using regular coins because the probabilities would be different. Since heads (60%) are more likely than tails (40%), you would expect to get more trials with one, two and three heads and fewer with 0 heads. This histogram would not be symmetric, or nearly symmetric, like the two histograms in steps 3 and 6.

12.6 Guided Practice (p. 742)

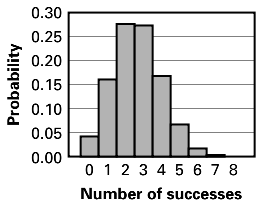
1. A binomial experiment is one in which each trial is independent and has only two outcomes—success or failure—with the probability of success constant.
2. No; the outcome cannot be reduced to 2 outcomes.
3. Symmetric; the values for 0 and 1 are the same as for 4 and 3, so the left and right halves will be mirror images.
4. $P(7 \text{ successes}) = {}_{12}C_7(0.7)^7(0.3)^5$
   $= 0.158$
5. $P(\text{at most 3 successes})$
   $= {}_{14}C_0(0.45)^0(0.55)^{14} + {}_{14}C_1(0.45)^1(0.55)^{13} + {}_{14}C_2(0.45)^2(0.55)^{12} + {}_{14}C_3(0.45)^3(0.55)^{11}$
   $= 0.00023 + 0.0027 + 0.0141 + 0.0462$
   $= 0.063$
6. $P(k = 0) = {}_6C_0(0.5)^0(0.5)^6 = 0.0156$
   $P(k = 1) = {}_6C_1(0.5)^1(0.5)^5 = 0.0938$
   $P(k = 2) = {}_6C_2(0.5)^2(0.5)^4 = 0.2344$
   $P(k = 3) = {}_6C_3(0.5)^3(0.5)^3 = 0.3125$
   $P(k = 4) = {}_6C_4(0.5)^4(0.5)^2 = 0.2344$
   $P(k = 5) = {}_6C_5(0.5)^5(0.5)^1 = 0.0938$
   $P(k = 6) = {}_6C_6(0.5)^6(0.5)^0 = 0.0156$



   The most likely number of successes is 3.
7. $P(k = 0) = {}_8C_0(0.33)^0(0.67)^8 = 0.0406$
   $P(k = 1) = {}_8C_1(0.33)^1(0.67)^7 = 0.16$
   $P(k = 2) = {}_8C_2(0.33)^2(0.67)^6 = 0.2758$
   $P(k = 3) = {}_8C_3(0.33)^3(0.67)^5 = 2717$
   $P(k = 4) = {}_8C_4(0.33)^4(0.67)^4 = 0.1673$
   $P(k = 5) = {}_8C_5(0.33)^5(0.67)^3 = 0.0659$
   $P(k = 6) = {}_8C_6(0.33)^6(0.67)^2 = 0.0162$
   $P(k = 7) = {}_8C_7(0.33)^7(0.67)^1 = 0.0023$
   $P(k = 8) = {}_8C_8(0.33)^8(0.67)^0 = 0.0001$

—CONTINUED—

7. **—CONTINUED—**



The most likely number of successes is 2.

8. $P(k = 0) = {}_{10}C_0(0.25)^0(0.75)^{10} = 0.0563$

$P(k = 1) = {}_{10}C_1(0.25)^1(0.75)^9 = 0.1877$

$P(k = 2) = {}_{10}C_2(0.25)^2(0.75)^8 = 0.2816$

$P(k = 3) = {}_{10}C_3(0.25)^3(0.75)^7 = 0.2503$

$P(k = 4) = {}_{10}C_4(0.25)^4(0.75)^6 = 0.1460$

$P(k = 5) = {}_{10}C_5(0.25)^5(0.75)^5 = 0.0584$

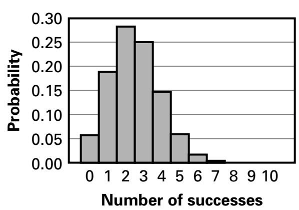
$P(k = 6) = {}_{10}C_6(0.25)^6(0.75)^4 = 0.0162$

$P(k = 7) = {}_{10}C_7(0.25)^7(0.75)^3 = 0.0031$

$P(k = 8) = {}_{10}C_8(0.25)^8(0.75)^2 = 0.0004$

$P(k = 9) = {}_{10}C_9(0.25)^9(0.75)^1 = 0.00003$

$P(k = 10) = {}_{10}C_{10}(0.25)^{10}(0.75)^0 = 0.000001$



The most likely number of successes is 2.

9. $P(k = 0) = {}_{15}C_0(0 \cdot 30)^0(0.70)^{15} = 0.0047$

$P(k = 1) = {}_{15}C_1(0 \cdot 30)^1(0.70)^{14} = 0.0305$

$P(k = 2) = {}_{15}C_2(0 \cdot 30)^2(0.70)^{13} = 0.0916$

$P(k = 3) = {}_{15}C_3(0 \cdot 30)^3(0.70)^{12} = 0.1700$

$P(k = 4) = {}_{15}C_4(0 \cdot 30)^4(0.70)^{11} = 0.2186$

$P(k \le 4)$

$= 0.0047 + 0.0305 + 0.0916 + 0.1700 + 0.2186$

$= 0.5154$

No; the probability of 4 or fewer students buying rings is much greater than 0.1 if the claim is true. Therefore, you should not reject the claim.

12.6 Practice and Applications (pp. 742–744)

10. $P(1 \text{ success}) = {}_{20}C_1(0.5)^1(0.5)^{19}$

$= 20(0.5)(0.5)^{19}$

$= 0.0000191$

11. $P(3 \text{ successes}) = {}_{20}C_3(0.5)^3(0.5)^{17}$

$= 0.00109$

12. $P(5 \text{ successes}) = {}_{20}C_5(0.5)^5(0.5)^{15}$

$= 0.0148$

13. $P(9 \text{ successes}) = {}_{20}C_9(0.5)^9(0.5)^{11}$

$= 0.160$

14. $P(10 \text{ successes}) = {}_{20}C_{10}(0.5)^{10}(0.5)^{10}$

$= 0.176$

15. $P(11 \text{ successes}) = {}_{20}C_{11}(0.5)^{11}(0.5)^9$

$= 0.160$

16. $P(\text{ 15 successes}) = {}_{20}C_{15}(0.5)^{15}(0.5)^5$

$= 0.0148$

17. $P(17 \text{ successes}) = {}_{20}C_{17}(0.5)^{17}(0.5)^3$

$= 0.00109$

18. $P(0 \text{ correct}) = {}_{30}C_0(0.25)^0(0.75)^{30}$

$= 0.000179$

19. $P(2 \text{ correct}) = {}_{30}C_2(0.25)^2(0.75)^{28}$

$= 0.00863$

20. $P(5 \text{ correct}) = {}_{30}C_5(0.25)^5(0.75)^{25}$

$= 0.105$

21. $P(10 \text{ correct}) = {}_{30}C_{10}(0.25)^{10}(0.75)^{20}$

$= 0.0909$

22. $P(15 \text{ correct}) = {}_{30}C_{15}(0.25)^{15}(0.75)^{15}$

$= 0.00193$

23. $P(20 \text{ correct}) = {}_{30}C_{20}(0.25)^{20}(0.75)^{10}$

$= 0.00000154$

24. $P(25 \text{ correct}) = {}_{30}C_{25}(0.25)^{25}(0.75)^5$

$= 3 \times 10^{-11}$

25. $P(30 \text{ correct}) = {}_{30}C_{30}(0.25)^{30}(0.75)^0$

$= 8.67 \times 10^{-19}$

26. $P(k \ge 3)$

$= {}_5C_3(0.2)^3(0.8)^2 + {}_5C_4(0.2)^4(0.8)^1 + {}_5C_5(0.2)^5(0.8)^0$

$= 0.0512 + 0.0064 + 0.00032$

$= 0.0579$

27. $P(k \le 2)$

$= {}_6C_0(0.5)^0(0.5)^6 + {}_6C_1(0.5)^1(0.5)^5 + {}_6C_2(0.5)^2(0.5)^4$

$= 0.015625 + 0.09375 + 0.234375$

$= 0.344$

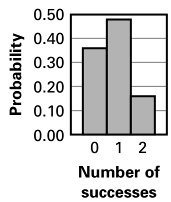
28. $P(k \le 1) = {}_9C_0(0.15)^0(0.85)^9 + {}_9C_1(0.15)^1(0.85)^8$

$= 0.2316 + 0.3679$

$= 0.599$

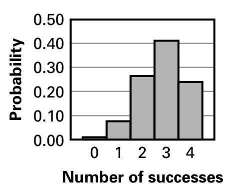
29. $P(k \le 5) = {}_{12}C_0(0.64)^0(0.36)^{12} + {}_{12}C_1(0.64)^1(0.36)^{11}$
$+ {}_{12}C_2(0.64)^2(0.36)^{10} + {}_{12}C_3(0.64)^3(0.36)^9$
$+ {}_{12}C_4(0.64)^4(0.36)^8 + {}_{12}C_5(0.64)^5(0.36)^7$
$= 0.0000047 + 0.000101 + 0.00099 + 0.00586 + 0.0234 + 0.0667$
$= 0.097$

30. $P(k = 0) = {}_2C_0(0.4)^0(0.6)^2 = 0.36$
$P(k = 1) = {}_2C_1(0.4)^1(0.6)^1 = 0.48$
$P(k = 2) = {}_2C_2(0.4)^2(0.6)^0 = 0.16$



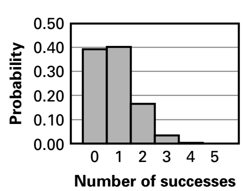
The most likely number of successes is 1.

31. $P(k = 0) = {}_4C_0(0.7)^0(0.3)^4 = 0.0081$
$P(k = 1) = {}_4C_1(0.7)^1(0.3)^3 = 0.0756$
$P(k = 2) = {}_4C_2(0.7)^2(0.3)^2 = 0.2646$
$P(k = 3) = {}_4C_3(0.7)^3(0.3)^1 = 0.4116$
$P(k = 4) = {}_4C_4(0.7)^4(0.3)^0 = 0.2401$



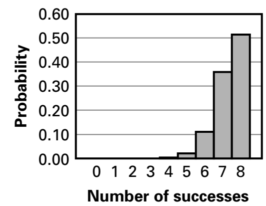
The most likely number of successes is 3.

32. $P(k = 0) = {}_5C_0(0.17)^0(0.83)^5 = 0.394$
$P(k = 1) = {}_5C_1(0.17)^1(0.83)^4 = 0.403$
$P(k = 2) = {}_5C_2(0.17)^2(0.83)^3 = 0.165$
$P(k = 3) = {}_5C_3(0.17)^3(0.83)^2 = 0.034$
$P(k = 4) = {}_5C_4(0.17)^4(0.83)^1 = 0.003$
$P(k = 5) = {}_5C_5(0.17)^5(0.83)^0 = 0.0001$



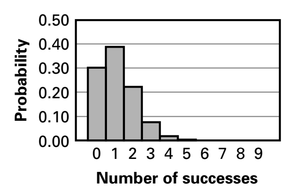
The most likely number of successes is 1.

33. $P(k = 0) = {}_8C_0(0.92)^0(0.08)^8 = 1.7 \times 10^{-9}$
$P(k = 1) = {}_8C_1(0.92)^1(0.08)^7 = 1.5 \times 10^{-7}$
$P(k = 2) = {}_8C_2(0.92)^2(0.08)^6 = 6.2 \times 10^{-6}$
$P(k = 3) = {}_8C_3(0.92)^3(0.08)^5 = 1.4 \times 10^{-4}$
$P(k = 4) = {}_8C_4(0.92)^4(0.08)^4 = 0.002$
$P(k = 5) = {}_8C_5(0.92)^5(0.08)^3 = 0.019$
$P(k = 6) = {}_8C_6(0.92)^6(0.08)^2 = 0.109$
$P(k = 7) = {}_8C_7(0.92)^7(0.08)^1 = 0.357$
$P(k = 8) = {}_8C_8(0.92)^8(0.08)^0 = 0.513$



The most likely number of successes is 8.

34. $P(k = 0) = {}_9C_0(0.125)^0(0.875)^9 = 0.301$
$P(k = 1) = {}_9C_1(0.125)^1(0.875)^8 = 0.387$
$P(k = 2) = {}_9C_2(0.125)^2(0.875)^7 = 0.221$
$P(k = 3) = {}_9C_3(0.125)^3(0.875)^6 = 0.074$
$P(k = 4) = {}_9C_4(0.125)^4(0.875)^5 = 0.016$
$P(k = 5) = {}_9C_5(0.125)^5(0.875)^4 = 0.002$
$P(k = 6) = {}_9C_6(0.125)^6(0.875)^3 = 0.0002$
$P(k = 7) = {}_9C_7(0.125)^7(0.875)^2 = 1.3 \times 10^{-5}$
$P(k = 8) = {}_9C_8(0.125)^8(0.875)^1 = 4.7 \times 10^{-7}$
$P(k = 9) = {}_9C_9(0.125)^9(0.875)^0 = 7.4 \times 10^{-9}$

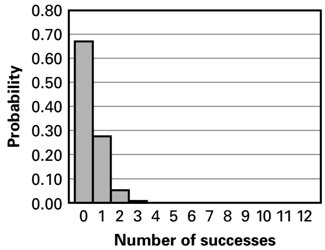


The most likely number of successes is 1.

35. $P(k = 0) = {}_{12}C_0(0.033)^0(0.967)^{12} = 0.669$
$P(k = 1) = {}_{12}C_1(0.033)^1(0.967)^{11} = 0.274$
$P(k = 2) = {}_{12}C_2(0.033)^2(0.967)^{10} = 0.051$
$P(k = 3) = {}_{12}C_3(0.033)^3(0.967)^9 = 0.0058$
$P(k = 4) = {}_{12}C_4(0.033)^4(0.967)^8 = 0.00045$
$P(k = 5) = {}_{12}C_5(0.033)^5(0.967)^7 = 0.000025$
$P(k = 6) = {}_{12}C_6(0.033)^6(0.967)^6 = 9.8 \times 10^{-7}$
$P(k = 7) = {}_{12}C_7(0.033)^7(0.967)^5 = 2.9 \times 10^{-9}$
$P(k = 8) = {}_{12}C_8(0.033)^8(0.967)^4 = 2 \times 10^{-11}$
$P(k = 9) = {}_{12}C_9(0.033)^9(0.967)^3 = 9.2 \times 10^{-12}$
$P(k = 10) = {}_{12}C_{10}(0.033)^{10}(0.967)^2 = 9.5 \times 10^{-14}$
$P(k = 11) = {}_{12}C_{11}(0.033)^{11}(0.967)^1 = 5.9 \times 10^{-16}$
$P(k = 12) = {}_{12}C_{12}(0.033)^{12}(0.967)^0 = 1.7 \times 10^{-18}$

—CONTINUED—

35. **—CONTINUED—**



The most likely number of successes is 0.

36. $P(12 \text{ successes}) = {}_{12}C_{12}(0.9)^{12}(0.1)^0$

$= 0.282$

37. $P = 0.0025$

$n = 20$

$P(k = 2) = {}_{20}C_2(0.0025)^2(0.9975)^{18}$

$= 0.00114$

38. $P(k = 0) = {}_8C_0(0.5)^0(0.5)^8 = 0.004$

$P(k = 1) = {}_8C_1(0.5)^1(0.5)^7 = 0.031$

$P(k = 2) = {}_8C_2(0.5)^2(0.5)^6 = 0.109$

$P(k = 3) = {}_8C_3(0.5)^3(0.5)^5 = 0.22$

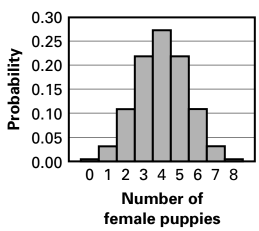
$P(k = 4) = {}_8C_4(0.5)^4(0.5)^4 = 0.27$

$P(k = 5) = {}_8C_5(0.5)^5(0.5)^3 = 0.22$

$P(k = 6) = {}_8C_6(0.5)^6(0.5)^2 = 0.109$

$P(k = 7) = {}_8C_7(0.5)^7(0.5)^1 = 0.031$

$P(k = 8) = {}_8C_8(0.5)^8(0.5)^0 = 0.004$



39. $P(k \geq 5) = 0.22 + 0.109 + 0.031 + 0.004$

$= 0.364$

40. $P(k = 0) = {}_7C_0(0.34)^0(0.66)^7 = 0.055$

$P(k = 1) = {}_7C_1(0.34)^1(0.66)^6 = 0.197$

$P(k = 2) = {}_7C_2(0.34)^2(0.66)^5 = 0.304$

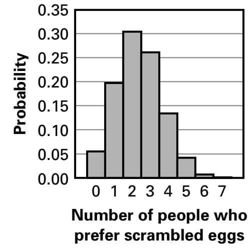
$P(k = 3) = {}_7C_3(0.34)^3(0.66)^4 = 0.261$

$P(k = 4) = {}_7C_4(0.34)^4(0.66)^3 = 0.134$

$P(k = 5) = {}_7C_5(0.34)^5(0.66)^2 = 0.042$

$P(k = 6) = {}_7C_6(0.34)^6(0.66)^1 = 0.007$

$P(k = 7) = {}_7C_7(0.34)^7(0.66)^0 = 0.00053$



41. $P(k \leq 3) = 0.055 + 0.197 + 0.304 + 0.261$

$= 0.817$

42. $P(k = 0) = {}_{12}C_0(0.07)^0(0.93)^{12} = 0.4186$

$P(k = 1) = {}_{12}C_1(0.07)^1(0.93)^{11} = 0.3781$

$P(k = 2) = {}_{12}C_2(0.07)^2(0.93)^{10} = 0.1565$

$P(k = 3) = {}_{12}C_3(0.07)^3(0.93)^9 = 0.0393$

$P(k = 4) = {}_{12}C_4(0.07)^4(0.93)^8 = 0.0067$

$P(k = 5) = {}_{12}C_5(0.07)^5(0.93)^7 = 0.0008$

$P(k = 6) = {}_{12}C_6(0.07)^6(0.93)^6 = 0.00007$

$P(k = 7) = {}_{12}C_7(0.07)^7(0.93)^5 = 4.5 \times 10^{-6}$

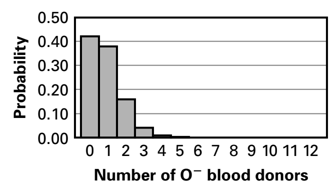
$P(k = 8) = {}_{12}C_8(0.07)^8(0.93)^4 = 2.1 \times 10^{-7}$

$P(k = 9) = {}_{12}C_9(0.07)^9(0.93)^3 = 7.1 \times 10^{-9}$

$P(k = 10) = {}_{12}C_{10}(0.07)^{10}(0.93)^2 = 1.6 \times 10^{-10}$

$P(k = 11) = {}_{12}C_{11}(0.07)^{11}(0.93)^1 = 2.2 \times 10^{-12}$

$P(k = 12) = {}_{12}C_{12}(0.07)^{12}(0.93)^0 = 1.4 \times 10^{-14}$



43. 0

44. $P(k = 0) = 0.001$

$P(k = 1) = 0.008$

$P(k = 2) = 0.0308$

$P(k = 3) = 0.075$

$P(k = 4) = 0.132$

$P(k = 5) = 0.176$

$P(k = 6) = 0.185$

$P(k = 7) = 0.159$

$P(k = 8) = 0.112$

$P(k = 9) = 0.067$

$P(k = 10) = 0.033$

$P(k = 11) = 0.014$

$P(k = 12) = 0.005$

$P(k = 13) = 0.002$

$P(k = 14) = 0.0004$

$P(k = 15) = 0.00009$

$P(k = 16) = 0.00002$

$P(k = 17) = 2.7 \times 10^{-6}$

$P(k = 18) = 3.5 \times 10^{-7}$

$P(k = 19) = 3.7 \times 10^{-8}$

$P(k = 20) = 3.1 \times 10^{-9}$

$P(k = 21) = 1.9 \times 10^{-10}$

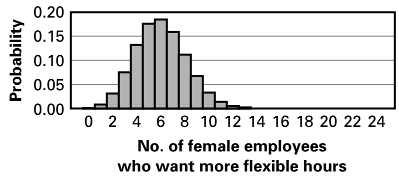
$P(k = 22) = 8.8 \times 10^{-12}$

$P(k = 23) = 2.6 \times 10^{-13}$

$P(k = 24) = 3.6 \times 10^{-15}$

—CONTINUED—

44. —CONTINUED—



45. 6

46. $P(k \geq 2) = 1 - [P(k = 0) + P(k = 1)]$
$= 1 - [{}_{30}C_0(0.05)^0(0.95)^{30} + {}_{30}C_1(0.05)^1(0.95)^{29}] = 0.446$

Do not reject the claim because the probability that two or more computers will fail is 0.446 which is much greater than 0.1.

47. $P(k \leq 3) = {}_{10}C_0(0.75)^0(0.25)^{10} + {}_{10}C_1(0.75)^1(0.25)^9$
$+ {}_{10}C_2(0.75)^2(0.25)^8 + {}_{10}C_3(0.75)^3(0.25)^7 = 0.00351$

Reject the claim because the probability of these findings is 0.0035, which is less than 0.01.

48. $P(k = 2) + P(k = 3) + \ldots + P(k = 12)$
$= 1 - [P(k = 13) + P(k = 14) + \ldots + P(k = 20)]$
$= 1 - [{}_{20}C_{13}(0.8)^{13}(0.2)^7 + {}_{20}C_{14}(0.8)^{14}(0.2)^6 + \ldots$
$+ {}_{20}C_{20}(0.8)^{20}(0.2)^0] = 0.032$

Reject the claim because the probability is 0.032 that 12 or fewer people would prefer the new juice and $0.032 < 0.1$.

49. $P(k \geq 3) = {}_5C_3(0.310)^3(0.69)^2 + {}_5C_4(0.310)^4(0.69)^1$
$+ {}_5C_5(0.310)^5(0.69)^0$
$= 0.1418 + 0.0319 + 0.0029 = 0.1766$

D

50. $P(k \leq 3) = {}_{10}C_3(0.60)^3(0.40)^7 + {}_{10}C_2(0.60)^2(0.40)^8$
$+ {}_{10}C_1(0.60)^1(0.40)^9 + {}_{10}C_{10}(0.60)^0(0.40)^{10}$
$= 0.0425 + 0.0106 + 0.0016 + 0.0001 = 0.055$

C

51. mean $= 2.221$, $\dfrac{\text{mean}}{h} = 0.370 =$ probability of success on any trial. So, mean $= n \times$ (probability of success) $= np$.

52. range: 2

deviation: 0.632

53. range: 11

deviation: 4.155

54. range: 0.6

deviation: 0.232

55. range: 19

deviation: 6.708

56. $x^2 + y^2 = 1$

$y = x$

$y^2 + y^2 = 1$

$2y^2 = 1$

$y^2 = \dfrac{1}{2}$

$y = \pm\dfrac{\sqrt{2}}{2}$

$\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right), \left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

$x^2 + \dfrac{1}{2} = 1$

$x^2 = \dfrac{1}{2}$

$x = \pm\dfrac{1}{2}$

$x = \pm\dfrac{\sqrt{2}}{2}$

57. $x^2 + y^2 = 74$

$x - y = -2$

$x = y - 2$

$(y - 2)^2 + y^2 = 74$

$y^2 - 4y + 4 + y^2 = 74$

$2y^2 - 4y - 70 = 0$

$y^2 - 2y - 35 = 0$

$(y - 7)(y + 5) = 0$

$y = 7 \quad y = -5$

Points of intersection $(-7, -5), (5, 7)$

58. $x^2 + 4y^2 = 1$

$y = x + 1$

$x^2 + 4(x + 1)^2 = 1$

$x^2 + 4x^2 + 8x + 4 = 1$

$5x^2 + 8x + 3 = 0$

$(5x + 3)(x + 1) = 0$

$x = -\frac{3}{5} \qquad x = -1$

$y = -\frac{3}{5} + 1 \qquad y = -1 + 1$

$y = \frac{2}{5} \qquad y = 0$

points of intersection: $\left(-\frac{3}{5}, \frac{2}{5}\right), (-1, 0)$

59. $5x^2 + y^2 = 10$

$(x^2 + y^2 = 9)$ Mult. by $-1 \rightarrow$

$$\begin{aligned} 5x^2 + y^2 &= 10 \\ -x^2 - y^2 &= -9 \\ \hline 4x^2 \quad &= 1 \end{aligned}$$

$x^2 = \dfrac{1}{4}$

$x = \pm\dfrac{1}{2}$

$5\left(\dfrac{1}{2}\right)^2 + y^2 = 10$

$y^2 = 10 - \dfrac{5}{4}$

$y^2 = \dfrac{35}{4}$

$y = \pm\dfrac{\sqrt{35}}{2}$

Points of intersection: $\left(\pm\dfrac{1}{2}, \pm\dfrac{\sqrt{35}}{2}\right)$

60. $x^2 - y^2 = 49$

$y = 7$

$x^2 - 49 = 49$

$x^2 = 98$

$x = \pm 9.899$

Points of intersection: $(\pm 9.899, 7)$

61. $2x^2 - 3y^2 = 6$

$y = 3x + 1$

$2x^2 - 3(3x + 1)^2 = 6$

$2x^2 - 27x^2 - 18x - 3 = 6$

$-25x^2 - 18x + 3 = 0$

$x = \dfrac{18 \pm \sqrt{324 - 4(-25)(-9)}}{2(-25)}$

$= \dfrac{18 \pm \sqrt{-576}}{-50}$

no solution

62. $a_1 = 3$

$a_n = a_{n-1} + 2(n - 1)$

63. $a_1 = 4$

$a_n = a_{n-1} \times 10$

64. $a_1 = 80$

$a_n = a_{n-1} - 20$

65. $a_1 = 1, a_2 = 3$

$a_n = a_{n-1} \times a_{n-2}$

66. $a_1 = 160$

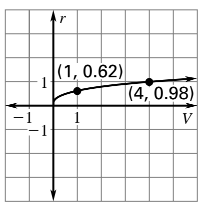
$a_n = \frac{1}{4} \times a_{n-1}$

67. $a_1 = 1, a_2 = 2$

$a_n = a_{n-1} + a_{n-2}$

68. $r = 0.62\sqrt[3]{V}$

$113 \text{ in.}^3 \approx V$



V	r
0	0
1	0.62
4	0.98
8	1.24

12.6 Technology Activity (p. 745)

1. $P(k = 0) = {}_{14}C_0(0.80)^0(0.20)^{14} = 1.6 \times 10^{-10}$

$P(k = 1) = {}_{14}C_1(0.80)^1(0.20)^{13} = 9.2 \times 10^{-9}$

$P(k = 2) = {}_{14}C_2(0.80)^2(0.20)^{12} = 2.4 \times 10^{-7}$

$P(k = 3) = {}_{14}C_3(0.80)^3(0.20)^{11} = 3.8 \times 10^{-6}$

$P(k = 4) = {}_{14}C_4(0.80)^4(0.20)^{10} = 4.2 \times 10^{-5}$

$P(k = 5) = {}_{14}C_5(0.80)^5(0.20)^9 = 3.4 \times 10^{-4}$

$P(k = 6) = {}_{14}C_6(0.80)^6(0.20)^8 = 0.002$

$P(k = 7) = {}_{14}C_7(0.80)^7(0.20)^7 = 0.009$

$P(k = 8) = {}_{14}C_8(0.80)^8(0.20)^6 = 0.032$

$P(k = 9) = {}_{14}C_9(0.80)^9(0.20)^5 = 0.086$

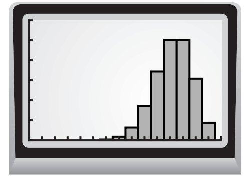
$P(k = 10) = {}_{14}C_{10}(0.80)^{10}(0.20)^4 = 0.172$

$P(k = 11) = {}_{14}C_{11}(0.80)^{11}(0.20)^3 = 0.25$

$P(k = 12) = {}_{14}C_{12}(0.80)^{12}(0.20)^2 = 0.25$

$P(k = 13) = {}_{14}C_{13}(0.80)^{13}(0.20)^1 = 0.15$

$P(k = 14) = {}_{14}C_{14}(0.80)^{14}(0.20)^0 = 0.04$



2. $P(k = 0) = {}_{10}C_0(0.21)^0(0.79)^{10} = 0.095$

$P(k = 1) = {}_{10}C_1(0.21)^1(0.79)^9 = 0.252$

$P(k = 2) = {}_{10}C_2(0.21)^2(0.79)^8 = 0.301$

$P(k = 3) = {}_{10}C_3(0.21)^3(0.79)^7 = 0.213$

$P(k = 4) = {}_{10}C_4(0.21)^4(0.79)^6 = 0.099$

$P(k = 5) = {}_{10}C_5(0.21)^5(0.79)^5 = 0.032$

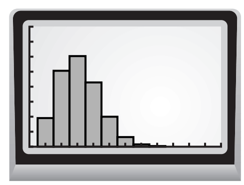
$P(k = 6) = {}_{10}C_6(0.21)^6(0.79)^4 = 0.007$

$P(k = 7) = {}_{10}C_7(0.21)^7(0.79)^3 = 0.001$

$P(k = 8) = {}_{10}C_8(0.21)^8(0.79)^2 = 0.0001$

$P(k = 9) = {}_{10}C_9(0.21)^9(0.79)^1 = 6.27 \times 10^{-6}$

$P(k = 10) = {}_{10}C_{10}(0.21)^{10}(0.79)^0 = 1.67 \times 10^{-7}$



Lesson 12.7

12.7 Guided Practice (p. 749)

1. normal distribution

2. the mean

3. 68%, 95%, 99.7%

4. $P = 2(0.34) + 2(0.135)$

$= 0.95$

5. $P = 2(0.34) + 2(0.135) + 2(0.0235)$

$= 0.997$

6. $P = 2(0.34) + (0.135)$

$= 0.815$

7. $P = 0.0015 + 0.0235 + 0.135 + 0.34$

$= 0.5$

8. $P = 0.0015 + 0.0235$

$= 0.025$

9. $P = 0.0015 + 0.0235 + 0.135$

$= 0.16$

10. $\bar{x} = 10(0.5)$

$= 5$

$\sigma = \sqrt{5(1 - 0.5)}$

$= 1.58$

11. $\bar{x} = 17(0.3)$
$= 5.1$
$\sigma = \sqrt{5.1(1 - 0.3)}$
$= 1.89$

12. $\bar{x} = 28(0.2)$
$= 5.6$
$\sigma = \sqrt{5.6(0.8)}$
$= 2.12$

13. $\bar{x} = 20(0.25)$
$= 5$
$\sigma = \sqrt{5(0.75)}$
$= 1.94$

14. $\bar{x} = 12(0.42)$
$= 5.04$
$\sigma = \sqrt{5.04(0.58)}$
$= 1.71$

15. $\bar{x} = 30(0.17)$
$= 5.1$
$\sigma = \sqrt{5.1(0.83)}$
$= 2.06$

16. $\bar{x} = 9.2$
$\sigma = \sqrt{9.2(0.98)}$
$= 3$
$P = 2(0.34) + 2(0.135) + 0.0235 + 0.0015$
$= 0.975$

17. $P = 0.135 + 0.0235 + 0.0015$
$= 0.16$

18. $P = 2(0.34) + 0.135 + 0.0235$
$= 0.839$

19. $P = 0.34 + 0.135 + 0.0235 + 0.0015$
$= 0.5$
50%

20. $P = 0.0235 + 0.135$
$= 0.1585$
15.85%

21. $P = 0.0015 + 0.0235$
$= 0.025$
2.5%

22. $P = 0.135 + 0.135$
$= 0.27$
27%

23. $\bar{x} = 22$ $\quad P = 0.34 + 0.34$
$\sigma = 3$ $\quad = 0.68$

24. $P = 0.34 + 0.135 + 0.0235$
$= 0.4985$

25. $P = 2(0.135) + 2(0.34) + 0.0235$
$= 0.9735$

26. $P = 2(0.34) + 0.135 + 0.0235 + 0.0015$
$= 0.84$

27. $P = 0.68 + 0.135 + 0.0235 + 0.0015$
$= 0.84$

28. $P = 0.84 + 0.135$
$= 0.975$

29. $P = 0.135 + 0.0235 + 0.0015$
$= 0.16$
$P(\text{all are 71 or greater}) = (0.16)^3$
≈ 0.004096

30. $P = 0.0015 + 0.0235$
$= 0.025$
$P(\text{all are less than 50}) = (0.025)^4$
≈ 0.00000039

31. $P = 0.68 + 0.135$
$= 0.815$
$P(\text{both are between 57 and 78}) = (0.815)^2$
≈ 0.665

32. $\bar{x} = 18(0.7)$
$= 12.6$
$\sigma = \sqrt{12.6(0.3)}$
$= 1.94$

33. $\bar{x} = 50(0.1)$
$= 5$
$\sigma = \sqrt{5(0.9)}$
$= 2.12$

34. $\bar{x} = 32(0.8)$
$= 25.6$
$\sigma = \sqrt{25.6(0.2)}$
$= 2.26$

35. $\bar{x} = 49(0.12)$
$= 5.88$
$\sigma = \sqrt{5.88(0.88)}$
$= 2.27$

36. $\bar{x} = 24(0.67)$
$= 16.08$
$\sigma = \sqrt{16.08(0.33)}$
$= 2.3$

37. $\bar{x} = 140(0.06)$
$= 8.4$
$\sigma = \sqrt{8.4(0.94)}$
$= 2.81$

38. $P = 0.135 + 0.0235 + 0.0015$
$= 0.16$
16%

39. $P = 2(0.135) + 2(0.34)$
$= 0.95$

40. $P = 0.0015$

41. $P = 0.34 + 0.135 + 0.0235 + 0.0015$
$= 0.5$
50%

42. $P = 0.0015 + 0.0235$
$= 0.025$

43. $P = 2(0.34) + 0.135 + 0.0235$
$= 0.839$

44. $P = 0.0015 + 0.0235$
$= 0.025$
$P(\text{all 3 are shorter than 58.6}) = (0.025)^3$
$= 0.0000156$

45. $P = 0.68$
$P(\text{5 are between 61.3 and 66.7}) = (0.68)^5$
$= 0.145$

46. $P = (0.0015)^4$
$= 5.06 \times 10^{-12}$

47. $P = (0.0235 + 0.0015)(0.0235 + 0.0015)$
$= 0.000625$

48. $P = (0.0015 + 0.0235 + 0.135)^5$
$= 0.000105$

49. $P = (0.68)^2$
$= 0.462$

50. $P = 0.0015$

51. $P = 0.0015 + 0.0235 + 0.135$
$= 0.16$

52. $P = 2(0.34) + 2(0.135) + 0.0235$
$= 0.974$

53. $P = 0.5 + 0.34$
$= 0.84$

54. $P = 2(0.34) + 2(0.135)$
$= 0.95$

55. $P = 0.5 + 0.34 + 0.135 + 0.0235$
$= 0.999$

56. a. $P = 0.5 + 0.34$
84%

b. $P = 0.5 + 0.34 + 0.135$
97.5%

c. ACT

d. You could translate ACT scores into percentiles of the normal distribution. Those percentiles could then be transformed into SAT scores of the equivalent percentile.

57. The normal distribution is a good approximation unless the binomial distribution is very skewed, such as when $p < 0.25$ or $p > 0.75$. The normal distribution approximates the binomial distribution best when $p = 0.5$.

12.7 Mixed Review (p. 752)

58. $\sqrt{9^3} = \sqrt{729}$
$= \pm 27$

59. $\sqrt[4]{256^3} = \sqrt[4]{16{,}777{,}216}$
$= \pm 64$

60. $\frac{1}{\sqrt{49}} = \pm\frac{1}{7}$

61. $\sqrt[3]{125} = 5$

62. $\sqrt{81} = \pm 9$

63. $\left(\sqrt[4]{625}\right)^2 = (5)^2 = 25$

64. $(0, \pm 4), (\pm 2, 0), \left(0, \pm 2\sqrt{3}\right)$

65. $(0, \pm 13), (\pm 12, 0), (0, \pm 5)$

66. $\left(0, \pm\sqrt{10}\right), \left(\pm\sqrt{5}, 0\right), \left(0, \pm\sqrt{5}\right)$

67. $\left(0, \pm\sqrt{21}\right), \left(\pm\sqrt{6}, 0\right), \left(0, \pm\sqrt{15}\right)$

68. $4x^2 + 9y^2 = 36$
$\frac{x^2}{9} + \frac{y^2}{4} = 1$
$(0, \pm 2), (\pm 3, 0), \left(\pm\sqrt{5}, 0\right)$

69. $10x^2 + 7y^2 = 70$
$\frac{x^2}{7} + \frac{y^2}{10} = 1$
$\left(0, \pm\sqrt{10}\right), \left(\pm\sqrt{7}, 0\right), \left(0, \pm\sqrt{3}\right)$

70. $P = 1 - \frac{3}{36}$
$= \frac{11}{12}$

71. $P = 1 - \frac{3}{36}$
$= \frac{11}{12}$

72. $P = 1 - \frac{3}{36}$
$= \frac{11}{12}$

73. $P = 1 - \frac{3}{36}$
$= \frac{11}{12}$

Quiz 3 (p. 752)

1. $P(k = 0) = {}_{50}C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{50}$
$= 0.000110$

2. $P(k = 1) = {}_{50}C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{49}$
$= 0.00110$

3. $P(k = 8) = {}_{50}C_8\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^{42}$
$= 0.151$

4. $P(k = 17) = {}_{50}C_{17}\left(\frac{1}{6}\right)^{17}\left(\frac{5}{6}\right)^{33}$
$= 0.00142$

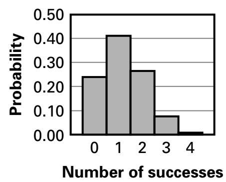
5. $P(k = 25) = {}_{50}C_{25}\left(\frac{1}{6}\right)^{25}\left(\frac{5}{6}\right)^{25}$
$= 4.66 \times 10^{-8}$

6. $P(k = 33) = {}_{50}C_{33}\left(\frac{1}{6}\right)^{33}\left(\frac{5}{6}\right)^{17}$
$= 9.29 \times 10^{-15}$

7. $P(k = 42) = {}_{50}C_{42}\left(\frac{1}{6}\right)^{42}\left(\frac{5}{6}\right)^{8}$
$= 2.59 \times 10^{-25}$

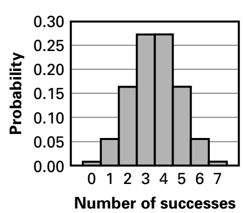
8. $P(k = 50) = {}_{50}C_{50}\left(\frac{1}{6}\right)^{50}\left(\frac{5}{6}\right)^{0}$
$= 1.24 \times 10^{-39}$

9. $P(k = 0) = {}_4C_0(0.3)^0(0.7)^4 = 0.2401$
$P(k = 1) = {}_4C_1(0.3)^1(0.7)^3 = 0.4116$
$P(k = 2) = {}_4C_2(0.3)^2(0.7)^2 = 0.2646$
$P(k = 3) = {}_4C_3(0.3)^3(0.7)^1 = 0.0756$
$P(k = 4) = {}_4C_4(0.3)^4(0.7)^0 = 0.0081$



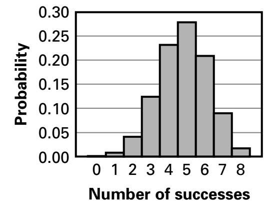
The most likely number of successes is 1.

10. $P(k = 0) = {}_7C_0(0.5)^0(0.5)^7 = 0.008$
$P(k = 1) = {}_7C_1(0.5)^1(0.5)^6 = 0.055$
$P(k = 2) = {}_7C_2(0.5)^2(0.5)^5 = 0.16$
$P(k = 3) = {}_7C_3(0.5)^3(0.5)^4 = 0.27$
$P(k = 4) = {}_7C_4(0.5)^4(0.5)^3 = 0.27$
$P(k = 5) = {}_7C_5(0.5)^5(0.5)^2 = 0.16$
$P(k = 6) = {}_7C_6(0.5)^6(0.5)^1 = 0.055$
$P(k = 7) = {}_7C_7(0.5)^7(0.5)^0 = 0.008$



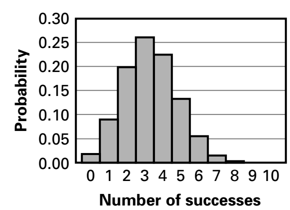
The most likely number of successes is 3 or 4 since 3 and 4 are equally likely.

11. $P(k = 0) = {}_8C_0(0.6)^0(0.4)^8 = 0.00066$
$P(k = 1) = {}_8C_1(0.6)^1(0.4)^7 = 0.008$
$P(k = 2) = {}_8C_2(0.6)^2(0.4)^6 = 0.04$
$P(k = 3) = {}_8C_3(0.6)^3(0.4)^5 = 0.12$
$P(k = 4) = {}_8C_4(0.6)^4(0.4)^4 = 0.23$
$P(k = 5) = {}_8C_5(0.6)^5(0.4)^3 = 0.28$
$P(k = 6) = {}_8C_6(0.6)^6(0.4)^2 = 0.21$
$P(k = 7) = {}_8C_7(0.6)^7(0.4)^1 = 0.09$
$P(k = 8) = {}_8C_8(0.6)^8(0.4)^0 = 0.02$



The most likely number of successes is 5.

12. $P(k = 0) = {}_{10}C_0(0.33)^0(0.67)^{10} = 0.018$
$P(k = 1) = {}_{10}C_1(0.33)^1(0.67)^9 = 0.09$
$P(k = 2) = {}_{10}C_2(0.33)^2(0.67)^8 = 0.20$
$P(k = 3) = {}_{10}C_3(0.33)^3(0.67)^7 = 0.26$
$P(k = 4) = {}_{10}C_4(0.33)^4(0.67)^6 = 0.23$
$P(k = 5) = {}_{10}C_5(0.33)^5(0.67)^5 = 0.13$
$P(k = 6) = {}_{10}C_6(0.33)^6(0.67)^4 = 0.05$
$P(k = 7) = {}_{10}C_7(0.33)^7(0.67)^3 = 0.015$
$P(k = 8) = {}_{10}C_8(0.33)^8(0.67)^2 = 0.0028$
$P(k = 9) = {}_{10}C_9(0.33)^9(0.67)^1 = 0.0003$
$P(k = 10) = {}_{10}C_{10}(0.33)^{10}(0.67)^0 = 0.000015$

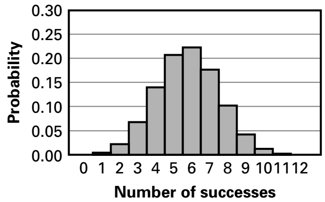


The most likely number of successes is 3.

13. $P(k = 0) = {}_{12}C_0(0.48)^0(0.52)^{12} = 0.00039$
$P(k = 1) = {}_{12}C_1(0.48)^1(0.52)^{11} = 0.004$
$P(k = 2) = {}_{12}C_2(0.48)^2(0.52)^{10} = 0.02$
$P(k = 3) = {}_{12}C_3(0.48)^3(0.52)^9 = 0.07$
$P(k = 4) = {}_{12}C_4(0.48)^4(0.52)^8 = 0.14$
$P(k = 5) = {}_{12}C_5(0.48)^5(0.52)^7 = 0.21$
$P(k = 6) = {}_{12}C_6(0.48)^6(0.52)^6 = 0.22$
$P(k = 7) = {}_{12}C_7(0.48)^7(0.52)^5 = 0.18$
$P(k = 8) = {}_{12}C_8(0.48)^8(0.52)^4 = 0.10$
$P(k = 9) = {}_{12}C_9(0.48)^9(0.52)^3 = 0.04$
$P(k = 10) = {}_{12}C_{10}(0.48)^{10}(0.52)^2 = 0.01$
$P(k = 11) = {}_{12}C_{11}(0.48)^{11}(0.52)^1 = 0.002$
$P(k = 12) = {}_{12}C_{12}(0.48)^{12}(0.52)^0 = 0.00015$

—CONTINUED—

13. —CONTINUED—



The most likely number of successes is 6.

14. $P(k = 0) = {}_{15}C_0(0.21)^0(0.79)^{15} = 0.03$

$P(k = 1) = {}_{15}C_1(0.21)^1(0.79)^{14} = 0.12$

$P(k = 2) = {}_{15}C_2(0.21)^2(0.79)^{13} = 0.22$

$P(k = 3) = {}_{15}C_3(0.21)^3(0.79)^{12} = 0.25$

$P(k = 4) = {}_{15}C_4(0.21)^4(0.79)^{11} = 0.20$

$P(k = 5) = {}_{15}C_5(0.21)^5(0.79)^{10} = 0.12$

$P(k = 6) = {}_{15}C_6(0.21)^6(0.79)^{9} = 0.05$

$P(k = 7) = {}_{15}C_7(0.21)^7(0.79)^{8} = 0.02$

$P(k = 8) = {}_{15}C_8(0.21)^8(0.79)^{7} = 0.005$

$P(k = 9) = {}_{15}C_9(0.21)^9(0.79)^{6} = 0.0010$

$P(k = 10) = {}_{15}C_{10}(0.21)^{10}(0.79)^{5} = 0.0002$

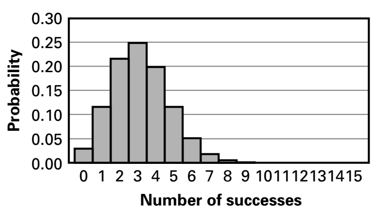
$P(k = 11) = {}_{15}C_{11}(0.21)^{11}(0.79)^{4} = 1.9 \times 10^{-5}$

$P(k = 12) = {}_{15}C_{12}(0.21)^{12}(0.79)^{3} = 1.7 \times 10^{-6}$

$P(k = 13) = {}_{15}C_{13}(0.21)^{13}(0.79)^{2} = 1.0 \times 10^{-7}$

$P(k = 14) = {}_{15}C_{14}(0.21)^{14}(0.79)^{1} = 3.8 \times 10^{-9}$

$P(k = 15) = {}_{15}C_{15}(0.21)^{15}(0.79)^{0} = 6.8 \times 10^{-11}$



The most likely number of successes is 3.

15. $P = 0.68$

16. $P = 0.34 + 0.135 + 0.0235 = 0.4985$

17. $P = 2(0.34) + 2(0.135) + 0.0235 = 0.9735$

18. $P = 0.50$

19. $P = 0.0015 + 0.0235 + 0.135 = 0.16$

20. $P = 0.0015$

21. $P = 1 - (0.015 + 0.067 + 0.15 + 0.21 + 0.21 + 0.16 + 0.10) = 0.088$

Yes; there is a 0.088 chance of getting 19 or fewer out of 26 and $0.088 < 0.1$ so reject the survey's findings.

22. $\bar{x} = 625(0.64)$

$= 400$

$P = 0.50$

Chapter 12 Extension *(pp. 753–754)*

1. Outcomes from A's perspective:

	A 1	A 2	A 3
B 1	0	1	−1
B 2	−1	0	1
B 3	1	−1	0

Expected value for A $= \frac{1}{3}(0) + \frac{1}{3}(1) + \frac{1}{3}(-1) = 0.$ Similarly for B. Game is fair.

2. Outcomes from A's perspective:

	A 1	A 2	A 3
B 1	4	−3	4
B 2	−3	4	−5
B 3	4	−5	4

Expected value for A $= \frac{5}{9}(4) + \frac{2}{9}(-3) + \frac{2}{9}(-5) = \frac{4}{9}.$

Expected value for B $= -\frac{4}{9}$. Game is not fair.

3. Expected value $=$

$\left(\frac{1}{42 \cdot {}_{49}C_5}\right)(45{,}000{,}000) + \left(1 - \frac{1}{42 \cdot {}_{49}C_5}\right)(-1) \approx -\$.44$

4. Expected value $= \$5(0.0002) + \$3{,}000(0.0000004) + \$7{,}000(0.00000008) + \$50{,}000(0.000000003) + \$1{,}000{,}000(0.000000002)$

$= \$.0049$

Chapter 12 Review *(pp. 756–758)*

1. zip codes $= 10^5 = 100{,}000$

2. ways $= 4 \cdot 3 \cdot 2 \cdot 1 = 24$

3. ${}_6P_6 = 720$

4. ${}_8P_4 = 1680$

5. ${}_5P_1 = 5$

6. ${}_9P_3 = 504$

7. ${}_{10}P_6 = 151{,}200$

8. ${}_4P_4 = 24$

9. ${}_9C_2 = 36$

10. ${}_7C_1 = 7$

11. ${}_5C_3 = 10$

12. ${}_8C_7 = 8$

13. ${}_{10}C_{10} = 1$

14. ${}_{13}C_5 = 1287$

15. $(x + 4)^3 = {}_3C_0x^3 + {}_3C_1x^2(4)^1 + {}_3C_2x^1(4)^2 + {}_3C_3(4)^3$

$= x^3 + 12x^2 + 48x + 64$

16. $(x - 10)^5 = {}_5C_0x^5 + {}_5C_1x^4(-10)^1 + {}_5C_2x^3(-10)^2 + {}_5C_3x^2(-10)^3 + {}_5C_4x^1(-10)^4 + {}_5C_5(-10)^5$

$= x^5 - 50x^4 + 1000x^3 - 10{,}000x^2 + 50{,}000x - 100{,}000$

17. $(x - 3y)^7 = {}_7C_0x^7 + {}_7C_1x^6(-3y)^1 + {}_7C_2x^5(-3y)^2 + {}_7C_3x^4(-3y)^3 + {}_7C_4x^3(-3y)^4 + {}_7C_5x^2(-3y)^5 + {}_7C_6x^1(-3y)^6 + {}_7C_7(-3y)^7$

$= x^7 - 21x^6y + 189x^5y^2 - 945x^4y^3 + 2835x^3y^4 - 5103x^2y^5 + 5103xy^6 - 2187y^7$

18. $(2x + y^2)^4 = {}_4C_0(2x)^4 + {}_4C_1(2x)^3(y^2)^1 + {}_4C_2(2x)^2(y^2)^2 + {}_4C_3(2x)^1(y^2)^3 + {}_4C_4(y^2)^4$

$= 16x^4 + 32x^3y^2 + 24x^2y^4 + 8xy^6 + y^8$

19. $P = \frac{3}{8}$

20. $P = \frac{7}{8}$

21. experimental probability $= \frac{90}{200}$

$= \frac{9}{20}$

theoretical probability $= \frac{100}{200}$

$= \frac{1}{2}$

The theoretical probability is 0.5. The experimental probability is 0.45. They are very similar.

22. $P = \dfrac{4^2 - 2^2}{4^2}$

$= 0.75$

23. $P(A \text{ or } B) = (0.25 + 0.2) - 0.15$

$= 0.3$

24. $\frac{1}{2} = \frac{2}{5} + \frac{1}{10} - P(A \text{ and } B)$

$P(A \text{ and } B) = 0$

25. $P(A') = 100\% - 99\%$

$= 1\%$

26. **a.** $P(A \text{ and } B) = \frac{4}{12} \cdot \frac{6}{12}$

$= 0.166$

b. $P(A \text{ and } B) = \frac{4}{12} \cdot \frac{6}{11}$

$= 0.182$

27. **a.** $P(A \text{ and } B) = \frac{2}{12} \cdot \frac{4}{12}$

$= 0.056$

b. $P(A \text{ and } B) = \frac{2}{12} \cdot \frac{4}{11}$

$= 0.061$

28. **a.** $P(A \text{ and } B) = \frac{4}{12} \cdot \frac{4}{12}$

$= 0.111$

b. $P(A \text{ and } B) = \frac{4}{12} \cdot \frac{3}{11}$

$= 0.0909$

29. $P(k = 3) = {}_{10}C_3(0.5)^3(1 - 0.5)^7$

$= 0.117$

30. $P(k = 5) = {}_{10}C_5(0.5)^5(0.5)^5$

$= 0.246$

31. $P(k = 9) = {}_{10}C_9(0.5)^1(0.5)^9$

$= 0.00977$

32. $P(k = 6) = {}_{10}C_6(0.5)^4(0.5)^6$

$= 0.205$

33. $P(k = 1) = {}_{10}C_6(0.5)^4(0.5)^6$

$= 0.00977$

34. $P(k = 10) = {}_{10}C_1(0.5)^9(0.5)^1$

$= 0.000977$

35. $P = 0.34 + 0.34$

$= 0.68$

36. $P = 0.5$

37. $P = 0.0235 + 0.0015$

$= 0.025$

38. $P = 0.0235$

Chapter 12 Test *(p. 759)*

1. ${}_4P_3 = 24$

2. ${}_{11}P_5 = 55{,}440$

3. ${}_{14}P_2 = 182$

4. ${}_9C_6 = 84$

5. ${}_{17}C_3 = 680$

6. ${}_5C_4 = 5$

7. $P = \dfrac{7!}{2! \cdot 2!}$

$= 1260$

8. $(x + 4)^6 = {}_6C_0x^6 + {}_6C_1x^5(4)^1 + {}_6C_2x^4(4)^2 + {}_6C_3x^3(4)^3 + {}_6C_4x^2(4)^4 + {}_6C_5x^1(4)^5 + {}_6C_6(4)^6$

$= x^6 + 24x^5 + 240x^4 + 1280x^3 + 3840x^2 + 6144x + 4096$

9. $(2x - 2)^5 = {}_5C_0(2x)^5 + {}_5C_1(2x)^4(-2)^1 + {}_5C_2(2x)^3(-2)^2 + {}_5C_3(2x)^2(-2)^3 + {}_5C_4(2x)^1(-2)^4 + {}_5C_5(-2)^5$

$= 32x^5 - 160x^4 + 320x^3 - 320x^2 + 160x - 32$

10. $(x + 8)^3 = {}_3C_0x^3 + {}_3C_1x^2(8)^1 + {}_3C_2x^1(8)^2 + {}_3C_3(8)^3$

$= x^3 + 24x^2 + 192x + 512$

11. $(x^2 + 1)^4 = {}_4C_0(x^2)^4 + {}_4C_1(x^2)^3 + {}_4C_2(x^2)^2 + {}_4C_3(x^2)^1 + {}_4C_4$

$= x^8 + 4x^6 + 6x^4 + 4x^2 + 1$

12. $(x + y^2)^5 = {}_5C_0x^5 + {}_5C_1x^4(y^2)^1 + {}_5C_2x^3(y^2)^2 + {}_5C_3x^2(y^2)^3 + {}_5C_4x^1(y^2)^4 + {}_5C_5(y^2)^5$

$= x^5 + 5x^4y^2 + 10x^3y^4 + 10x^2y^6 + 5xy^8 + y^{10}$

13. $(3x - y)^3 = {}_3C_0(3x)^3 + {}_3C_1(3x)^2(-y)^1 + {}_3C_2(3x)^1(-y)^2 + {}_3C_3(-y)^3$

$= 27x^3 - 27x^2y + 9xy^2 - y^3$

14. $P = \frac{26}{52} = 0.5$

15. $P = \frac{4}{52} = 0.0769$

16. $P = \frac{2}{52} = 0.0385$

17. $P = \frac{4}{52} = 0.0769$

18. $P = \frac{13}{52} = 0.25$

19. $P = \frac{1}{52} = 0.0192$

20. $100 = 80 + 20 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0\%$

21. $0.82 = P(A) + 0.7 - 0.05$

$P(A) = 0.17$

22. $P(A') = 1 - \frac{1}{4} = \frac{3}{4}$

23. $P(A \text{ and } B) = 0.25(0.75) = 0.1875$

24. $P(A \text{ and } B) = 0.3(0.4) = 12\%$

25. $0.32 = P(A)(0.8)$

$P(A) = 0.4$

26. $P(k \geq 7) = {}_{10}C_7(0.5)^3(0.5)^7 + {}_{10}C_8(0.5)^8(0.5)^2 + {}_{10}C_9(0.5)^9(0.5)^1 + {}_{10}C_{10}(0.5)^{10} \approx 0.172$

27. 68%; 95%

28. choices $= (2)(4)(2) = 16$

29. $\text{ways} = \dfrac{9!}{4! \cdot 5!} = 126$

30. $\text{land} = \dfrac{57\text{ million}}{197\text{ million}} = 0.289$

$\text{water} = \dfrac{140\text{ million}}{197\text{ million}} = 0.711$

31. $P(k \le 2) = 0$

Reject the claim because the probability of this many no-shows is less than 0.05.

32. $P = 0.0015 + 0.0235 = 0.025$

Chapter 12 Standardized Test *(pp. 760–761)*

1. $\text{ways} = {}_{20}P_2 = 380$

D

2. $\text{ways} = {}_{20}C_2 = 380$

D

3. $\text{coeffiecient} = {}_8C_3(2x)^5(5)^3 = 224{,}000$

E

4. $P = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{2}{5}$ B

5. $P = \frac{1}{2}(25\pi) \div 100 = \frac{\pi}{8}$ C

6. $P(A \text{ or } B) = 0.7 + 0.23 - 0.1 = 0.83$ D

7. $P(\text{at least 2 are the same}) = 1 - P(\text{all are different}) =$

$1 - \frac{5}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \approx 0.96$ D

8. $P(A \text{ and } B) = 0.8(0.7) = 0.56$ C

9. $P(A \text{ and } B) = 0.5(0.5) = 0.25$ A

10. $P = \frac{{}_5C_2}{2^5} = \frac{10}{32} = \frac{5}{16}$ D

11. $P = 0.0235 + 0.0015 = 0.025$ B

12. $\sigma = \sqrt{119(0.7)(0.3)} \approx 5$ B

13. A **14.** C

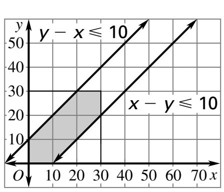
15. a. $0 \le x \le 30$

$0 \le y \le 30$

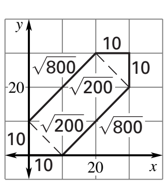
b. $x - y \le 10$

$y - x \le 10$

c.



d.



$$P = \frac{\left(\sqrt{800} \cdot \sqrt{200}\right) + 2\left(\frac{1}{2}(10)(10)\right)}{30 \cdot 30}$$

$$P = \frac{400 + 100}{900} = \frac{5}{9}$$

16. a. $z = \dfrac{x_{\text{old}} - 70}{8}$

b. $z = \dfrac{x_{\text{new}} - 85}{5}$

c. $\dfrac{x_{\text{old}} - 70}{8} = \dfrac{x_{\text{new}} - 85}{5}$

$\dfrac{5(x_{\text{old}} - 70)}{8} = x_{\text{new}} - 85$

$x_{\text{new}} = \dfrac{5 \times (x_{\text{old}} - 70)}{8} + 85$

d. $x_{\text{new}} = \dfrac{5 \times (70 - 70)}{8} + 85$

$x_{\text{new}} = 85$

This makes sense because your score was the mean originally (70), so it should also be the mean (85) in the new distribution.

e. 81.5%; 75 to 90; You could use the normal distribution having a mean of 85 and standard deviation of 5. The range is from two standard deviations below the mean to one standard deviation above the mean.

Cumulative Practice, Chs. 1–12 *(pp. 762–763)*

1. $-4x + 5 = 33$

$-4x = 28$

$x = -7$

2. $\frac{1}{4}(x - 7) = 2$

$x - 7 = 8$

$x = 15$

3. $|x - 3| = 11$

$x - 3 = 11$ $x - 3 = -11$

$x = 14$ $x = -8$

4. $|8 - 3x| = 1$

$8 - 3x = 1$ $8 - 3x = -1$

$-3x = -7$ $-3x = -9$

$x = \frac{7}{3}$ $x = 3$

5. $x^2 + 7x + 10 = 0$

$(x + 2)(x + 5) = 0$

$x + 2 = 0$ $x + 5 = 0$

$x = -2$ $x = -5$

6. $5x^2 - 13 = 32$

$5x^2 = 45$

$x^2 = 9$

$x = \pm 3$

7. $-x^2 = 16$

$x^2 = -16$

$x = \pm 4i$

8. $x^2 + 6x - 5 = 0$

$x = \dfrac{-6 \pm \sqrt{36 - 4(1)(-5)}}{2(1)}$

$= \dfrac{-6 \pm \sqrt{56}}{2}$

$= -3 \pm \sqrt{14}$

9. $4x^2 - x + 1 = 0$

$x = \dfrac{1 \pm \sqrt{1 - 4(4)(1)}}{2(4)}$

$= \dfrac{1 \pm \sqrt{-15}}{8}$

$= \dfrac{1 \pm i\sqrt{15}}{8}$

10. $x^3 - 27 = 0$

$x^3 = 27$

$x = 3$

11. $x^3 + x^2 - 4x - 4 = 0$

$x^2(x + 1) - 4(x + 1) = 0$

$(x - 2)(x + 2)(x + 1) = 0$

$x - 2 = 0 \quad x + 2 = 0 \quad x + 1 = 0$

$x = 2 \quad x = -2 \quad x = -1$

12. $\sqrt{x + 5} = 7$

$x + 5 = 49$

$x = 44$

13. $8(x - 3)^{\frac{3}{2}} = 1$

$64(x - 3)^3 = 1$

$(x - 3)^3 = \frac{1}{64}$

$x - 3 = \frac{1}{4}$

$x = 3\frac{1}{4}$

14. $4^{x+1} = 64$

$(x + 1)\log 4 = \log 64$

$x + 1 = 3$

$x = 2$

15. $\log 4x = 2$

$\log 4 + \log x = 2$

$\log x = 2 - \log 4$

$x = 10^{2 - \log 4}$

$x = 25$

16. $\dfrac{1}{x - 4} = \dfrac{6}{x + 6}$

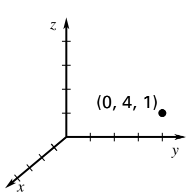
$x + 6 = 6(x - 4)$

$x + 6 = 6x - 24$

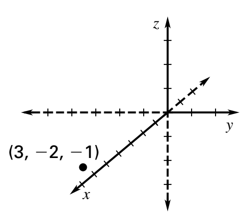
$30 = 5x$

$x = 6$

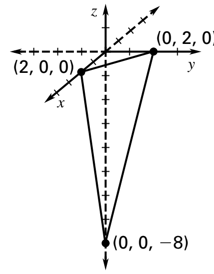
17.



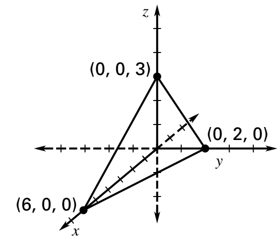
18.



19. $4x + 4y - z = 8$



20. $2x + 6y + 4z = 12$



21. $20 - 2 = 18$

22. $-9 + 60 = 51$

23. $3(5) + 4(-2) - (-2)(-1) = 15 - 8 - 2 = 5$

24. $-1(6) + 4(3) - 18(-5) = -6 + 12 + 90 = 96$

25. $x = -2, y = 20$

$y = \dfrac{k}{x}$

$20 = \dfrac{k}{-2}$

$-40 = k$

$y = \dfrac{-40}{x}$

when $x = 2, y = -20$

26. $y = \dfrac{k}{x}$

$9 = \dfrac{k}{\frac{1}{3}}$

$3 = k$

$y = \dfrac{3}{x}$

when $x = 2, y = \dfrac{3}{2}$

27. $y = \dfrac{k}{x}$

$-\dfrac{4}{5} = \dfrac{k}{20}$

$-16 = k$

$y = -\dfrac{16}{x}$

when $x = 2, y = -8$

28. $y = \dfrac{k}{x}$

$4 = \dfrac{k}{1}$

$4 = k$

$y = \dfrac{4}{x}$

when $x = 2, y = 2$

29. $z = kxy$

$-4 = k(2)(3)$

$-\dfrac{2}{3} = k$

$z = -\dfrac{2}{3}xy$

when $x = -1$ and $y = 5, z = \frac{10}{3}$

30. $z = kxy$

$24 = k(-2)(6)$

$-2 = k$

$z = -2xy$

when $x = -1$ and $y = 5, z = 10$

31. $z = kxy$

$\dfrac{3}{8} = k\left(\dfrac{1}{2}\right)\left(\dfrac{1}{4}\right)$

$3 = k$

$z = 3xy$

when $x = -1$ and $y = 5, z = -15$

32. $d = \sqrt{(-9-0)^2 + (2-0)^2}$

$= \sqrt{81 + 4}$

$= \sqrt{85}$

≈ 9.22

midpoint $= \left(-\frac{9}{2}, 1\right)$

33. $d = \sqrt{(5-0)^2 + (0-8)^2}$

$= \sqrt{25 + 64}$

$= \sqrt{89}$

≈ 9.43

midpoint $= \left(\frac{5}{2}, 4\right)$

34. $d = \sqrt{(-5-3)^2 + (14+8)^2}$

$= \sqrt{64 + 484}$

≈ 23.41

midpoint $= (-1, 3)$

35. $d = \sqrt{(-2-5)^2 + (-3-1)^2}$

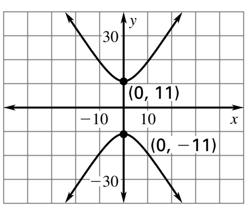
$= \sqrt{49 + 16}$

$= \sqrt{65}$

≈ 8.06

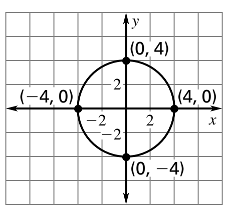
midpoint $= \left(\frac{3}{2}, -1\right)$

36.



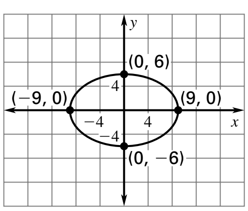
$\frac{y^2}{121} - \frac{x^2}{49} = 1$

37.



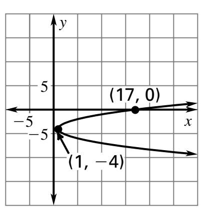
$x^2 + y^2 = 16$

38.



$\frac{x^2}{81} + \frac{y^2}{36} = 1$

39.



$(y + 4)^2 + 1 = x$

40. $y^2 = 8x$

41. $(x - 2)^2 + (y + 2)^2 = 9$

42. $\frac{x^2}{64} + \frac{y^2}{25} = 1$

43. $\frac{y^2}{4} - \frac{x^2}{5} = 1$

44. $16x^2 + y^2 - 24y + 80 = 0$

$16x^2 + 25y^2 - 400 = 0$

$16x^2 = -25y^2 + 400$

$-25y^2 + 400 + y^2 - 24y + 80 = 0$

$-24y^2 - 24y + 480 = 0$

$y^2 + y - 20 = 0$

$(y + 5)(y - 4) = 0$

$y = -5 \qquad y = 4$

$(0, 4)$

45. $x^2 + y^2 + 36x - 10y + 324 = 0 \qquad x^2 + y^2 + 36x - 10y + 324 = 0$

$x^2 + y^2 + 36x - 20y + 324 = 0$ Mult by (-1) $-x^2 - y^2 - 36x + 20y - 324 = 0$

$10y = 0$

$y = 0$

$x^2 + 36x + 324 = 0$

$(x + 18)(x + 18) = 0$

$x = -18 \qquad$ solution $(-18, 0)$

46. $x^2 + y^2 - 4x + 2y = 20$

$y^2 - 5x + 34 = 0$

$(x^2 - 4x) + (y^2 + 2y) = 20$

$x(x - 4) + y(y + 2) = 20$

$(x + y)(x - 4)(y + 2) = 20$

$x + y = 20 \qquad x = 24 \qquad y = 18$

no solution

47. $x^2 - y - 2 = 0 \qquad x^2 = y + 2$

$x^2 + 4y^2 - 3y - 4 = 0$

$y + 2 + 4y^2 - 3y - 4 = 0$

$4y^2 - 2y - 2 = 0$

$(2y + 1)(2y - 2) = 0$

$2y + 1 = 0 \qquad 2y - 2 = 0$

$y = -\frac{1}{2} \qquad y = 1$

points of intersection:

$\left(-\frac{\sqrt{6}}{2}, -\frac{1}{2}\right), \left(\frac{\sqrt{6}}{2}, -\frac{1}{2}\right), \left(-\sqrt{3}, 1\right), \left(\sqrt{3}, 1\right)$

48. Arithmetic; the rule for the next term is $a_n = a_{n-1} + 6$

49. Geometric; the rule for the next term is $a_n = 3a_{n-1}$

50. Neither; each term is found by adding the next odd integer to the previous term.

51. Geometric; the rule for the next term is $a_n = (0.1)a_{n-1}$

52. 3, 8, 13, 18, 23

53. 9, 6, 1, −6, −15

54. 5, 11, 17, 23, 29

55. 1, 5, 14, 30, 55

56. $a_n = 5(2)^{n-1}$

$a_1 = 5, a_n = 2a^{n-1}$

57. $a_n = 7 - 6n$;

$a_1 = 1, a_n = a_{n-1} - 6$

58. $a_n = 2n + 1$;

$a_1 = 3, a_n = a_{n-1} + 2$

59. $a_n = 243\left(\frac{1}{3}\right)^{n-1}$;

$a_1 = 243, a_n = \frac{a_n - 1}{3}$

60. $40\left(\frac{1 + 40}{2}\right) = 820$

61. $5\left(\frac{8 + 12}{2}\right) = 50$

62. $1\left(\frac{1 - (0.75)^6}{1 - 0.75}\right) \approx 3.288$

63. $\frac{8}{1 - 0.5} = 16$

64. 6720

65. 720

66. 380

67. 70

68. 1

69. 21

70. $(x+4)^5 = {}_5C_0x^5 + {}_5C_1x^4(4) + {}_5C_2x^3(4)^2 + {}_5C_3x^2(4)^3 + {}_5C_4x(4)^4 + {}_5C_5(4)^5$

$= x^5 + 20x^4 + 160x^3 + 640x^2 + 1280x + 1024$

71. $(2x+5)^3 = {}_3C_0(2x)^3 + {}_3C_1(2x)^2(5) + {}_3C_2(2x)(5)^2 + {}_3C_3(5)^3$

$= 8x^3 + 60x^2 + 150x + 125$

72. $(x+y)^6 = {}_6C_0x^6 + {}_6C_1x^5y + {}_6C_2x^4y^2 + {}_6C_3x^3y^3 + {}_6C_4x^2y^4 + {}_6C_5xy^5 + {}_6C_6y^6$

$= x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

73. $(3x-1)^4 = {}_4C_0(3x)^4 + {}_4C_1(3x)^3(-1) + {}_4C_2(3x)^2(-1)^2 + {}_4C_3(3x)(-1)^3 + {}_4C_4(-1)^4$

$= 81x^4 - 108x^3 + 54x^2 - 12x + 1$

74. $(x+2)^4 = {}_4C_0x^4 + {}_4C_1x^3(2) + {}_4C_2x^2(2)^2 + {}_4C_3x(2)^3 + {}_4C_4(2)^4$

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

75. $(x^2-4)^3 = {}_3C_0(x^2)^3 + {}_3C_1(x^2)^2(-4) + {}_3C_2(x^2)(-4)^2 + {}_3C_3(-4)^3$

$= x^6 - 12x^4 + 48x^2 - 64$

76. $0.75 = 0.3 + 0.5 - P(A \text{ and } B)$

$P(A \text{ and } B) = 0.05$

77. $P(A \text{ and } B) = 0.4(0.5)$

$= 0.2$

78. $P(A \text{ and } B) = 0.9(0.1)$

$= 0.09$

9%

79. $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$

80. $\frac{5}{32}$

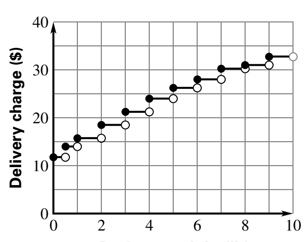
81. $\frac{10}{32} = \frac{5}{16}$

82. $\frac{10}{32} = \frac{5}{16}$

83. $\frac{5}{32}$

84. $\frac{1}{32}$

85.



86. $B = 1000\left(1 + \frac{0.04}{12}\right)^{5.12}$

$B \approx \$1221.00$

87. $a_n = 0.8a_{n-1} + 1000$

It approaches 5000.

88. ways $= 36^4$

$= 1,679,616$

89. $\frac{4}{10} = \frac{2}{5}$

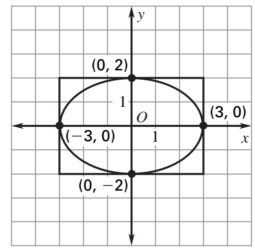
Project, Ch. 12 *(pp. 764–765)*

Investigation

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1$

2. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ Answers may vary.

3.



4. Answers may vary.

5. Answers may vary.

6. Answers may vary.

7. The greater the number of points, the more accurate the estimated area is.