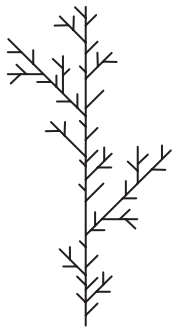
CHAPTER 11

Think & Discuss *(p. 649)*

1.



Stage 4

2. Answers may vary.

3. *Sample answer:* Add new segments to the tips of existing segments at each stage.

Skill Review *(p. 650)*

1. $n + 4$ **2.** $3n$ **3.** $\frac{n}{2}$ **4.** $3(3) - 7 = 2$ **5.** $\frac{8}{8+1} = \frac{8}{9}$

6. $3(2)^{4-1} = 3(2)^3 = 24$

7.

	$f(0)$	$f(1)$	$f(2)$
function values	3	0	−9
1st order differences	−3	−9	−15
2nd order differences	−6	−6	−6

$f(3)$	$f(4)$	$f(5)$	$f(6)$
−24	−45	−72	−105
−21	−27	−33	
−6	−6		

8.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
function values	3	16	45	96	175	288
1st order differences	13	29	51	79	113	
2nd order differences	16	22	28	34		
3rd order differences	6	6	6			

9.

	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$
function values	−3	7	67	237	601	1267
1st order differences	10	60	170	364	666	
2nd order differences	50	110	194	302		
3rd order differences	60	84	108			
4th order differences	24	24				

10. $4^x = 16{,}384$

$x = 7$

11. $2^{x-1} = 32$

$2^{x-1} = 2^5$

$x = 6$

12. $10 = \frac{5}{1-x}$

$10 - 10x = 5$

$10x = 5$

$x = \frac{1}{2}$

13. $24 = \frac{2}{1+x}$

$24 + 24x = 2$

$24x = -22$

$x = -\frac{22}{24} = -\frac{11}{12}$

Lesson 11.1

11.1 Guided Practice (p. 655)

1. A sequence is a function whose domain is a set of consecutive integers. A series is the sum of a sequence.

2. **a.** The sum from k equal 3 to 10 of $k + 2$ **b.** k

c. 3 **d.** 10

3. $a_n = 2n$

$a_1 = 2 \cdot 1 = 2$

$a_2 = 2 \cdot 2 = 4$

$a_3 = 2 \cdot 3 = 6$

$a_4 = 2 \cdot 4 = 8$

$a_5 = 2 \cdot 5 = 10$

$a_6 = 2 \cdot 6 = 12$

4. $a_n = 6 - n$

$a_1 = 6 - 1 = 5$

$a_2 = 6 - 2 = 4$

$a_3 = 6 - 3 = 3$

$a_4 = 6 - 4 = 2$

$a_5 = 6 - 5 = 1$

$a_6 = 6 - 6 = 0$

5. $a_n = 3n + 1$

$a_1 = 3 \cdot 1 + 1 = 4$

$a_2 = 3 \cdot 2 + 1 = 7$

$a_3 = 3 \cdot 3 + 1 = 10$

$a_4 = 3 \cdot 4 + 1 = 13$

$a_5 = 3 \cdot 5 + 1 = 16$

$a_6 = 3 \cdot 6 + 1 = 19$

6. $f(n) = 2^{n+3}$

$f(1) = 2^{1+3} = 2^4 = 16$

$f(2) = 2^{2+3} = 2^5 = 32$

$f(3) = 2^{3+3} = 2^6 = 64$

$f(4) = 2^{4+3} = 2^7 = 128$

$f(5) = 2^{5+3} = 2^8 = 256$

$f(6) = 2^{6+3} = 2^9 = 512$

7. $\sum_{k=3}^{10} (k + 2) = (3 + 2) + (4 + 2) + (5 + 2) + (6 + 2) + (7 + 2) + (8 + 2) + (9 + 2) + (10 + 2)$

$= 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12$

$= 68$

8. $\sum_{n=1}^{12} n^2 = \frac{12(12+1)(24+1)}{6} = \frac{12 \cdot 13 \cdot 25}{6} = 650$

9. $a_n = n + 1$

$a_1 = 1 + 1 = 2$

$a_2 = 2 + 1 = 3$

$a_3 = 3 + 1 = 4$

$a_4 = 4 + 1 = 5$

$a_5 = 5 + 1 = 6$

$a_6 = 6 + 1 = 7$

10. $a_n = n^2$

$a_1 = 1^2 = 1$

$a_2 = 2^2 = 4$

$a_3 = 3^2 = 9$

$a_4 = 4^2 = 16$

$a_5 = 5^2 = 25$

$a_6 = 6^2 = 36$

11. $a_n = 3 - n$

$a_1 = 3 - 1 = 2$

$a_2 = 3 - 2 = 1$

$a_3 = 3 - 3 = 0$

$a_4 = 3 - 4 = -1$

$a_5 = 3 - 5 = -2$

$a_6 = 3 - 6 = -3$

12. $a_n = n^3 - 1$

$a_1 = 1^3 - 1 = 0$

$a_2 = 2^3 - 1 = 7$

$a_3 = 3^3 - 1 = 26$

$a_4 = 4^3 - 1 = 63$

$a_5 = 5^3 - 1 = 124$

$a_6 = 6^3 - 1 = 215$

13. $a_n = (n + 1)^2$

$a_1 = (1 + 1)^2 = 4$

$a_2 = (2 + 1)^2 = 9$

$a_3 = (3 + 1)^2 = 16$

$a_4 = (4 + 1)^2 = 25$

$a_5 = (5 + 1)^2 = 36$

$a_6 = (6 + 1)^2 = 49$

14. $a_n = (-n)^3$

$a_1 = (-1)^3 = -1$

$a_2 = (-2)^3 = -8$

$a_3 = (-3)^3 = -27$

$a_4 = (-4)^3 = -64$

$a_5 = (-5)^3 = -125$

$a_6 = (-6)^3 = -216$

15. $a_n = n^2 + 3$

$a_1 = 1^2 + 3 = 4$

$a_2 = 2^2 + 3 = 7$

$a_3 = 3^2 + 3 = 12$

$a_4 = 4^2 + 3 = 19$

$a_5 = 5^2 + 3 = 28$

$a_6 = 6^2 + 3 = 39$

16. $a_n = (n - 1)^2$

$a_1 = (1 - 1)^2 = 0$

$a_2 = (2 - 1)^2 = 1$

$a_3 = (3 - 1)^2 = 4$

$a_4 = (4 - 1)^2 = 9$

$a_5 = (5 - 1)^2 = 16$

$a_6 = (6 - 1)^2 = 25$

17. $f(n) = \dfrac{n}{n + 1}$

$f(1) = \dfrac{1}{1 + 1} = \dfrac{1}{2}$

$f(2) = \dfrac{2}{2 + 1} = \dfrac{2}{3}$

$f(3) = \dfrac{3}{3 + 1} = \dfrac{3}{4}$

$f(4) = \dfrac{4}{4 + 1} = \dfrac{4}{5}$

$f(5) = \dfrac{5}{5 + 1} = \dfrac{5}{6}$

$f(6) = \dfrac{6}{6 + 1} = \dfrac{6}{7}$

18. $f(n) = \dfrac{n^2}{2n}$

$f(1) = \dfrac{1}{2} = 0.5$

$f(2) = \dfrac{4}{4} = 1$

$f(3) = \dfrac{9}{6} = 1.5$

$f(4) = \dfrac{16}{8} = 2$

$f(5) = \dfrac{25}{10} = 2.5$

$f(6) = \dfrac{36}{12} = 3$

19. $f(n) = \dfrac{n + 2}{2n}$

$f(1) = \dfrac{3}{2}$

$f(2) = \dfrac{4}{4} = 1$

$f(3) = \dfrac{5}{6}$

$f(4) = \dfrac{6}{8} = \dfrac{3}{4}$

20. $f(n) = \dfrac{3}{-n}$

$f(1) = \dfrac{3}{-1} = -3$

$f(2) = \dfrac{3}{-2}$

$f(3) = \dfrac{3}{-3} = -1$

$f(4) = \dfrac{3}{-4}$

$f(5) = \dfrac{7}{10}$

$f(6) = \dfrac{8}{12} = \dfrac{2}{3}$

$f(5) = \dfrac{3}{-5}$

$f(6) = -\dfrac{3}{6} = -\dfrac{1}{2}$

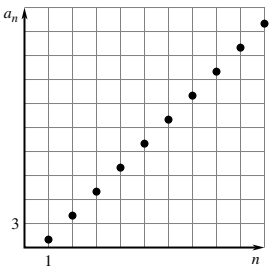
11.1 Practice and Applications (pp. 655–657)

21. 9; $2n - 1$ **22.** 10,000; 10^{n-1} **23.** -16; $a_n = 3_n - 1$ if n is odd or $2 - 3n$ if n is even **24.** -25; $(-1)^n(5n)$

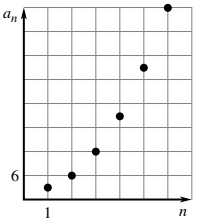
25. $-\dfrac{1}{10}$; $-\dfrac{1}{2n}$ **26.** $\dfrac{6}{9}$; $\dfrac{n}{(n + 3)}$ **27.** $\dfrac{6}{3}$; $\dfrac{n}{3}$

28. $\dfrac{5}{60}$; $\dfrac{n}{(n + 1)10}$ **29.** 5.9; $1.1 + 0.8n$

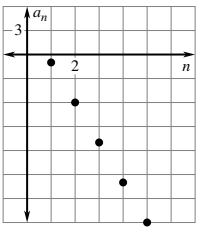
30.



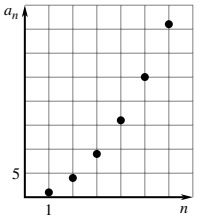
31.



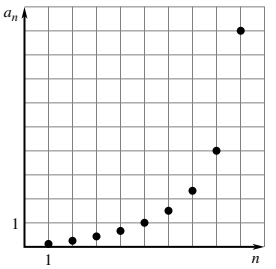
32.



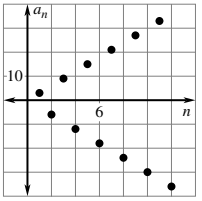
33.



34.



35.



36. $\sum_{n=1}^{5} (4n - 3)$ **37.** $\sum_{n=1}^{5} 4n$ **38.** $\sum_{n=0}^{\infty} 3(2n - 3)$

39. $\sum_{n=1}^{\infty} (-1)^{n+1} n$ **40.** $\sum_{n=1}^{5} -(n + 6)$ **41.** $\sum_{n=5}^{\infty} \dfrac{n}{n + 1}$

42. $\sum_{n=1}^{4} \dfrac{1}{10^{k-1}}$ **43.** $\sum_{n=1}^{6} n^2$

44. $\sum_{n=1}^{6} 3n = 3(1) + 3(2) + 3(3) + 3(4) + 3(5) + 3(6)$

$= 3 + 6 + 9 + 12 + 15 + 18$

$= 63$

45. $\sum_{i=0}^{5} 12i = 12(1) + 12(2) + 12(3) + 12(4) + 12(5)$

$= 12 + 24 + 36 + 48 + 60$

$= 180$

46. $\sum_{n=0}^{4} n^2 = \frac{4(5)(9)}{6} = 30$

47. $\sum_{n=1}^{3} 4n^3 = 4(1)^3 + 4(2)^3 + 4(3)^3$

$= 4 + 32 + 108$

$= 144$

48. $\sum_{k=1}^{5} (k^2 - 1) = (1 - 1) + (4 - 1) + (9 - 1)$

$+ (16 - 1) + (25 - 1)$

$= 0 + 3 + 8 + 15 + 24$

$= 50$

49. $\sum_{n=0}^{4} (2n^2 + 1) = (0 + 1) + (2 + 1) + (8 + 1)$

$+ (18 + 1) + (32 + 1)$

$= 1 + 3 + 9 + 19 + 33$

$= 65$

50. $\sum_{k=1}^{4} k(k + 2) = 1(1 + 2) + 2(2 + 2) + 3(3 + 2)$

$+ 4(4 + 2)$

$= 1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + 4 \cdot 6$

$= 3 + 8 + 15 + 24$

$= 50$

51. $\sum_{n=2}^{10} \frac{2}{n} = \frac{2}{2} + \frac{2}{3} + \frac{2}{4} + \frac{2}{5} + \frac{2}{6} + \frac{2}{7} + \frac{2}{8} + \frac{2}{9} + \frac{2}{10}$

$= 3.858$

52. $\sum_{n=2}^{12} \frac{1}{n-1} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}$

$+ \frac{1}{10} + \frac{1}{11} = 3.0199$

53. $\sum_{n=1}^{5} \frac{n}{n+1} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = 3.55$

54. $\sum_{i=2}^{6} \frac{i}{i-1} = \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \frac{5}{4} + \frac{6}{5} = 7.283$

55. $\sum_{n=1}^{\infty} \left(\frac{n}{n^2} - \frac{1}{n}\right) = 0$ **56.** $\sum_{i=1}^{42} 1 = 42$

57. $\sum_{n=1}^{5} n = \frac{5(6)}{2} = 15$ **58.** $\sum_{i=1}^{18} i = \frac{18(19)}{2} = 171$

59. $\sum_{k=1}^{20} k = \frac{20(21)}{2} = 210$ **60.** $\sum_{n=1}^{6} n^2 = \frac{6(7)(13)}{6} = 91$

61. $\sum_{i=1}^{10} i^2 = \frac{10(11)(21)}{6} = 385$

62. $\sum_{i=1}^{12} i^2 = \frac{12(13)(25)}{6} = 650$

63. $\sum_{k=1}^{35} k^2 = \frac{35(36)(71)}{6} = 14{,}910$

64. 36°, 60°, 77.1°, 90°, 100°, 108°

65. $1 + 2 + 3 + 4 + 5 = 15$ ft

66. $15 + 4 + 3 + 2 + 1 = 25$ ft **67.** $2^n - 1$; $2^6 - 1 = 63$

68. $a_5 = \frac{1}{2}5^2 + \frac{1}{2}5 = \frac{25}{2} + \frac{5}{2} = 15$

$\sum_{n=1}^{5} \frac{1}{2}n^2 + \frac{1}{2}n = \left(\frac{1}{2} + \frac{1}{2}\right) + (2 + 1) + \left(\frac{9}{2} + \frac{3}{2}\right) + (8 + 2)$

$+ \left(\frac{25}{2} + \frac{5}{2}\right)$

$= 1 + 3 + 6 + 10 + 15 = 35$

69. B **70.** A

71. a. $\sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$ true

$\sum_{i=1}^{n} ka_i = ka_1 + ka_2 + ka_3 + \cdots + ka_n$

$= k(a_1 + a_2 + a_3 + \cdots a_n)$

$= k\sum_{i=1}^{n} a_i$

b. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$ true

$\sum_{i=1}^{n} (a_i + b_i) = (a_1 + b_1) + (a_2 + b_2) + \cdots$

$+ (a_n + b_n)$

$= (a_1 + a_2 + \cdots + a_n)$

$+ (b_1 + b_2 + \cdots + b_n)$

$= \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$

c. $\sum_{i=1}^{n} a_i b_i = \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} b_i\right)$ false

$\sum_{i=1}^{3} i(i + 1) = 1(2) + 2(3) + 3(4) = 20$

$\left(\sum_{i=1}^{3} i\right)\left(\sum_{i=1}^{3} i + 1\right) = (1 + 2 + 3)(2 + 3 + 4)$

$= (6)(9) = 54$

d. $\sum_{i=1}^{n} (a_i)^k = \left(\sum_{i=1}^{n} a_i\right)^k$ false

$\sum_{i=1}^{3} (i + 1)^3 = 2^3 + 3^3 + 4^3 = 99$

$\left(\sum_{i=1}^{3} (i + 1)\right)^3 = [2 + 3 + 4]^3 = 729$

72. $\sum_{i=1}^{n} \left(\frac{1}{2}i^2 + \frac{1}{2}i\right) = \sum_{i=1}^{n} \frac{1}{2}i^2 + \sum_{i=1}^{n} \frac{1}{2}i = \frac{1}{2}\sum_{i=1}^{n} i^2 + \frac{1}{2}\sum_{i=1}^{n} i$

$= \frac{1}{2}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{1}{2}\left(\frac{n(n+1)}{2}\right)$

$= \frac{n}{2}\left[\frac{(2n^2 + 3n + 1) + (3n + 3)}{6}\right]$

$= \frac{n(n^2 + 3n + 2)}{6}$

$= \frac{n(n+1)(n+2)}{6}$

11.1 Mixed Review (p. 657)

73. $17 = 3x + 5$

$3x = 12$

$x = 4$

74. $18 = -7 + x$

$25 = x$

75. $15 = -1 + 8x$

$8x = 16$

$x = 2$

76. $9 = 4 - 5x$

$5x = -5$

$x = -1$

77. $5 = 6 - 2x$

$-1 = -2x$

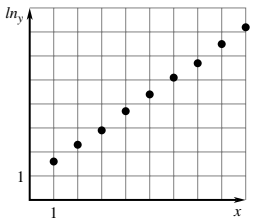
$x = \frac{1}{2}$

78. $24 = 10 + 7x$

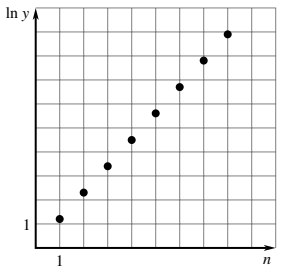
$7x = 14$

$x = 2$

79. $y = (2.5)2^x$



80. $y = \left(\frac{16}{15}\right)3^x$



81. $\sqrt{(0+4)^2 + (0+6)^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

82. $\sqrt{(1+3)^2 + (4+9)^2} = \sqrt{16+169} = \sqrt{185}$

83. $\sqrt{(5+1)^2 + (2-8)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$

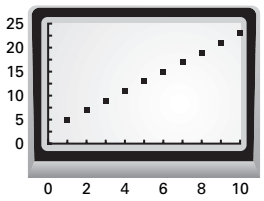
84. $\sqrt{(9-2)^2 + (-1-9)^2} = \sqrt{49+100} = \sqrt{149}$

85. $\sqrt{(3-11)^2 + (-3+4)^2} = \sqrt{64+1} = \sqrt{65}$

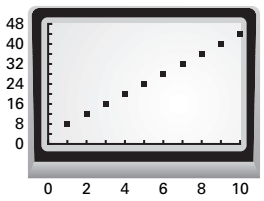
86. $\sqrt{(10-40)^2 + (30+20)^2} = \sqrt{900+2500} = 10\sqrt{34}$

Technology Activity 11.1 (p. 658)

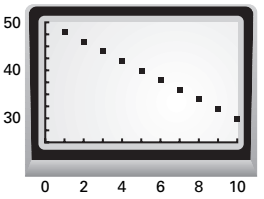
1. 5, 7, 9, 11, 13, 15, 17, 19, 21, 23; 140



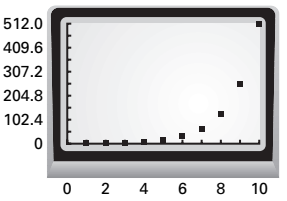
2. 8, 12, 16, 20, 24, 28, 32, 36, 40, 44; 260



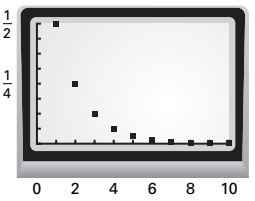
3. 48, 46, 44, 42, 40, 38, 36, 34, 32, 30; 390



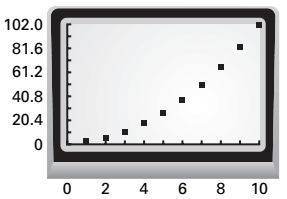
4. 1, 2, 4, 8, 16, 32, 64, 128, 256, 512; 1023



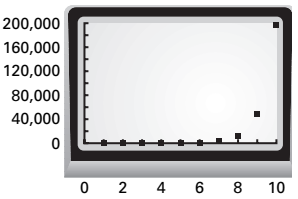
5. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}, \frac{1}{512}, \frac{1}{1024}; \frac{1023}{1024}$



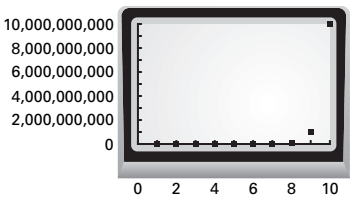
6. 3, 6, 11, 18, 27, 38, 51, 66, 83, 102; 405



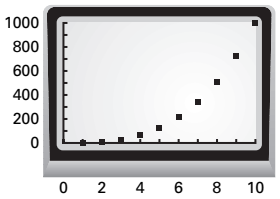
7. $\frac{3}{4}$, 3, 12, 48, 192, 768, 3072, 12,288, 49,152, 196,608; 262,143.75



8. 12, 102, 1002, 10,002, 100,002, 1,000,002, 10,000,002, 100,000,002, 1,000,000,002, 10,000,000,002; 11,111,111,130



9. $\frac{4}{3}, 8\frac{1}{3}, 27\frac{1}{3}, 64\frac{1}{3}, 125\frac{1}{3}, 216\frac{1}{3}, 343\frac{1}{3}, 512\frac{1}{3}, 729\frac{1}{3}, 1000\frac{1}{3}$
$= 3028\frac{1}{3}$



Lesson 11.2

11.2 Guided Practice (p. 663)

1. arithmetic series

2. An arithmetic sequence is a function whose domain is a set of consecutive integers and whose consecutive terms differ by a constant amount. An arithmetic series is the sum of an arithmetic sequence.

3. $S_n = n\left(\frac{a_1 + a_n}{2}\right)$ **4.** $a_n = 5 + 2(n - 1) = 2n + 3$

5. $a_2 = a_1 + (-3)(2 - 1)$
$18 = a_1 - 3$
$21 = a_1$
$a_n = 21 - 3(n - 1)$
$a_n = 24 - 3n$

6. $20 = a_1 + \left(\frac{1}{2}\right)(5 - 1)$
$20 = a_1 + 2$
$18 = a_1$
$a_n = 18 + \frac{1}{2}(n - 1)$
$a_n = \frac{35}{2} + \frac{1}{2}n$

7. $61 = a_1 + 14d$
$12 = a_1 + 7d$
$49 = 7d$
$7 = d$
$61 = a_1 + 98$
$-37 = a_1$
$a_n = -37 + 7n - 7$
$a_n = 7n - 44$

8. $24 = a_1 + 11k$
$10 = a_1 + 4d$
$14 = 7d$
$2 = d$
$10 = a_1 + 8$
$2 = a_1$
$a_n = 2 + 2n - 2$
$a_n = 2n$

9. $32 = a_1 + 15d$
$8 = a_1 + 9d$
$24 = 6d$
$4 = d$
$8 = a_1 + 36$
$-28 = a_1$
$a_n = -28 + 4n - 4$
$a_n = -32 + 4n$

10. $a_1 = 2 \quad d = 4$
$a_n = 2 + 4n - 4 = 4n - 2$
$a_{10} = 40 - 2 = 38$
$S_{10} = 10\left(\frac{2 + 38}{2}\right) = 200$

11. $a_1 = 3 \quad d = \frac{1}{2}$
$a_n = 3 + \frac{1^n}{2} - \frac{1}{2} = \frac{5}{2} + \frac{1^n}{2}$
$a_{10} = \frac{5}{2} + \frac{10}{2} = \frac{15}{2}$
$S_{10} = 10\left(\frac{3 + \frac{15}{2}}{2}\right) = \frac{105}{2}$

12. $a_1 = 6 \quad d = -3$
$a_n = 6 - 3n + 3 = 9 - 3n$
$a_{10} = 9 - 30 = -21$
$S_{10} = 10\left(\frac{6 + (-21)}{2}\right) = -75$

13. $a_1 = 0.7 \quad d = 1.2$
$a_n = 0.7 + 1.2n - 1.2 = 1.2n - 0.5$
$a_{10} = 12 - 0.5 = 11.5$
$S_{10} = 10\left(\frac{0.7 + 11.5}{2}\right) = 61$

14. $a_1 = 29 \quad d = 2$
$a_n = 29 + 2n - 2 = 2n + 27$
$a_{42} = 84 + 27 = 111$
$S_{42} = 42\left(\frac{29 + 111}{2}\right) = 2940$

11.1 Practice and Applications (pp. 663–665)

15. yes; $d = -3$ **16.** no; d is not constant
17. no; d is not constant **18.** yes; $d = \frac{1}{2}$
19. no; d is not constant **20.** yes; $d = \frac{2}{3}$

21. $a_1 = 1 \quad d = 2$
$a_n = 1 + 2n - 2 = 2n - 1$
$a_{25} = 50 - 1 = 49$

22. $a_1 = 6 \quad d = 8$
$a_n = 6 + 8n - 8 = 8n - 2$
$a_{25} = 200 - 2 = 198$

23. $a_1 = 9 \quad d = 14$
$a_n = 9 + 14n - 14 = 14n - 5$
$a_{25} = 350 - 5 = 345$

24. $a_1 = -1 \quad d = 1$
$a_n = -1 + n - 1 = n - 2$
$a_{25} = 25 - 2 = 23$

25. $a_1 = 4 \quad d = -3$
$a_n = 4 - 3n + 3 = 7 - 3n$
$a_{25} = 7 - 75 = -68$

26. $a_1 = \frac{1}{2} \quad d = \frac{5}{2}$

$a_n = \frac{1}{2} + \frac{5n}{2} - \frac{5}{2} = \frac{5n}{2} - 2$

$a_{25} = \frac{125}{2} - \frac{4}{2} = \frac{121}{2}$

27. $a_1 = \frac{11}{2} \quad d = -\frac{4}{3}$

$a_n = \frac{11}{2} - \frac{4n}{3} + \frac{4}{3} = -\frac{4n}{3} + \frac{41}{6}$

$a_{25} = -\frac{100}{3} + \frac{41}{6} = -\frac{159}{6} = -\frac{53}{2}$

28. $a_1 = \frac{5}{2} \quad d = -\frac{4}{6}$

$a_n = \frac{5}{2} - \frac{4}{6}n + \frac{4}{6} = \frac{19}{6} - \frac{2}{3}n$

$a_{25} = \frac{19}{6} - \frac{100}{6} = -\frac{81}{6} = -\frac{27}{2}$

29. $a_1 = 1.6 \quad d = 2.4$

$a_n = 1.6 + 2.4n - 2.4 = 2.4n - 0.8$

$a_{25} = 60 - 0.8 = 59.2$

30. $d = 4 \quad a_{14} = 46$

$46 = a_1 + 13(4)$

$-6 = a_1$

$a_n = -6 + 4n - 4$

$a_n = -10 + 4n$

31. $d = -12 \quad a_1 = 80$

$a_n = 80 - 12n + 12$

$a_n = 92 - 12n$

32. $d = \frac{5}{3} \quad a_8 = 24$

$24 = a_1 + 7\left(\frac{5}{3}\right)$

$\frac{37}{3} = a_1$

$a_n = \frac{37}{3} + \frac{5}{3}n - \frac{5}{3}$

$a_n = \frac{32}{3} + \frac{5}{3}n$

33. $a_5 = 17 \quad a_{15} = 77$

$77 = a_1 + 14d$

$17 = a_1 + 4d$

$60 = 10d$

$6 = d$

$17 = a_1 + 24$

$a_1 = -7$

$a_n = -7 + 6n - 6$

$a_n = -13 + 6n$

34. $d = -6 \quad a_{12} = -4$

$-4 = a_1 - 66$

$62 = a_1$

$a_n = 62 - 6n + 6$

$a_n = 68 - 6n$

35. $a_2 = -28 \quad a_{20} = 52$

$52 = a_1 + 19d$

$-28 = a_1 + d$

$80 = 18d$

$\frac{40}{9} = d$

$-28 = a_1 + \frac{40}{9}$

$-\frac{292}{9} = a_1$

$a_n = -\frac{292}{9} + \frac{40}{9}n - \frac{40}{9}$

$a_n = -\frac{332}{9} + \frac{40}{9}n$

36. $a_1 = -2 \quad a_9 = -\frac{1}{6}$

$-\frac{1}{6} = a_1 + 8d$

$-2 = a_1$

$\frac{11}{6} = 8d$

$\frac{11}{48} = d$

$a_n = -2 + \frac{11}{48}n - \frac{11}{48}$

$a_n = -\frac{107}{48} + \frac{11}{48}n$

37. $a_7 = 34 \quad a_{18} = 122$

$122 = a_1 + 17d$

$34 = a_1 + 6d$

$88 = 11d$

$8 = d$

$34 = a_1 + 48$

$-14 = a_1$

$a_n = -14 + 8d - 8$

$a_n = -22 + 8d$

38. $d = -4.1 \quad a_{16} = 48.2$

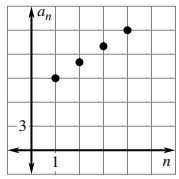
$48.2 = a_1 + 15(-4.1)$

$109.7 = a_1$

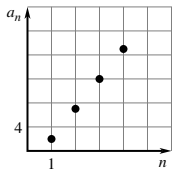
$a_n = 109.7 - 4.1n + 4.1$

$a_n = 113.8 - 4.1n$

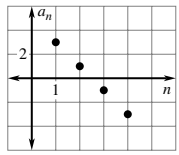
39.



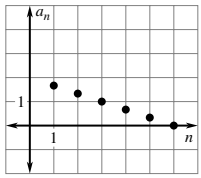
40.



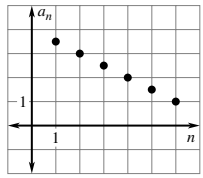
41.



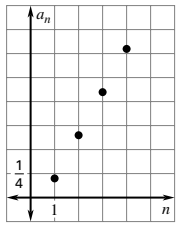
42.



43.



44.



45. **a.** $n = 20$

$a_n = 3 + 5n - 5 = -2 + 5n$

$a_{20} = -2 + 5 \cdot 20 = 98$

$S_{20} = 20\left(\frac{3 + 98}{2}\right) = 1010$

b. $366 = n\left(\frac{3 + (-2 + 5n)}{2}\right)$

$732 = n + 5n^2$

$5n^2 + n - 732 = 0$

$(5n + 61)(n - 12) = 0$

$n = 12$

46. **a.** $n = 40$

$a_n = 50 - 8n + 8 = 58 - 8n$

$a_{40} = 58 - 8 \cdot 40 = -262$

—CONTINUED—

46. —CONTINUED—

$$S_{40} = 40\left(\frac{50 + (-262)}{2}\right) = -4240$$

b. $$182 = n\left(\frac{50 + (58 - 8n)}{2}\right)$$

$$364 = 108n - 8n^2$$

$$8n^2 - 108n + 364 = 0$$

$$4(2n + 13)(n - 7) = 0$$

$$n = 7$$

47. a. $n = 19$

$$a_n = -10 + 5n - 5 = -15 + 5n$$

$$a_{19} = -15 + 5 \cdot 19 = 80$$

$$S_{19} = 19\left(\frac{-10 + 80}{2}\right) = 665$$

b. $$375 = n\left(\frac{-10 + (-15 + 5n)}{2}\right)$$

$$750 = -25n + 5n^2$$

$$5n^2 - 25n - 750 = 0$$

$$5(n + 10)(n - 15) = 0$$

$$n = 15$$

48. a. $n = 32$

$$a_n = 34 - 3n + 3 = 37 - 3n$$

$$a_{32} = 37 - 3(32) = -59$$

$$S_{32} = 32\left(\frac{34 + (-59)}{2}\right) = -400$$

b. $$-12 = n\left(\frac{34 + (37 - 3n)}{2}\right)$$

$$-24 = 71n - 3n^2$$

$$3n^2 - 71n - 24 = 0$$

$$(3n + 1)(n - 24) = 0$$

$$n = 24$$

49. a. $n = 68$

$$a_n = 2 + 7n - 7 = 7n - 5$$

$$a_{68} = 7(68) - 5 = 471$$

$$S_{68} = 68\left(\frac{2 + 471}{2}\right) = 16{,}082$$

b. $$1661 = n\left(\frac{2 + (7n - 5)}{2}\right)$$

$$3322 = -3n + 7n^2$$

$$7n^2 - 3n - 3322 = 0$$

$$(7n + 151)(n - 22) = 0$$

$$n = 22$$

50. a. $n = 24$

$$a_n = 2 + 14n - 14 = -12 + 14n$$

$$a_{24} = -12 + 14(24) = 324$$

$$S_{24} = 24\left(\frac{2 + 324}{2}\right) = 3912$$

b. $$2178 = n\left(\frac{2 + (-12 + 14n)}{2}\right)$$

$$4356 = -10n + 14n^2$$

$$14n^2 - 10n - 4356 = 0$$

$$2(7n + 121)(n - 18) = 0$$

$$n = 18$$

51. $\sum_{i=1}^{20} (3 + 5i) = 20\left(\frac{8 + 103}{2}\right) = 1110$

52. $\sum_{i=1}^{34} (1 + 8i) = 34\left(\frac{9 + 273}{2}\right) = 4794$

53. $\sum_{i=1}^{15} (-10 - 3i) = 15\left(\frac{-13 - 55}{2}\right) = -510$

54. $\sum_{i=1}^{22} \left(6 - \frac{3}{4}i\right) = 22\left(\frac{\frac{21}{4} - \frac{42}{4}}{2}\right) = -57\frac{3}{4}$

55. $\sum_{i=1}^{45} (11 + 4i) = 45\left(\frac{15 + 191}{2}\right) = 4635$

56. $\sum_{i=1}^{18} (8.1 + 4.4i) = 18\left(\frac{12.5 + 87.3}{2}\right) = 898.2$

57. a. $a_n = 6n$ **b.** $S_9 = 9\left(\frac{6 + 54}{2}\right) + 1 = 271$

58. $a_1 = 15 \quad d = -1 \quad a_7 = 21 \quad S_7 = 7\left(\frac{15 + 21}{2}\right) = 126$

59. $\left(4\sum_{n=1}^{4} 2i\right) + 1 = \left(4\left(\frac{8 + 32}{2}\right) + 1\right) = 81 \text{ ft}^2$

60. $24\left(\sum_{i=1}^{11} 24 + n\right) + 18\left(\sum_{i=1}^{11} 35 + n\right) + 12\left(\sum_{i=1}^{10} 46 + n\right)$

$$= 24\left[11\left(\frac{25 + 35}{2}\right)\right] + 18\left[11\left(\frac{36 + 46}{2}\right)\right] + 12\left[10\left(\frac{47 + 56}{2}\right)\right]$$

$$= 24 \cdot 330 + 18 \cdot 451 + 12 \cdot 515$$

$$= 7920 + 8118 + 6180 = \$22{,}218$$

61. Answers may vary.

62. a.

n	d_n (in.)	l_n (in.)
1	3	3π
2	3.008	3.008π
3	3.016	3.016π
4	3.024	3.024π

b. It is an arithmetic sequence;

$$a_n = 3\pi + 0.008\pi n - 0.008\pi$$

$$a_n = 2.992\pi + 0.008\pi n$$

c. $7 = 3 + 0.008n$

$$4 = 0.008n$$

$$n = 500$$

—CONTINUED—

62. —CONTINUED—

$$S_{500} = 500\left(\frac{3\pi + 6.992\pi}{2}\right) \approx 7847.7 \text{ in. or } 654 \text{ ft}$$

d. $S_{1000} = 1000\left(\frac{3\pi + 10.992\pi}{2}\right) \approx 21{,}978.6 \text{ in. or } 1831.5 \text{ ft}$

Sample answer: The cost of the 7 in. roll is \$15 and the 11 in. roll has 2.8 times as much paper so the 11 in. roll should cost about \$42.

63. $\frac{7}{16} + \frac{9}{16} + \frac{11}{16} + \frac{13}{16} + \frac{15}{16} + \frac{17}{16} + \frac{19}{16} + \frac{21}{16} + \frac{23}{16} + \frac{25}{16} = 10$

11.2 Mixed Review (p. 665)

64. $x^{\frac{1}{2}} = 5$

$x = 25$

65. $2x^{3/4} = 54$

$x^{3/4} = 27$

$x = 81$

66. $x^{2/3} + 10 = 19$

$x^{2/3} = 9$

$x = 27$

67. $(8x)^{1/2} + 6 = 0$

$(8x)^{1/2} = -6$

no solution

68. $x^{1/3} - 11 = 0$

$x^{1/3} = 11$

$x = 1331$

69. $(2x)^{1/2} = x - 4$

$2x = x^2 - 8x + 16$

$x^2 - 10x + 16 = 0$

$(x - 8)(x - 2) = 0$

$x = 8$

70. $2^x = 4.5$

$x = \log_2 4.5 \approx 2.170$

71. $4^x - 3 = 5$

$x = \frac{3}{2}$

72. $10^{3x} + 7 = 15$

$x = \log 2$

≈ 0.3010

73. $6^x - 5 = 1$

$6^x = 6$

$x = 1$

74. $25^x - 28 = 97$

$25^x = 125$

$x = \frac{3}{2}$

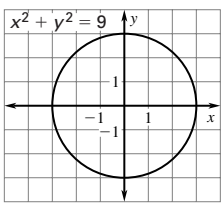
75. $5(2)^{2x} - 4 = 13$

$5(2)^{2x} = 17$

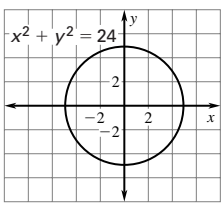
$(2)^{2x} = 3.4$

$x = \log_4 3.4 \approx 0.8828$

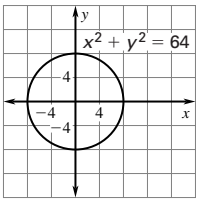
76. $x^2 + y^2 = 9$



77. $x^2 + y^2 = 24$

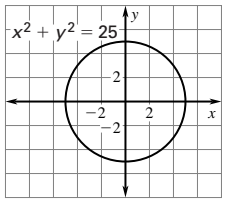


78. $x^2 + y^2 = 64$



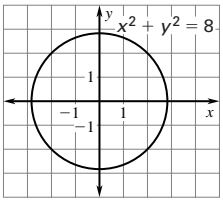
79. $6x^2 + 6y^2 = 150$

$x^2 + y^2 = 25$



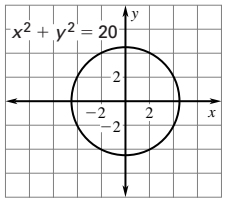
80. $\frac{1}{2}x^2 + \frac{1}{2}y^2 = 4$

$x^2 + y^2 = 8$



81. $20x^2 + 20y^2 = 400$

$x^2 + y^2 = 20$



Lesson 11.3

11.3 Guided Practice (p. 670)

1. common; r

2. If there is some quantity r such that the kth term is r times the $(k - 1)$th term for every value of k

3. $S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

4. 3 **5.** 6 **6.** -3

7. 2 **8.** $-\frac{1}{2}$ **9.** $\frac{1}{2}$

10. 81; 3^{n-1} **11.** 512; $2(4)^{n-1}$ **12.** 1296; $(-6)^{n-1}$

13. 0.6; $375\left(-\frac{1}{5}\right)^{n-1}$ **14.** $\frac{1}{32}$; $\frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n = \frac{1}{2^n}$

15. $\frac{7}{8}$; $(-28)\left(-\frac{1}{2}\right)^{n-1}$ **16.** $a_n = 2(3)^{n-1}$

17. $a_n = 6(-2)^{n-1}$ **18.** $a_n = 12(-3)^{n-1}$

19. $\frac{1}{4} = a_1$

$6 = a_1 r^2$

$\frac{6}{a_1} = r^2$

$24 = r^2$

$2\sqrt{6} = r$

$a_n = \frac{1}{4}\left(2\sqrt{6}\right)^{n-1}$

20. $5 = a_1 r$

$\frac{5}{r} = a_1$

$\frac{1}{5} = a_1 r^3$

$\frac{1}{5} = \frac{5}{r}(r^3)$

$\frac{1}{5} = 5r^2$

$\frac{1}{25} = r^2$

$\frac{1}{5} = r$

$25 = a_1$

$a_n = 25\left(\frac{1}{5}\right)^{n-1}$

21. $28 = a_1 r$

$\frac{28}{r} = a_1$

$-1792 = a_1 r^4$

$-64 = r^3$

$-4 = r$

$a_n = -7(-4)^{n-1}$

22. $S_8 = 1\left(\frac{1 - 8^8}{1 - 8}\right)$

$S_8 = 2{,}396{,}745$

23. $a_8 = (80.9)(0.914)^7$

$= \$43.11$

24. geometric; 4

25. neither; no common ratio or difference

26. arithmetic; 9

27. arithmetic; -4

28. arithmetic; 4

29. goemetric; 3

30. arithmetic; $\frac{1}{3}$

31. neither; no common ratio or difference

32. neither; no common ratio or difference

33. 4

34. 2

35. -2

36. 8

37. $\frac{1}{2}$

38. $-\frac{1}{2}$

39. $a_n = (-4)^{n-1}$

$a_6 = (-4)^5 = -1024$

40. $a_n = 5(2)^{n-1}$

$a_6 = 5(2)^5 = 160$

41. $a_n = 2(7)^{n-1}$

$a_6 = 2(7)^5 = 33{,}614$

42. $a_n = 6(-5)^{n-1}$

$a_6 = 6(-5)^5 = -18{,}750$

43. $a_n = 5\left(-\frac{1}{3}\right)^{n-1}$

$a_6 = 5\left(-\frac{1}{3}\right)^5 = -\frac{5}{243}$

44. $a_n = 2\left(\frac{2}{3}\right)^{n-1}$

$a_6 = 2\left(\frac{2}{3}\right)^5 = \frac{64}{243}$

45. $a_n = 4(3)^{n-1}$

46. $a_n = 45\left(\frac{1}{3}\right)^{n-1}$

47. $72 = a_1 r^2$

$72 = a_1(36)$

$2 = a_1$

$a_n = 2(6)^{n-1}$

48. $a_n = 4\left(\frac{1}{8}\right)^{n-1}$

49. $a_n = -2(8)^{n-1}$

50. $-\frac{1}{2} = a_1$

$-16 = a_1 r^3$

$32 = r^3$

$\sqrt[3]{32} = r$

$a_n = -\frac{1}{2}\left(\sqrt[3]{32}\right)^{n-1}$

51. $10 = a_1 r^2$

$\frac{10}{r^2} = a_1$

$300 = a_1 r^5$

$30 = r^3$

$\sqrt[3]{30} = r$

$a_1 = \frac{10}{\sqrt[3]{900}}$

$a_n = \frac{10}{\sqrt[3]{900}}\left(\sqrt[3]{30}\right)^{n-1}$

52. $-20 = a_1 r$

$-\frac{20}{r} = a_1$

$-5 = a_1 r^3$

$\frac{1}{4} = r^2$

$\frac{1}{2} = r$

$-40 = a_1$

$a_n = -40\left(\frac{1}{2}\right)^{n-1}$

53. $-30 = a_1 r$

$-\frac{30}{r} = a_1$

$3750 = a_1 r^4$

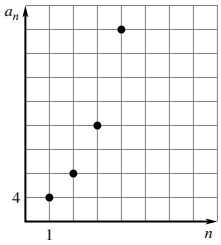
$-125 = r^3$

$-5 = r$

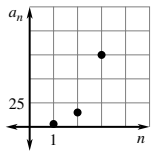
$6 = a_1$

$a_n = 6(-5)^{n-1}$

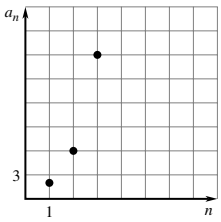
54.



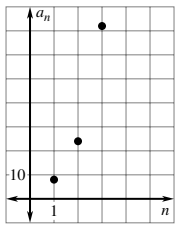
55.



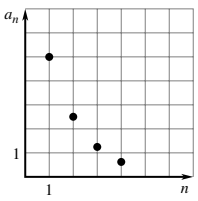
56.



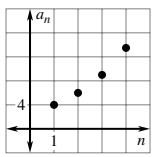
57.



58.



59.



60. a. $n = 14$

$S_{14} = 1\left(\frac{1 - 4^{14}}{1 - 4}\right)$

$= 89{,}478{,}485$

b. $S_n = 341$

$341 = \left(\frac{1 - 4^n}{1 - 4}\right)$

$-1023 = 1 - 4^n$

$1024 = 4^n$

$n = 5$

61. a. $n = 10$

$$S_{10} = 1\left(\frac{1 - 9^{10}}{1 - 9}\right)$$

$$= 435{,}848{,}050$$

b. $S_n = 820$

$$820 = \left(\frac{1 - 9^n}{-8}\right)$$

$$6561 = 9^n$$

$$n = 4$$

62. a. $n = 18$

$$S_{18} = 7\left(\frac{1 - (-3)^{18}}{1 + 3}\right)$$

$$= -677{,}985{,}854$$

b. $S_n = 3829$

$$3829 = 7\left(\frac{1 - (-3)^n}{4}\right)$$

$$2188 = 1 - (-3)^n$$

$$-2187 = (-3)^n$$

$$n = 7$$

63. a. $n = 16$

$$S_{16} = -90\left(\frac{1 - \left(-\frac{1}{3}\right)^{16}}{1 + \frac{1}{3}}\right)$$

$$= -67.5$$

b. $S_n = -66.67$

$$-66.67 = -90\left(\frac{1 - \left(-\frac{1}{3}\right)^n}{\frac{4}{3}}\right)$$

$$-\frac{20}{27} = \left(\frac{1 - \left(-\frac{1}{3}\right)^n}{\frac{4}{3}}\right)$$

$$\frac{80}{81} = 1 - \left(-\frac{1}{3}\right)^n$$

$$\frac{1}{81} = \left(-\frac{1}{3}\right)^n$$

$$n = 4$$

64. $\sum_{i=1}^{10} 6(2)^{i-1} = 6\left(\frac{1 - 2^{10}}{1 - 2}\right)$

$$= 6\left(\frac{1 - 1024}{-1}\right)$$

$$= -6(-1023)$$

$$= 6138$$

65. $\sum_{i=1}^{8} 5(4)^{i-1} = 5\left(\frac{1 - 4^8}{1 - 4}\right)$

$$= 5\left(\frac{1 - 65{,}536}{-3}\right)$$

$$= 5\left(\frac{-65{,}535}{-3}\right)$$

$$= 109{,}225$$

66. $\sum_{i=0}^{9} 12\left(-\frac{1}{2}\right)^i = 12\left(\frac{1 - \left(-\frac{1}{2}\right)^{10}}{1 - \left(-\frac{1}{2}\right)}\right)$

$$= 12\left(\frac{1 - \frac{1}{1024}}{\frac{3}{2}}\right)$$

$$= 12\left(\frac{\frac{1023}{1024}}{\frac{3}{2}}\right)$$

$$= 7.99$$

67. $\sum_{i=1}^{10} 8\left(\frac{3}{4}\right)^{i-1} = 8\left(\frac{1 - \left(\frac{3}{4}\right)^{10}}{\frac{1}{4}}\right)$

$$= 8\left(\frac{1 - 0.0563}{0.25}\right)$$

$$= 8\left(\frac{0.9437}{0.25}\right)$$

$$= 30.198$$

68. $\sum_{i=0}^{6} 4\left(\frac{3}{2}\right)^i = 4\left(\frac{1 - \left(\frac{3}{2}\right)^7}{1 - \frac{3}{2}}\right)$

$$= 4\left(\frac{1 - 17.0859375}{-0.5}\right)$$

$$= 4\left(\frac{-16.0895375}{-0.5}\right)$$

$$= 128.6875$$

69. $\sum_{i=1}^{12} (-2)^{i-1} = 1\left(\frac{1 - (-2)^{12}}{1 + 2}\right)$

$$= \left(\frac{1 - 4096}{3}\right)$$

$$= \frac{-4095}{3}$$

$$= -1365$$

70. $64\left(\frac{1}{2}\right)^{n-1}$; for $1 \le n \le 7$

71. $S_7 = 64\left(\frac{1 - \left(\frac{1}{2}\right)^7}{1 - \frac{1}{2}}\right) = 64\left(\frac{\frac{127}{128}}{\frac{1}{2}}\right) = 64 \cdot \frac{127}{64} = 127$

72. $1000\left(\frac{1}{2}\right)^n$

73. 1000, 500, 250, 125, 62.5, 31.25, 15.625, 7.8125, 3.91, 1.95

74. $a_n = 118(1.2)^{n-1}$

75. $a_3 = 118(1.2)^2 = \$169.92$ million

76. $a_n = 300$

$300 = 118(1.2)^{n-1}$

$2.54 = (1.2)^{n-1}$

$n - 1 = \frac{\log 2.54}{\log 1.2} \approx 5.1$

$n = 6.1$; 1995

77. $S_7 = 118\left(\frac{1 - (1.2)^7}{1 - 1.2}\right)$

$= \$1.524$ billion

78. $a_n = 3^{n-1}$

$a_{10} = 3^9 = 19{,}683$

79. $b_n = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{3}{4}\right)^n$

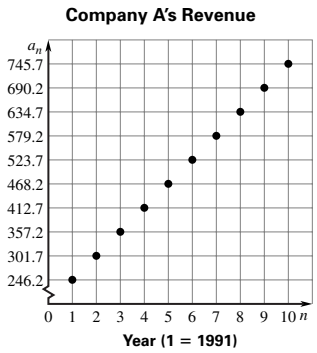
$b_{15} = \left(\frac{\sqrt{3}}{4}\right)\left(\frac{3}{4}\right)^{15} = 0.006$

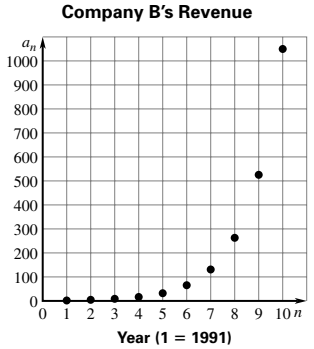
80. Answers may vary.

81. **a.** Company A: $a_n = 190.7 + 55.5n$

Company B: $a_n = 2.05(2)^{n-1}$

b.





c. Company A: $S_{10} = 10\left(\frac{246.2 + 745.7}{2}\right)$

$= \$4959.5$ million

Company B: $S_{10} = 2.05\left(\frac{1 - 2^{10}}{1 - 2}\right)$

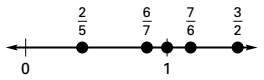
$= \$2097.2$ million

d. 2002

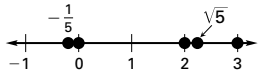
82. **a.** $\frac{x^5 - 1}{x - 1}$ **b.** $\frac{3x(16x^8 - 1)}{2x^2 - 1}$

11.3 Mixed Review (p. 673)

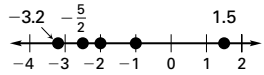
83. $\frac{2}{5}, \frac{6}{7}, 1, \frac{7}{6}, \frac{3}{2}$



84. $-\frac{1}{5}, 0, 2, \sqrt{5}, 3$



85. $-3.2, -\frac{5}{2}, -2, -1, 1.5$



86. $x^2 + x - 2 \geq 0$

$(x + 2)(x - 1) \geq 0$

$x \leq -2$ or $x \geq 1$

87. $x^2 - 6x - 7 \leq 0$

$(x - 7)(x + 1) \leq 0$

$-1 \leq x \leq 7$

88. $x^2 < 36$

$-6 < x < 6$

89. $-x^2 - 8x < 20$

$0 < x^2 + 8x + 20$

all reals

90. $3x^2 - 9x + 6 > 0$

$3(x - 1)(x - 2) > 0$

$x < 1$ or $x > 2$

91. $\frac{1}{2}x^2 + 5x \leq -12$

$\left(\frac{1}{2}x + 3\right)(x + 4) \leq 0$

$-6 \leq x \leq -4$

92. $\frac{3}{x + 1} = 8$

$3 = 8 + 8x$

$-5 = 8x$

$x = -\frac{5}{8}$

93. $\frac{4}{1 - x} = 10$

$10 - 10x = 4$

$-10x = -6$

$x = \frac{3}{5}$

94. $\frac{-12}{x + 4} = -x$

$-12 = -x^2 - 4x$

$x^2 + 4x - 12 = 0$

$(x + 6)(x - 2) = 0$

$x = -6, x = 2$

95. $\frac{-24}{x} - x = 11$

$-24 - x^2 = 11x$

$x^2 + 11x + 24 = 0$

$(x + 3)(x + 8) = 0$

$x = -3, x = -8$

96. $\frac{x}{x - 8} = \frac{x}{24}$

$24x = x^2 - 8x$

$x^2 - 32x = 0$

$x(x - 32) = 0$

$x = 0, x = 32$

97. $x + 10 = \frac{x^2}{x - 5}$

$x^2 + 5x - 50 = x^2$

$5x = 50$

$x = 10$

Quiz 1 (p. 673)

1. 8; $2(n - 1)$ **2.** 243; 3^n **3.** $\frac{1}{80}$; $\left(\frac{1}{5}\right)\left(-\frac{1}{2}\right)^{n-1}$

4. $\sum_{k=0}^{4} k^4 = 0 + 1 + 16 + 81 + 256 = 354$

5. $\sum_{m=1}^{6} (m^2 + 5) = 6 + 9 + 14 + 21 + 30 + 41 = 121$

6. $\sum_{n=1}^{5} (n^3 - 1) = 0 + 7 + 26 + 63 + 124 = 220$

7. $a_n = -3 + 4n$

$a_{12} = 45$

8. $a_n = 43 - 9n$

$a_{12} = -65$

9. $a_n = \frac{n}{2}$

$a_{12} = 6$

10. $a_n = 1.4 + 1.5n - 1.5$

$= -0.1 + 1.5n$

$S_{30} = 30\left(\frac{1.4 + 44.9}{2}\right)$

$= 694.5$

11. $a_n = 2(5)^{n-1}$

$a_{15} = 2(5)^{14} = 1.22 \times 10^{10}$

12. $a_n = -3(-4)^{n-1}$

$a_{15} = -3(-4)^{14} = -805{,}306{,}368$

13. $a_n = 12\left(\frac{1}{3}\right)^{n-1}$

$a_{15} = 12\left(\frac{1}{3}\right)^{14} = 2.509 \times 10^{-6}$

14. $a_n = 2^{n-1}$

$S_{10} = 1\left(\frac{1 - 2^{10}}{1 - 2}\right)$

$= 1023$

Lesson 11.4

Developing Concepts Activity 11.4 (p. 674)

1. 1 **2.** A_n approaches 1

11.4 Guided Practice (p. 678)

1. infinite **2.** if $-1 < r < 1$

3. the first term and common ratio

4. $S_n = \left(\frac{5}{1 - \frac{1}{4}}\right) = \frac{5}{\frac{3}{4}} = \frac{20}{3}$

5. $S_n = \frac{-2}{1 + \frac{1}{4}} = -\frac{2}{\frac{5}{4}} = -\frac{8}{5}$

6. $6 = \frac{1}{1 - r}$

$6 - 6r = 1$

$-6r = -5$

$r = \frac{5}{6}$

7. $12 = \frac{2}{1 - r}$

$12 - 12r = 2$

$-12r = -10$

$r = \frac{5}{6}$

8. $\frac{21}{2} = \frac{\frac{1}{2}}{1 - r}$

$\frac{21}{2} - \frac{21}{2}r = \frac{1}{2}$

$-\frac{21}{2}r = -\frac{20}{2}$

$r = \frac{20}{21}$

9. $0.555 \ldots = 5(0.1) + 5(0.01) + \ldots$

$= \frac{5(0.1)}{1 - 0.1}$

$= \frac{0 - 0.5}{0.9} = \frac{5}{9}$

10. $0.1212 \ldots = 12(0.01) + 12(0.01)^2 + \ldots$

$= \frac{12(0.01)}{1 - 0.01}$

$= \frac{0.12}{0.99} = \frac{12}{99} = \frac{4}{33}$

11. $245.245245 = 245{,}000(0.001) + (245{,}000)(0.001)^2 + (245{,}000)(0.001)^3 + \ldots$

$= \frac{245{,}000(0.001)}{1 - 0.001}$

$= \frac{245}{0.999} = \frac{245{,}000}{999}$

12. a. $d = 5 + 2[5(0.5)] + 2[5(0.5)^2] + \ldots$

$= 5 + 10(0.5) + 10(0.5)^2 + \ldots$

$= 5 + \frac{10(0.5)}{1 - 0.5}$

$= 5 + 10 = 15$ ft

b. $0.75(15) = 5 + 10\left(\frac{1 - (0.5)^n}{1 - 0.5}\right)$

$6.25 = 10\left(\frac{1 - (0.5)^n}{0.5}\right)$

$\frac{\log 0.25}{\log 0.5} = 2$

11.4 Practice and Applications (pp. 678–680)

13. no; $|r| = \frac{3}{2}, \frac{3}{2} > 1$ **14.** yes; $|r| = \frac{1}{5}, \frac{1}{5} < 1$

15. yes; $|r| = \frac{1}{3}, \frac{1}{3} < 1$ **16.** no; $|r| = \frac{4}{3}, \frac{4}{3} > 1$

17. $\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n = \frac{1}{1 - 0.5} = \frac{1}{\frac{1}{2}} = 2$

18. $\sum_{n=0}^{\infty} 3\left(\frac{2}{3}\right)^n = \frac{3}{1 - \frac{2}{3}} = \frac{3}{\frac{1}{3}} = 9$

19. $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1} = \frac{1}{1 + \frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

20. $\sum_{n=0}^{\infty}\frac{2}{7}(2)^n =$ no sum

21. $\sum_{n=0}^{\infty} 4\left(\frac{1}{4}\right)^n = \frac{4}{1 - \frac{1}{4}} = \frac{4}{\frac{3}{4}} = \frac{16}{3}$

22. $\sum_{n=1}^{\infty}\left(\frac{1}{10}\right)^{n-1} = \frac{1}{1 - \frac{1}{10}} = \frac{1}{\frac{9}{10}} = \frac{10}{9}$

23. $\sum_{n=0}^{\infty} 2\left(\frac{6}{5}\right)^n =$ no sum

24. $\sum_{n=0}^{\infty} 4\left(\frac{3}{7}\right)^n = \frac{4}{1 - \frac{3}{7}} = \frac{4}{\frac{4}{7}} = 7$

25. $\sum_{n=0}^{\infty} -\frac{1}{8}\left(-\frac{1}{2}\right)^n = \frac{-\frac{1}{8}}{1 + \frac{1}{2}} = \frac{-\frac{1}{8}}{\frac{3}{2}} = -\frac{1}{12}$

26. $\sum_{n=1}^{\infty}\frac{1}{2}\left(-\frac{2}{5}\right)^{n-1} = \frac{\frac{1}{2}}{1 + \frac{2}{5}} = \frac{\frac{1}{2}}{\frac{7}{5}} = \frac{5}{14}$

27. $\sum_{n=0}^{\infty}\frac{1}{12}\left(-\frac{3}{25}\right)^n = \frac{\frac{1}{12}}{1 + \frac{3}{25}} = \frac{\frac{1}{12}}{\frac{28}{25}} = \frac{25}{336}$

28. $\sum_{n=1}^{\infty} -\left(-\frac{2}{11}\right)^{n-1} = \frac{-1}{1 + \frac{2}{11}} = \frac{-1}{\frac{13}{11}} = -\frac{11}{13}$

29. $4 = \dfrac{1}{1 - r}$

$4 - 4r = 1$

$-4r = -3$

$r = \dfrac{3}{4}$

30. $10 = \dfrac{1}{1 - r}$

$10 - 10r = 1$

$-10r = -9$

$r = \dfrac{9}{10}$

31. $12 = \dfrac{3}{1 - r}$

$12 - 12r = 3$

$-12r = -9$

$r = \dfrac{3}{4}$

32. $8 = \dfrac{2}{1 - r}$

$8 - 8r = 2$

$-8r = -6$

$r = \dfrac{3}{4}$

33. $6 = \dfrac{2}{1 - r}$

$6 - 6r = 2$

$-6r = -4$

$r = \dfrac{2}{3}$

34. $50 = \dfrac{4}{1 - r}$

$50 - 50r = 4$

$-50r = -46$

$r = \dfrac{23}{25}$

35. $-\dfrac{1}{9} = \dfrac{-\frac{1}{6}}{1 - r}$

$-\dfrac{1}{9} + \dfrac{1}{9}r = -\dfrac{1}{6}$

$\dfrac{1}{9}r = -\dfrac{1}{18}$

$r = -\dfrac{1}{2}$

36. $-\dfrac{11}{13} = \dfrac{-1}{1 - r}$

$-\dfrac{11}{13} + \dfrac{11}{13}r = -1$

$\dfrac{11}{13}r = -\dfrac{2}{13}$

$r = -\dfrac{2}{11}$

37. $\dfrac{20}{9} = \dfrac{4}{1 - r}$

$\dfrac{20}{9} - \dfrac{20}{9}r = 4$

$-\dfrac{20}{9}r = \dfrac{16}{9}$

$r = -\dfrac{4}{5}$

38. $0.444 \ldots = 4(0.1) + 4(0.1)^2 + \ldots$

$= \dfrac{4(0.1)}{1 - 0.1}$

$= \dfrac{0.4}{0.9} = \dfrac{4}{9}$

39. $0.777 \ldots = 7(0.1) + 7(0.1)^2 + \ldots$

$= \dfrac{7(0.1)}{1 - 0.1}$

$= \dfrac{0.7}{0.9} = \dfrac{7}{9}$

40. $0.999 = 9(0.1) + 9(0.1)^2 + \ldots$

$= \dfrac{9(0.1)}{1 - 0.1}$

$= \dfrac{0.9}{0.9} = 1$

41. $0.515151 \ldots = 51(0.01) + 51(0.01)^2 + \ldots$

$= \dfrac{51(0.01)}{1 - 0.01}$

$= \dfrac{0.51}{0.99} = \dfrac{17}{33}$

42. $0.2323 \ldots = 23(0.01) + 23(0.01)^2 + \ldots$

$= \dfrac{23(0.01)}{1 - 0.01}$

$= \dfrac{0.23}{0.99} = \dfrac{23}{99}$

43. $0.1616 \ldots = 16(0.01) + 16(0.01)^2 + \ldots$

$= \dfrac{16(0.01)}{1 - 0.01}$

$= \dfrac{0.16}{0.99} = \dfrac{16}{99}$

44. $63.6363 \ldots = 6300(0.01) + 6300(0.01)^2 + \ldots$

$= \dfrac{6300(0.01)}{1 - 0.01}$

$= \dfrac{63}{0.99} = \dfrac{6300}{99}$

45. $120.120120 \ldots$

$= (120{,}000)(0.001) + 120{,}000(0.001)^2 + \ldots$

$= \dfrac{120{,}000(0.001)}{1 - 0.001}$

$= \dfrac{120}{0.999} = \dfrac{120{,}000}{999}$

46. $297.297297 = 297{,}000(0.001) + 297{,}000(0.001)^2 + \ldots$

$= \dfrac{297{,}000(0.001)}{1 - 0.001}$

$= \dfrac{297}{0.999} = \dfrac{297{,}000}{999}$

47. $S = \dfrac{18}{1 - 0.9} = 180$ in. ;

$(180)(0.8) = 18 + 36\left(\dfrac{1 - 0.9^n}{1 - 0.9}\right)$

$144 = 18 + 3.6(1 - 0.9^n)$

$126 = 3.6(1 - 0.9^n)$

$35 = 1 - 0.9^n$

$34 = 0.9^n$

$n = 33$; on the 16th pass

48. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^n = \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

49. total distance $= \dfrac{20}{1 - \frac{1}{2}} = 40$ ft

—CONTINUED—

49.—CONTINUED—

$$\text{total time} = \frac{1}{1 - \frac{1}{2}} = 2 \text{ sec}$$

50. $\sum_{n=0}^{\infty} M\$1(0.805)^n = \frac{M\$1}{1 - 0.805} = \frac{M\$1}{0.195} = M\$5.13$

51. $\frac{M\$4.72}{0.198} = M\$24.21$

52. *Sample answer:*

$1 + \frac{2}{3} + \left(\frac{2}{3}\right)^2 + \ldots$

$4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \ldots$

53. B **54.** D

55. a.

	1	2	3
Number of new triangles	1	3	12
Area of each new triangle	$\frac{\sqrt{3}}{4}$	$\frac{\sqrt{3}}{36}$	$\frac{\sqrt{3}}{324}$
Total new area	$\frac{\sqrt{3}}{4}$	$\frac{\sqrt{3}}{12}$	$\frac{\sqrt{3}}{27}$

	4	. . .	n
Number of new triangles	48	. . .	$3(4)^{n-2}$
Area of each new triangle	$\frac{\sqrt{3}}{2916}$	. . .	$\frac{\sqrt{3}}{4 \cdot 9^{n-1}}$
Total new area	$\frac{\sqrt{3}}{60.75}$	. . .	$\left(\frac{3\sqrt{3}}{16}\right)\left(\frac{4}{9}\right)^{n-1}$

b. 0.69282

11.4 Mixed Review (p. 680)

56. x-axis; y-axis; domain: $x \neq 0$; range: $y \neq 0$

57. x-axis; y-axis; domain: $x \neq 0$; range: $y \neq 0$

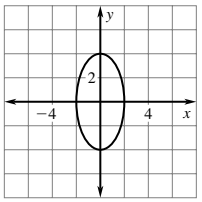
58. $y = -3$; y-axis; domain: $x \neq 0$; range: $y \neq -3$

59. $y = 1$; $x = -7$; domain: $x \neq -7$; range: $y \neq 1$

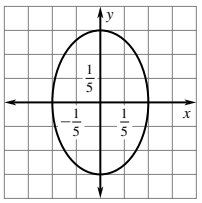
60. $y = -2$; $x = \frac{17}{2}$; domain: $x \neq \frac{17}{2}$; range: $y \neq -2$

61. $y = 2.2$; $x = 0.7$; domain: $x \neq 0.7$; range: $y \neq 2.2$

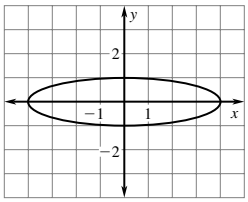
62. $\frac{x^2}{4} + \frac{y^2}{16} = 1$



63. $\frac{25x^2}{4} + \frac{25y^2}{9} = 1$



64. $\frac{x^2}{16} + y^2 = 1$



65. $a_n = -3 + 4n$

66. $a_n = -10 + 8n$

67. $a_n = 13 - 2n$

68. $a_n = -5(2)^{n-1}$

69. $a_n = 4\left(\frac{1}{2}\right)^{n-1}$

70. $a_n = -10\left(\frac{7}{8}\right)^{n-1}$

Lesson 11.5

Activity (p. 681)

1. a. 3, 8, 13, 18, 23 **b.** 3, 6, 12, 24, 48

2. a. arithmetic **b.** geometric

11.5 Guided Practice (p. 684)

1. $n!$

2. *Sample answer*: An explicit rule for a sequence gives an expression for the nth term as a function of n. A recursive rule gives the value of one or more beginning terms and an expression for the nth term as a function of one or more preceding terms.

3. $a_n = n^2 - 4n$;
$a_1 = 2$;
$a_n = a_{n-1} + n$

4. 1, 2, 3, 4, 5

5. 2, 8, 32, 128, 512

6. 1, −1, −3, −5, −7

7. −1, 3, −9, 27, −81

8. 2, 1, −1, −5, −13

9. 3, 10, 101, 10,202, 104,080,805

10. $a_1 = 21$
$a_n = a_{n-1} - 4$

11. $a_1 = 2$
$a_n = (3)a_{n-1}$

12. $a_1 = \frac{1}{2}$
$a_n = (\frac{1}{2})a_{n-1}$

13. $a_n = (0.7)a_{n-1} + 750$
$a_1 = 5200$
$a_5 = (0.7)a_4 + 750$
$a_5 = 3148$

11.5 Practice and Application (pp. 684–686)

14. 1, 5, 9, 13, 17

15. 4, 12, 21, 31, 42

16. 0, −1, −5, −14, −30

17. −4, −12, −20, −28, −36

18. 2, 6, 38, 1446, 2,090,918

19. 5, −4, 8, 1, 15

20. 10, 30, 90, 270, 810

21. 2, 1, 7, 8, 16

22. 3, 7, 47, 2207, 4,870,847

23. 48, 26, 15, 9.5, 6.75

24. 4, 2, −2, −4, −2

25. 1, 3, 3, 9, 27

26. $a_n = 2 \cdot 10^{n-1}$
$a_1 = 2$
$a_n = 10 \cdot a_{n-1}$

27. $a_n = -7 + 10n$
$a_1 = 3$
$a_n = a_{n-1} + 10$

28. $a_n = 10 \cdot 2^{n-1}$
$a_1 = 10$
$a_n = 2 \cdot a_{n-1}$

29. $a_n = 2 + 3n$
$a_1 = 5$
$a_n = a_{n-1} + 3$

30. $a_n = 1 - n$
$a_1 = 0$
$a_n = a_{n-1} - 1$

31. $a_n = 5(2.5)^{n-1}$
$a_1 = 5$
$a_n = (2.5)a_{n-1}$

32. $a_n = 13.5 + \left(\frac{1}{2}\right)n$
$a_1 = 14$
$a_n = a_{n-1} + \frac{1}{2}$

33. $a_n = \frac{1}{2}(4^{n-1})$
$a_1 = \frac{1}{2}$
$a_n = 4a_{n-1}$

34. $a_n = \frac{1}{2} - \left(\frac{3}{2}\right)n$
$a_1 = -1$
$a_n = a_{n-1} - \frac{3}{2}$

35. $a_1 = 1$
$a_n = a_{n-1} + 6$

36. $a_1 = 66$
$a_n = \frac{1}{2}a_{n-1}$

37. $a_1 = 41$
$a_n = a_{n-1} - 9$

38. $a_1 = 3$
$a_n = (a_{n-1})^2 - 1$

39. $a_1 = 33$
$a_n = \frac{1}{3}a_{n-1}$

40. $a_1 = 7.2$
$a_n = a_{n-1} - 4$

41. $a_1 = 2$
$a_2 = 5$
$a_n = (a_{n-1})(a_{n-2})$

42. $a_1 = 6$
$a_n = \left(\sqrt{2}\right)a_{n-1}$

43. $a_1 = 48$
$a_n = \frac{1}{10}a_{n-1}$

44. $a_1 = 150$
$a_n = a_{n-1} + 25$
$500 - a_{12} = 500 - 425 = \$75$

45. 1, 2, 4, 8, 16, 32, 64; geometric

46. $a_n = 2^{n-1}$;
$a_1 = 1$
$a_n = (2)a_{n-1}$

47. $a_1 = 32$
$a_n = (0.6)a_{n-1} + 14$
$a_6 = 34.77$

48. 35 oz

49. $a_1 = 9000$
$a_n = (0.9)a_{n-1} + 800$
$a_3 = 8729$

50. 8000

51. $a_1 = 20$
$a_n = (0.7)a_{n-1} + 20$

52. about 66.7 mg

53. no

54. yes

55. *Sample answer:* $a_1 = 1$; $a_2 = 2$; $a_3 = 3$;
$a_n = a_{n-1} + a_{n-3}$; 1, 2, 3, 4, 6, 9, 13, 19

56. $a_5 = 2a_4 + 9 = 295$; D

57. $a_n = (-1.65)a_{n-1}$; B

58. **a.** 7, 22, 11, 34, 17, 52, 26, 13, 40, 20

b. *Sample answer:* Let $a_1 = 1$. The first 10 terms will be 1, 4, 2, 1, 4, 2, 1, 4, 2, 1. Let $a_1 = 2$. The first 10 terms will be 2, 1, 4, 2, 1, 4, 2, 1, 4, 2. Let $a_1 = 3$. The first 10 terms will be 3, 10, 5, 16, 8, 4, 2, 1, 4, 2. It appears that no matter what value you start with, eventually the series repeats the value 1, 4, 2.

11.5 Mixed Review (p. 686)

59. $2^5 = 32$

60. $6^4 = 1296$

61. $8^4 = 4096$

62. $12^3 = 1728$

63. $26^3 = 17{,}576$

64. $10^5 = 100{,}000$

65. $18^3 = 5832$

66. $3^7 = 2187$

67. $\frac{3}{5x} + \frac{3}{7x} = \frac{21 + 15}{35x} = \frac{36}{35x}$

68. $-\frac{2}{7x} - \frac{5}{3x} = \frac{-6 - 35}{21x} = \frac{-41}{21x}$

69. $\frac{x + 1}{(x^2 - 9)} - \frac{5}{x - 3} = \frac{x + 1 - 5x - 15}{(x + 3)(x - 3)} = \frac{-4x - 14}{x^2 - 9}$

70. $\frac{2x^2}{3x + 5} - \frac{14}{x + 7} = \frac{2x^3 + 14x^2 - 42x - 70}{3x^2 + 26x + 35}$

71. $\frac{4x + 1}{x^2 - 4} - \frac{3}{x - 2} = \frac{x - 5}{x^2 - 4}$

72. $\frac{x^2 - 1}{x + 2} - \frac{3}{x + 1} = \frac{x^3 + x^2 - 4x - 7}{x^2 + 3x + 2}$

73. $x^2 + y^2 = 4$
$2x + y = -1$
$(-1.272, 1.544)$
$(0.472, -1.944)$

74. $x^2 + y^2 = 25$
$y = x - 1$
$(4, 3)$
$(-3, -4)$

75. $x^2 + 4y^2 = 16$
$y = 3x + 1$
$(-0.980, -1.939)$
$(0.331, 1.993)$

76. $x^2 + y^2 = 10$
$4x + y = 6$
$(0.731, 3.077)$
$(2.093, -2.371)$

77. $x^2 + y^2 = 30$
$y = x + 2$
$(-4.742, -2.742)$
$(2.742, 4.742)$

78. $16x^2 + y^2 = 32$
$\frac{1}{4}x - \frac{1}{2}y = 2$
$(-0.877, -4.438)$
$(1.123, -3.439)$

79. 7, 6, 5, 4, 3, 2

80. 1, 16, 81, 256, 625, 1296

81. 10, 13, 18, 25, 34, 45

82. 16, 25, 36, 49, 64, 81

83. $\frac{1}{5}, \frac{1}{3}, \frac{3}{7}, \frac{1}{2}, \frac{5}{9}, \frac{3}{5}$

84. $2, \frac{5}{3}, \frac{3}{2}, \frac{7}{5}, \frac{4}{3}, \frac{9}{7}$

Quiz 2 (p. 687)

1. $\sum_{n=0}^{\infty} 4\left(\frac{1}{9}\right)^n = \frac{4}{1 - \frac{1}{9}} = \frac{9}{2}$

2. $\sum_{n=1}^{\infty} 5\left(-\frac{6}{7}\right)^{n-1} = \frac{5}{1 + \frac{6}{7}} = \frac{35}{13}$

3. $\sum_{n=0}^{\infty} -\frac{3}{8}\left(\frac{4}{7}\right)^n = \frac{-\frac{3}{8}}{1 - \frac{4}{7}} = -\frac{7}{8}$

4. $\sum_{n=0}^{\infty} \frac{4}{5}\left(\frac{5}{4}\right)^n$ = no sum

5. $5 = \dfrac{1}{1 - r}$

$5 - 5r = 1$

$-5r = -4$

$r = \dfrac{4}{5}$

6. $12 = \dfrac{1}{1 - r}$

$12 - 12r = 1$

$-12r = -11$

$r = \dfrac{11}{12}$

7. $24 = \dfrac{3}{1 - r}$

$24 - 24r = 3$

$-24r = -21$

$r = \dfrac{7}{8}$

8. $0.888\ldots = 8(0.1) + 8(0.1)^2 + \ldots$

$= \dfrac{8(0.1)}{1 - 0.1} = \dfrac{0.8}{0.9} = \dfrac{8}{9}$

9. $0.151515\ldots = 15(0.01) + 15(0.01)^2 + \ldots$

$= \dfrac{15(0.01)}{1 - 0.01} = \dfrac{0.15}{0.99} = \dfrac{5}{33}$

10. $126.126126\ldots = 126000(0.001) + 126000(0.001)^2 + \ldots$

$= \dfrac{126000(0.001)}{1 - .001} = \dfrac{126}{0.999} = \dfrac{126{,}000}{999}$

11. 5, 8, 11, 14, 17

12. 1, 4, 16, 64, 256

13. 17, 19, 22, 26, 31

14. 1, 2, 1, −1, −2

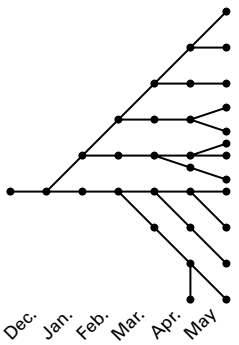
15. 2, 4, 8, 32, 256

16. 10, 10, 20, 30, 50

17. $18\frac{2}{3}$ ft

Math & History (p. 687)

1.



2. 377 pairs of rabbits for a total of 754 rabbits; increases without bound

Technology Activity 11.5 (p. 688)

1.

5100	
4465	
3893.50	
3379.15	
2916.24	
2499.61	
2124.65	
1787.19	
1483.47	
1210.12	
964.11	
742.70	

2.

6150	
4843	
3771.26	
2892.43	
2171.80	
1580.87	
1096.32	
698.98	
373.16	
105.99	
−113.09	
−292.73	

3.

3500	
2925	
2436	
2021	
1668	
1368	
1112	
896	
711	
555	
421	
308	

4.

7500	
5638	
4194	
3075	
2208	
1537	
1016	
612	
299	
57	
−131	
−276	

5. $a_1 = 8000$

$a_n = (1.01)a_{n-1} - 125$

$n = 103$ months

Chapter 11 Extension *(p. 690)*

1. $\dfrac{1(1 + 1)(2 \cdot 1 + 1)}{6} = \dfrac{6}{6} = 1 = 1^2$, so the formula is true for $n = 1$. Suppose $1^2 + 2^2 + \ldots + k^2 = \dfrac{k(k + 1)(2k + 1)}{6}$ is true.

Then $1^2 + 2^2 + \ldots + k^2 + (k + 1)^2 = \dfrac{k(k + 1)(2k + 1)}{6} + (k + 1)^2 =$

$\dfrac{k(k + 1)(2k + 1) + 6(k + 1)^2}{6} =$

$\dfrac{(k + 1)(2k^2 + 7k + 6)}{6} = \dfrac{(k + 1)(k + 2)(2k + 3)}{6} =$

$\dfrac{(k + 1)[(k + 1) + 1][2(k + 1) + 1]}{6}$, and the formula is true for $n = k + 1$. Therefore, the formula is true for all positive integers.

2. $1(1 + 2) = 3 = 2 \cdot 1 + 1$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k}(2i + 1) = k(k + 2)$ is true.

Then $\sum_{i=1}^{k+1}(2i + 1) = k(k + 2) + [2(k + 1) + 1] =$

$k^2 + 4k + 3 = (k + 1)(k + 3) = (k + 1)[(k + 1) + 2]$, and the formula is true for $n = k + 1$. Therefore, the formula is true for all positive integers.

3. $\dfrac{a_1(1 - r^1)}{1 - r} = a_1 = a_1 \cdot r^{1-1}$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k} a_1 r^{i-1} = \dfrac{a_1(1 - r^k)}{1 - r}$ is true. Then

$\sum_{i=1}^{k+1} a_1 r^{i-1} = \frac{a_1(1-r^k)}{1-r} + a_1 r^{k+1-1} =$
$\frac{a_1(1-r^k) + a_1(r^k)(1-r)}{1-r} = \frac{a_1[(1-r^k) + r^k(1-r)]}{1-r} =$
$\frac{a_1(1-r^{k+1})}{1-r}$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

4. $\frac{2(1)(1+1)(2\cdot 1+1)}{3} = \frac{12}{3} = 4 = (2\cdot 1)^2$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k}(2i)^2 =$ $\frac{2k(k+1)(2k+1)}{3}$ is true. Then $\sum_{i=1}^{k+1}(2i)^2 =$ $\frac{2k(k+1)(2k+1)}{3} + (2k+2)^2 =$ $\frac{2k(k+1)(2k+1) + 3(2k+2)^2}{3} =$ $\frac{(2k+2)[k(2k+1) + 3(2k+2)]}{3} =$ $\frac{2(k+1)(2k^2+7k+6)}{3} = \frac{2(k+1)(2k+3)(k+2)}{3} =$ $\frac{2(k+1)[(k+1)+1][2(k+1)+1]}{3}$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

5. $\frac{5^{1+1}-5}{4} = \frac{25-5}{4} = \frac{20}{4} = 5 = 5^1$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k} 5^i = \frac{5^{k+1}-5}{4}$ is true. Then $\sum_{i=1}^{k+1} 5^i = \frac{5^{k+1}-5}{4} + 5^{k+1} = \frac{[(5^{k+1}-5) + 4(5^{k+1})]}{4} =$ $\frac{5(5^{k+1})-5}{4} = \frac{5^{(k+1)+1}-5}{4}$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

6. $\left(\frac{1}{2}\right)^1 = \frac{1}{2} = 1 - \left(\frac{1}{2}\right)^1$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k}\left(\frac{1}{2}\right)^i = 1 - \left(\frac{1}{2}\right)^k$ is true. Then $\sum_{i=1}^{k+1}\left(\frac{1}{2}\right)^i =$ $1 - \left(\frac{1}{2}\right)^k + \left(\frac{1}{2}\right)^{k+1} = 1 - \left(\frac{1}{2}\right)^k + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^k =$ $1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^k = 1 - \left(\frac{1}{2}\right)^{k+1}$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

7. A recursive formula for the nth pentagonal number is $P_n = p_{n-1} + 3n - 2$. $\frac{1(3\cdot 1-1)}{2} = 1 = P_1$, so the formula is true for $n = 1$. Suppose $P_k = \frac{k(3k-1)}{2}$ is true. Then $P_{k+1} = \frac{k(3k-1)}{2} + 3(k+1) - 2 =$ $\frac{3k^2-k+6k+6-4}{2} = \frac{3k^2+5k+2}{2} =$ $\frac{(k+1)(3k+2)}{2} = \frac{(k+1)[3(k+1)-1]}{2}$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

8. The Fibonocci sequence is 1, 1, 2, 3, 5, 8, 13, ... , and for each $n > 2$, $f_n = f_{n-1} + f_{n-2}$. Proof:

$f_1 = 1 = 2 - 1 = f_3 - 1$, so the formula is true for $n = 1$. Suppose $\sum_{i=1}^{k} f_i = f_{n+2} - 1$ is true. Then $\sum_{i=1}^{k+1} f_i =$ $f_{k+2} - 1 + f_{k+1} = (f_{k+2} + f_{k+1}) - 1 = f_{k+3} - 1 =$ $f_{(k+1)+2} - 1$, and the formula is true for $n = k+1$. Therefore, the formula is true for all positive integers.

Chapter 11 Review *(pp. 692–694)*

1. 6, 9, 14, 21, 30, 41

2. 8, 27, 64, 125, 216, 343

3. 4, 2, 0, -2, -4, -6

4. $\frac{1}{4}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}, \frac{5}{8}, \frac{2}{3}$

5. 10; $a_n = 2n$

6. -48; $a_n = -3(-2)^{n-1}$

7. $\frac{1}{243}$; $a_n = \left(\frac{1}{3}\right)^n$

8. $\sum_{i=1}^{4} 4i$

9. $\sum_{i=1}^{\infty} i$

10. $\sum_{i=0}^{4} 3i$

11. $\sum_{n=1}^{25} n^2 = \frac{25(26)(51)}{6} = 5525$

12. $\sum_{n=4}^{10} n(2n-1)$
$= 28 + 45 + 66 + 91 + 120 + 153 + 190 = 693$

13. $\sum_{i=1}^{12} i = \frac{12(13)}{2} = 78$

14. $\sum_{k=1}^{30} 4 = 4 \cdot 30 = 120$

15. $a_n = -5 + 6n$

16. $a_n = 2 + 2n$

17. $a_n = 4 - \left(\frac{1}{2}\right)n$

18. $a_n = 8 + 5n$

19. $a_n = 21 - 2n$

20. $a_n = 5n$

21. $a_n = -4 + 12n$
$S_{14} = 14\left(\frac{8 + 164}{2}\right) = 1204$

22. $a_n = -10 + 4n$
$S_{20} = 20\left(\frac{-6 + (70)}{2}\right) = 640$

23. $a_n = 0.1 + 0.4n$

$$S_{54} = 54\left(\frac{0.5 + 21.7}{2}\right) = 599.4$$

24. $a_n = -16 + 4n$

$$S_{40} = 40\left(\frac{-12 + 144}{2}\right) = 2640$$

25. $a_n = 64\left(\frac{1}{2}\right)^{n-1}$

26. $a_n = 6 \cdot 2^{n-1}$

27. $a_n = 200\left(\frac{1}{10}\right)^{n-1}$

28. $a_n = 6(3)^{n-1}$

29. $a_n = -64\left(-\frac{1}{4}\right)^{n-1}$

30. $a_n = 500\left(\frac{1}{10}\right)^{n-1}$

31. $\sum_{i=1}^{5} 16(2)^{i-1} = 16\left(\frac{1 - 2^5}{1 - 2}\right) = 16(31) = 496$

32. $\sum_{i=1}^{10} 20(0.2)^{i-1} = 20\left(\frac{1 - (0.2)^{10}}{1 - 0.2}\right)$

$$= 20(1.24999) \approx 25$$

33. $\sum_{i=0}^{6} 10\left(\frac{1}{2}\right)^i = 10\left(\frac{1 - (0.5)^6}{1 - 0.5}\right)$

$$= 10(1.96875) = 19.6875$$

34. $\sum_{i=1}^{8} 2\left(\frac{3}{5}\right)^{i-1} = 2\left(\frac{1 - \left(\frac{3}{5}\right)^8}{1 - \frac{3}{5}}\right)$

$$= 2(2.4580096) = 4.9160192$$

35. $\sum_{n=1}^{\infty} 15\left(\frac{2}{9}\right)^{n-1} = \frac{15}{1 - \frac{2}{9}} = \frac{15}{\frac{7}{9}} = \frac{135}{7}$

36. $\sum_{n=1}^{\infty} 3\left(\frac{3}{4}\right)^{n-1} = \frac{3}{1 - \frac{3}{4}} = \frac{3}{\frac{1}{4}} = 12$

37. $\sum_{n=1}^{\infty} 5(0.8)^{n-1} = \frac{5}{1 - 0.8} = \frac{5}{0.2} = 25$

38. $\sum_{n=1}^{\infty} 4(-0.2)^{n-1} = \frac{4}{1 + 0.2} = \frac{10}{3}$

39. $18 = \frac{12}{1 - r}$

$18 - 18r = 12$

$-18r = -6$

$r = \frac{1}{3}$

40. $2 = \frac{0.5}{1 - r}$

$2 - 2r = 0.5$

$-2r = -1.5$

$r = \frac{3}{4}$

41. $20 = \frac{4}{1 - r}$

$20 - 20r = 4$

$-20r = -16$

$r = \frac{4}{5}$

42. $-5 = \frac{-2}{1 - r}$

$-5 + 5r = -2$

$5r = 3$

$r = \frac{3}{5}$

43. $-10 = \frac{-3}{1 - r}$

$-10 + 10r = -3$

$10r = 7$

$r = \frac{7}{10}$

44. $6 = \frac{\frac{1}{3}}{1 - r}$

$6 - 6r = \frac{1}{3}$

$-6r = \frac{-17}{3}$

$r = \frac{17}{18}$

45. $\frac{1}{4} = \frac{\frac{1}{16}}{1 - r}$

$\frac{1}{4} - \frac{1}{4}r = \frac{1}{16}$

$-\frac{1}{4}r = \frac{-3}{16}$

$r = \frac{3}{4}$

46. $\frac{10}{3} = \frac{6}{1 - r}$

$\frac{10}{3} - \frac{10}{3}r = 6$

$-\frac{10}{3}r = \frac{8}{3}$

$r = -\frac{4}{5}$

47. $0.2222 \ldots = 2(0.1) + 2(0.1)^2 + \ldots$

$$= \frac{2(.1)}{1 - 0.1} = \frac{0.2}{0.9} = \frac{2}{9}$$

48. $0.4545 \ldots = 45(0.01) + 45(0.01)^2 + \ldots$

$$= \frac{45(0.01)}{1 - 0.01} = \frac{0.45}{0.99} = \frac{5}{11}$$

49. $39.3939 \ldots = 3900(0.01) + 3900(0.01)^2 + \ldots$

$$= \frac{3900(0.01)}{1 - 0.01} = \frac{39}{0.99} = \frac{1300}{33}$$

50. $0.001001 \ldots = 1(0.001) + 1(0.001)^2 + \ldots$

$$= \frac{1(0.001)}{1 - 0.001} = \frac{0.001}{0.999} = \frac{1}{999}$$

51. 10, 40, 160, 640, 2560, 10,240

52. 1; 2; 6; 24; 120; 720

53. 2; 0; −3; −7; −12; −18

54. −1; 4; 19; 364; 132, 499; 17, 555, 985, 004

55. $a_1 = 7$

$a_n = 2a_{n-1}$

56. $a_1 = 4$

$a_n = a_{n-1} + n + 2$

57. $a_1 = 1$

$a_n = a_{n-1} + 5$

58. $a_1 = 200$

$a_n = \left(\frac{1}{2}\right)a_{n-1}$

59. $a_1 = 1$

$a_n = (a_{n-1})^2 + 1$

60. $a_1 = -2$

$a_n = a_{n-1} - 2n$

Chapter 11 Test *(p. 695)*

1. arithemic; $d = 2$

2. neither

3. geometric; $r = \frac{1}{2}$

4. geometric; $r = 3$

5. 2, 5, 10, 17, 26, 37

6. −2, 1, 4, 7, 10, 13

7. 4, 6, 9, 13, 18, 24

8. 1, 2, 4, 8, 16, 32

9. $a_5 = 32$

$a_n = 2^n$

10. $a_5 = 24$

$a_n = 5n - 1$

11. $a_5 = 1250$

$a_n = (2)5^{n-1}$

12. $a_5 = -13$

$a_n = -8 - n$

13. $a_5 = \frac{5}{16}$

$a_n = 5\left(-\frac{1}{2}\right)^{n-1}$

14. $a_5 = \frac{6}{7}$

$a_n = \frac{n + 1}{n + 2}$

15. $a_5 = \frac{7}{10}$

$a_n = \frac{n + 2}{2n}$

16. $a_5 = 5.5$

$a_n = 1.1n$

17. $a_1 = 4$
$a_n = 0.3a_{n-1}$

18. $a_1 = 1$
$a_n = a_{n-1} + 4$

19. $a_1 = 40$
$a_n = \left(\frac{1}{2}\right)a_{n-1}$

20. $a_1 = 2$
$a_n = a_{n-1} + 4n - 2$

21. $\sum_{i=1}^{100} i = \frac{(100)(101)}{2} = 5050$

22. $\sum_{i=2}^{5} \frac{1}{2}i^2 = \frac{4}{2} + \frac{9}{2} + \frac{16}{2} + \frac{25}{2} = \frac{54}{2} = 27$

23. $\sum_{i=1}^{6} (i - 10) = -9 - 8 - 7 - 6 - 5 - 4 = -39$

24. $\sum_{i=1}^{20} (3i + 2) = 3\sum_{i=1}^{20} i + \sum_{i=1}^{20} 2 = 3\left[\frac{20(21)}{2}\right] + 2 \cdot 20$
$= 630 + 40 = 670$

25. $\sum_{i=1}^{5} 7(-2)^{i-1} = 7 - 14 + 28 - 56 + 112 = 77$

26. $\sum_{i=0}^{9} 5\left(\frac{1}{4}\right)^i = 5\left(\frac{1 - \left(\frac{1}{4}\right)^{10}}{1 - \frac{1}{4}}\right) = 5\left(\frac{1 - \left(\frac{1}{4}\right)^{10}}{\frac{3}{4}}\right) = 6.667$

27. $\sum_{i=1}^{\infty} 64\left(-\frac{1}{2}\right)^{i-1} = \frac{64}{1 + \frac{1}{2}} = \frac{64}{\frac{3}{2}} = \frac{128}{3}$

28. $\sum_{i=1}^{\infty} 100\left(\frac{7}{10}\right)^{i-1} = \frac{100}{1 - \frac{7}{10}} = \frac{100}{\frac{3}{10}} = \frac{1000}{3}$

29. $11 = a_1 + 2d$
$3 = a_1$
$11 = 3 + 2d$
$8 = 2d$
$4 = d$
$a_n = 3 + 4n - 4$
$a_n = 4n - 1$
$a_{30} = 119$
$S_{30} = 30\left(\frac{3 + 119}{2}\right) = 1830$

30. $a_n = 2\left(\frac{1}{2}\right)^{n-1}$
$S = \frac{2}{1 - \frac{1}{2}} = \frac{2}{\frac{1}{2}} = 4$

31. $\sum_{i=1}^{6} (2i - 1)$

32. $0.7575\ldots = 75(0.01) + 75(0.01)^2 + \ldots$
$= \frac{75(0.01)}{1 - 0.01} = \frac{0.75}{0.99} = \frac{25}{33}$

33. $a_n = -4.9 + 9.8n$
$a_{10} = 93.1$
$S_{10} = 10\left(\frac{4.9 + 93.1}{2}\right)$
$= 490m$

34. $a_n = 2^{n-1}$
$S_9 = \left(\frac{1 - 2^9}{1 - 2}\right)$
$= 511$

35. yes;
$\sum_{i=1}^{\infty} 20\left(\frac{9}{10}\right)^{i-1} = \frac{20}{1 - \frac{9}{10}} = \frac{20}{\frac{1}{10}}$
$= 200$

Chapter 11 Standardized Test *(pp. 696–697)*

1. C **2.** A **3.** B **4.** E

5. $9 = a_1 + 13.2$
$-17 = a_1$
$a_n = -17 + 2n - 2$
$a_n = -19 + 2n$
E

6. $a_n = 2 + 15n - 15$
$a_n = -13 + 15n$
$S_{50} = 50\left(\frac{2 + 737}{2}\right)$
$= 18{,}475$
C

7. $-12 = a_1(3)^2$
$-\frac{4}{3} = a_1$
$a_n = -\frac{4}{3}(3)^{n-1}$
A

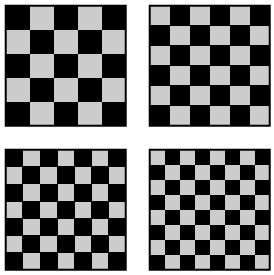
8. $\sum_{i=0}^{9} 20\left(\frac{1}{2}\right)^i = 20\left(\frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}}\right) = 39.96$
D

9. E **10.** E **11.** D **12.** C **13.** A **14.** B

15. a. the number of squares on a side

b. the number of blue squares

c.



d.

n	1	2	3	4	5	6	7	8
a_n	1	2	5	8	13	18	25	32

e. neither geometric or arithmetic

16. a. $8\left(\frac{4 + 25}{2}\right) = 116$

b. 116; Multiply the sum of the first and last terms by 4.

c. $\sum_{n=1}^{8} (1 + 3n)$

d. $\sum_{n=0}^{7} (1 + 3(n + 1))$

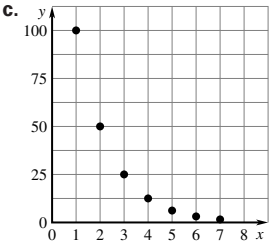
e. $\sum_{n=4}^{11} (1 + 3(n - 3))$

f. Students' answers may vary on style .

17. a. geometric; infinite

b. 6.25, 3.125, 1.5625

c.



d. $a_n = 100\left(\frac{1}{2}\right)^{n-1}$

e. $a_1 = 100$
$a_n = \left(\frac{1}{2}\right)a_{n-1}$

f. $a_{12} = 100\left(\frac{1}{2}\right)^{11} = \frac{25}{512}$
the explicit rule