# Honors Algebra 2 A
# Semester Exam Review
# 2015–2016

Exam Formulas

General Exponential Equation: $y = ab^x$

Exponential Growth: $A(t) = A_0(1+r)^t$

Exponential Decay: $A(t) = A_0(1-r)^t$

Continuous Growth: $A(t) = A_0e^{rt}$

Continuous Decay: $A(t) = A_0e^{-rt}$

Compound Interest ($n$ compoundings per year): $F(t) = P\left(1+\frac{r}{n}\right)^{nt}$

Compound Interest (continuous compounding): $F(t) = Pe^{rt}$

$\log_b N = p$ if and only if $b^p = N$

The average rate of change for a function $f$ on the interval $[a,b]$ : $\frac{f(b)-f(a)}{b-a}$

If $ax^2+bx+c=0,$ then $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Factor Theorem: $P(a)=0$ if and only if $(x-a)$ is a factor of $P(x)$ .

Remainder Theorem: The remainder when a polynomial function $P(x)$ is divided by $(x-a)$ is $P(a)$.

**Note: On this exam review, items marked with [calculator icon] are items for which a calculator may or may not be allowed.**

**Unit 1, Topic 1**

1. Let $f$ and $g$ be functions that are inverses of each other.

   Complete the following statements.

   a. If the point $(a,b)$ is on the graph of *f*, then the point ________ is on the graph of *g*.

   b. If $f(3)=7$, then $g(7)=$ _____ .

   c. The graphs of *f* and *g* are symmetric with respect to the line ______________.

   d. The range of *f* is the same as the ______________ of *g*.

   e. The domain of *f* is the same as the _________________ of *g*.

2. Let $f$ and $g$ be functions that are inverses of each other.

   a. Give an numerical example showing why if $f(x)=x^2$, then $g(x)\neq\frac{1}{x^2}$.

   b. Give a numerical example showing why if $f(x)=3x$, then $g(x)\neq-3x$.

3. Let $f$ and g be functions that are inverses of each other.

   a. If $f(x)=3x-2$, then $g(x)=$ ____________________ .

   b. If $f(x)=2x^3+9$, then $g(x)=$ ____________________ .

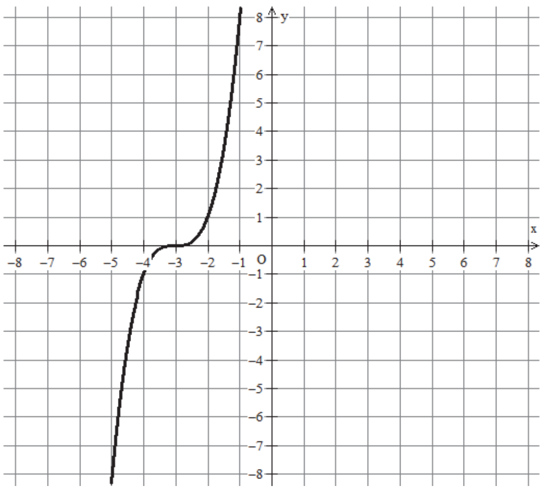
   c. If $f(x)=\sqrt[3]{5x}$, then $g(x)=$ ____________________ .

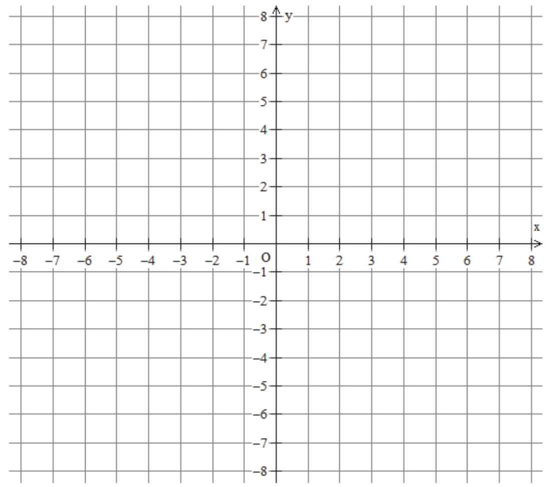
   d. If $f(x)=10^x$, then $g(x)=$ ____________________ .

   e. If $f(x)=\ln x$, then $g(x)=$ ____________________ .

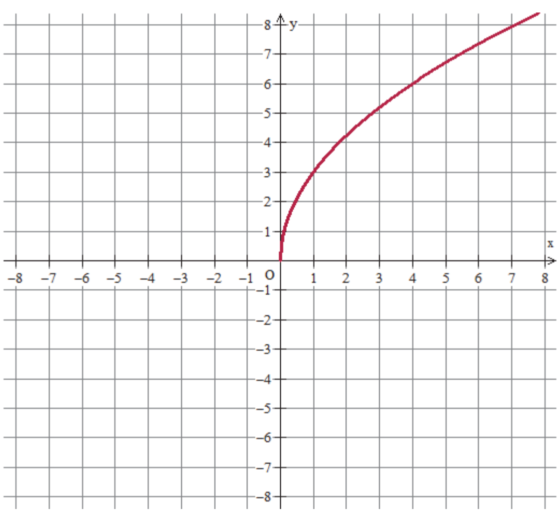
4. For each graph below, sketch the inverse function on the graph to its right.

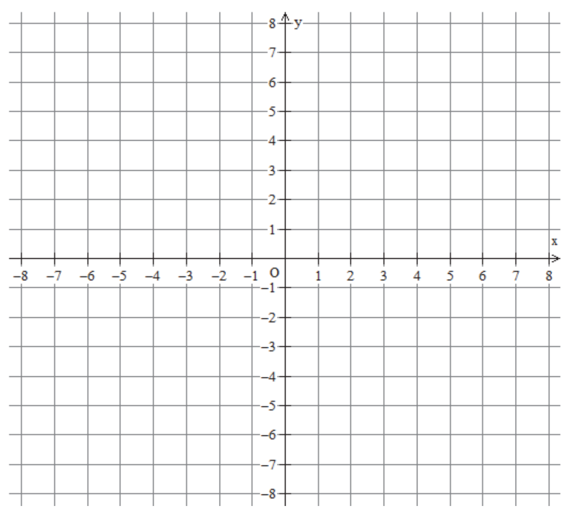
a.





b.





5. Jill sells lemonade. The profit, *p*, in dollars is a function of the number of glasses of lemonade, *g*, that she sells. The function that represents this relationship is $p(g) = 2g - 18$.

   a. Write the function that gives the number of glasses that she will need to sell to earn a profit of *p* dollars.

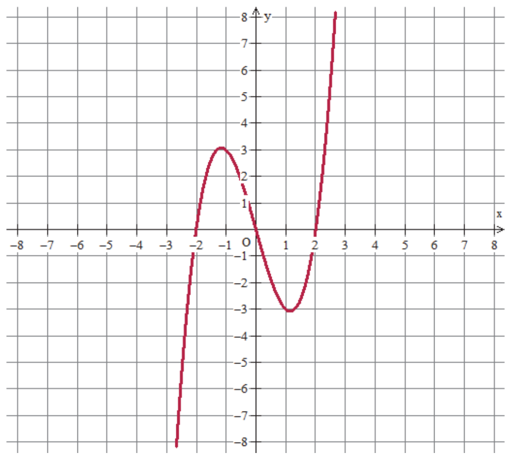
   b. If Jill made a profit of $32, how many glasses did she sell?

6. On a national test, a student receives a score based on the number of correct items. The score, *s*, in points is a function of the number of correct items *c*. The function that represents this relationship is $s(c) = 200 + 2.5c$

   a. Write a function that gives the number of correct items that it will take to receive a score of *s*.

   b. A student received a score of 325. How many items did the student get correct?

7. Look at the graph of a function below.



a. Explain why the inverse of this function is NOT a function.

b. If the domain of the function is restricted to $x \geq 1$, would the inverse be a function?

**Unit 1, Topic 2**

8. Write the following expression as a radical.

a. $x^{\frac{1}{5}}$  b. $y^{-\frac{1}{3}}$  c. $z^{\frac{2}{3}}$

9. Determine the exponent that goes into the box.

a. $\frac{1}{\sqrt{x}} = x^{\square}$  b. $\sqrt[5]{x^4} = x^{\square}$  c. $\left(\frac{x^{\frac{2}{3}}}{x^{\frac{1}{6}}}\right)^{18} = x^{\square}$

d. $\left(\sqrt{x}\right)^6 = x^{\square}$  e. $\frac{x^{\square}}{x^{\frac{1}{6}}} = x^{\frac{2}{3}}$

10. Compute.

a. $81^{-\frac{1}{2}}$  b. $27^{\frac{2}{3}}$  c. $\left(\frac{1}{10000}\right)^{-\frac{1}{4}}$

11. Determine the value of $n$.

a. $\left(\sqrt[4]{x}\right)^n = x^8$

b. $\left(\sqrt[n]{3}\right)^4 = 9$

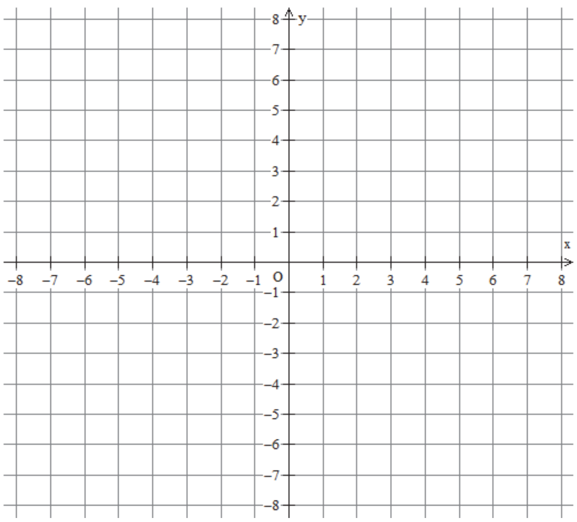
12. Sally solves the radical equation $3\sqrt{x} = -15$ and obtains the solution $x = 25$. Is this solution extraneous? Justify your answer.

13. Giacomo solves the radical equation $\sqrt{3x+4} = x$ and obtains the solutions $x = -1$ and $x=4$. Determine if either of the solutions are extraneous.

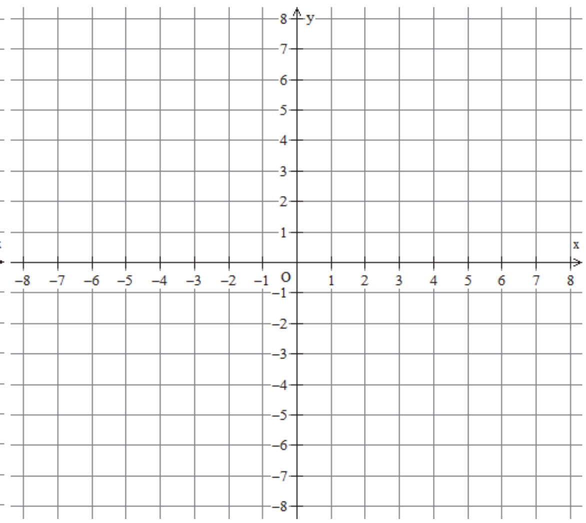
14. Johnny solves the radical equation $\sqrt[3]{x} = -2$ and obtains the solution $x = -8$. Is this solution extraneous? Justify your answer.

15. Sketch the equations below.

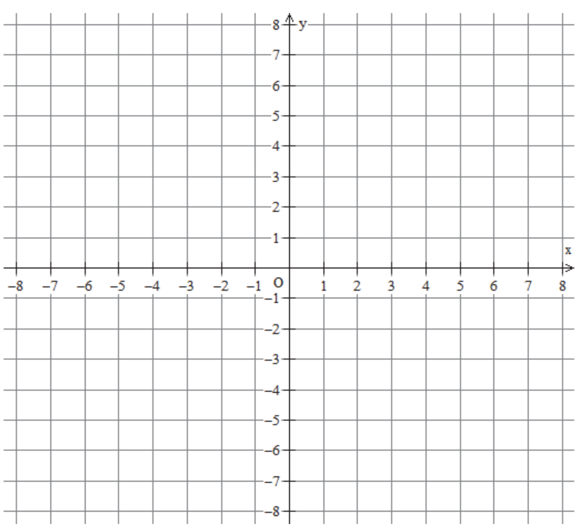
a. $y = -\sqrt{x}$



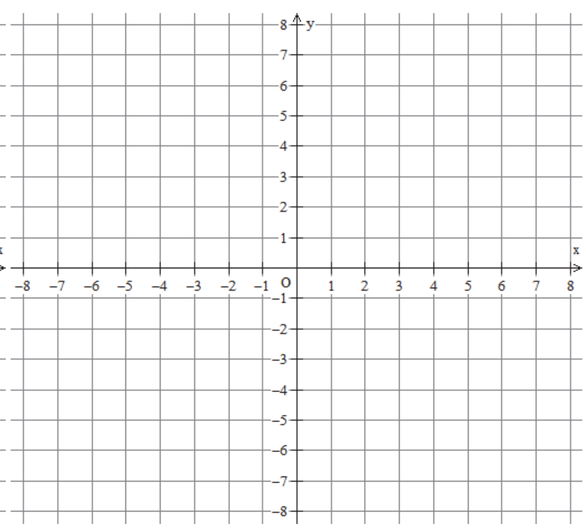
b. $y = \sqrt[3]{x}$



c. $y = \sqrt{x+2} - 3$



d. $y = -\sqrt[3]{x} + 2$



16. Let $f(x) = -\sqrt{x} + 7$.

a. What is the domain of $f$? ______________________________

b. What is the range of $f$? ________________________________

c. On what interval is the function decreasing? _____________________

d. What is the endpoint of the graph? __________

17. Solve the equation $\sqrt[3]{5x} = 10$.

18. The number of square meters of sod, $s$, that can be bought and delivered for $c$ dollars is given by the function $s(c) = \sqrt{\frac{c-5}{2}}$. If a person needs 10 square meters of sod, how much will it cost?

19. The population of a town can be modeled by the function $P(x) = 60{,}000\sqrt[3]{x-1970}$, where $x$ is the year. In what year was the population 120,000?

20. A <u>radical function</u> $g$ has the following properties:

- The domain is $x \geq 3$.
- The range is $y \leq 1$
- The function is decreasing for its entire domain.

Write the function equation $g(x) =$ ______________________

**Unit 1, Topic 3**

21. There are four functions represented by tables below. They are either exponential or logarithmic. For each table, write a function equation.

| a. $f(x) =$ | |
|---|---|
| $x$ | $f(x)$ |
| 0 | 1 |
| 1 | $\frac{1}{3}$ |
| 2 | $\frac{1}{9}$ |
| 3 | $\frac{1}{27}$ |
| 4 | $\frac{1}{81}$ |
| 5 | $\frac{1}{243}$ |

| b. $g(x) =$ | |
|---|---|
| $x$ | $g(x)$ |
| 5 | 1 |
| 25 | 2 |
| 125 | 3 |
| 625 | 4 |
| 3125 | 5 |
| 15625 | 6 |

| c. $h(x) =$ | |
|---|---|
| $x$ | $h(x)$ |
| –3 | $\frac{1}{8}$ |
| –2 | $\frac{1}{4}$ |
| –1 | $\frac{1}{2}$ |
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |

| d. $m(x) =$ | |
|---|---|
| $x$ | $m(x)$ |
| 1 | 0 |
| $\frac{1}{10}$ | 1 |
| $\frac{1}{100}$ | 2 |
| $\frac{1}{1000}$ | 3 |
| $\frac{1}{10000}$ | 4 |
| $\frac{1}{100000}$ | 5 |

22. For each equation in column 1, choose the interval in which the solution lies from column 2.

| Column 1 | Column 2 |
|---|---|
| a. $\log_2 18 = x$ | $x$ is between 0 and 1 |
| b. $3^x = 10$ | $x$ is between 1 and 2 |
| c. $\log 20$ | $x$ is between 2 and 3 |
| d. $5^{-x} = \frac{1}{6}$ | $x$ is between 4 and 5 |

23. Write the logarithmic equation that is equivalent to each exponential equation

a. $4^2 = 16$ b. $10^3 = 1000$ c. $e^0 = 1$

24. Write the exponential equation that is equivalent to each logarithmic equation.

a. $\log \frac{1}{100} = -2$ b. $\ln \frac{1}{e} = -1$ c. $\log_9 81 = 2$

25. Evaluate the following logarithms

a. $\log_3 9$ ______ b. $\log_4 \frac{1}{16} =$ ______ c. $\log 10{,}000$ ____ d. $\ln e =$ ______

e. $\log_6 1$ ______ f. $\ln \frac{1}{e} =$ ______ g. $\log_4 8 =$ ______ h. $\log_{100} 10 =$ ______

26. Does each function below represent exponential growth or decay?

a. $f(x) = \left(\frac{1}{2}\right)^x$ b. $g(x) = 3^{-x}$ c. $h(x) = 5^x$

27. Write an exponential function in terms of time *t* (*t* in years) for each situation.

a. There are 300 bacteria at time 0. The bacteria has a continuous growth rate of 70% per year.

b. The population of a town is currently 2000. The population is growing at an annual rate of 11% per year.

c. Jack puts $500 into a savings account. It earns interest at a nominal annual rate of 6% per year, compounded monthly.

d. The number of deer in a forest is decreasing at an annual rate of 8%. There are currently 700 deer in the forest.

e. The number of gnats in a swamp decreases at a continuous decay rate of 12% per year. There are currently 4 billion gnats in the swamp.

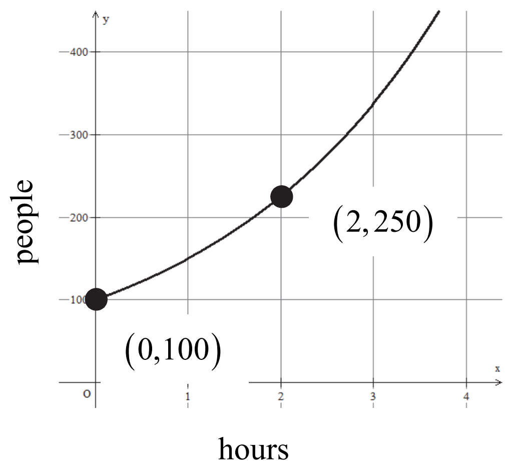
28. Solve each equation. Your answer must be exact or expressed as a decimal to at least three places beyond the decimal point.

a. $10^{3t-2} = 9800$

b. $5e^{t+4} = 275$

29. The graph below shows the number of people, $y$, who have heard a rumor about a snowstorm $x$ hours after midnight.




a. What is the average rate of change on the time interval $[0,2]$ ? Be sure to specify the correct units.

b. Assuming that the graph represents an exponential function, write an equation in the form $y = ab^x$ for this information.

30. The number of apples that have been harvested from an orchard has been increasing at an exponential rate. The table below shows the number of apples harvested in thousands, $y$, where $x$ is the number of years after 2000.

| Years since 2000 ($x$) | Number of apples in thousands ($y$) |
|---|---|
| 0 | 5 |
| 3 | 40 |

Write an equation in the form $y = ab^x$ that represents the number of apples harvested x years after 2000.

31. The number of tons of flour that a company manufactured can be described by the function $F(t) = 50e^{0.25t}$, where $t$ is the number of years since the beginning of 2010.

    a. What is the continuous percentage rate of increase of this function?

    b. How many tons of flour were produced at the beginning of 2012?

    c. To the nearest hundredth of a year, when will the company have manufactured twice as much flour as they had at the beginning of 2010?

32. Sammy deposits $2,500 in an account that pays a nominal interest rate of 12%, compounded monthly. How much money will be in the account after 4 years?

33. Look at the functions below.

$f(x) = 2^x$ $g(x) = \log_3 x$

$h(x) = 10^{-x}$ $p(x) = -\ln x$

Several properties are listed below. For each property, write the function(s) that have this property. You may use *f*, *g*, *h*, or *p* as your answers.

a. __________________ The graph of the function has a horizontal asymptote of $y = 0$.

b. __________________ The function has a range of all real numbers.

c. __________________ The function is increasing on its entire domain.

d. __________________ The graph of the function has a *y*-intercept at the point $(0,1)$.

e. __________________ The domain of the function is the positive real numbers.

f. __________________ The graph of the function has a vertical asymptote of $x = 0$.

g. __________________ The function has a domain of all real numbers.

h. __________________ The function is decreasing on its entire domain.

i. __________________ The graph of the function has an *x*-intercept at the point $(1,0)$.

j. __________________ The range of the function is the positive real numbers.

34. Each function below is a transformation of the function $f(x)=e^x$. After each given transformation, write the function rule.

a. The graph of function *g* is the graph of $f(x)=e^x$ translated one unit to the right.
$g(x)=$ ________________

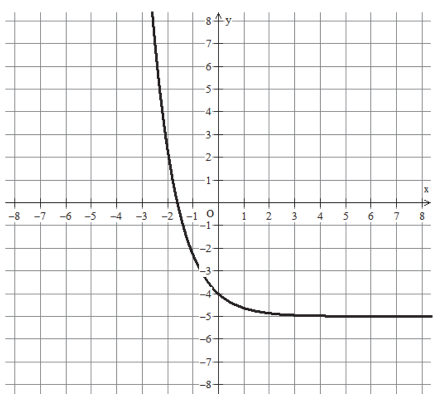
b. The asymptote of the graph of function *h* has the equation $y=-4$.
$h(x)=$ _________________________

c. The graph of function *p* is the graph of $f(x)=e^x$ reflected across the *x*-axis.
$p(x)=$ __________________

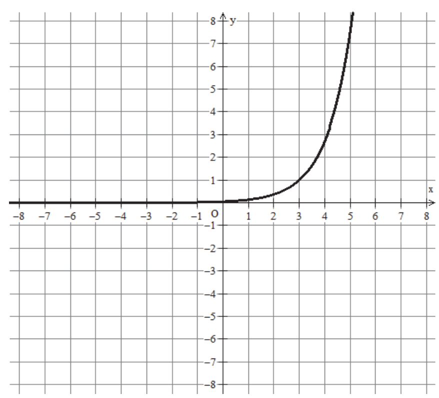
d. The graph of function *s* is the graph of $f(x)=e^x$ reflected across the *y*-axis.
$s(x)=$ ______________________

e. The graph of function *w* is the graph of $f(x)=e^x$ dilated by a factor of 2 with respect to the *x*-axis. $w(x)=$ __________________________

f. $t(x)=$ __________________



g. $z(x)=$ _____________



35. Each function below is a transformation of the function $f(x) = \log_2 x$. After each given transformation, write the function rule.

a. The graph of function *g* is the graph of $f(x) = \log_2 x$ translated two units to the left. $g(x) =$ ________________

b. The asymptote of the graph of function *h* has the equation $x = 2$.
$h(x) =$ _________________________

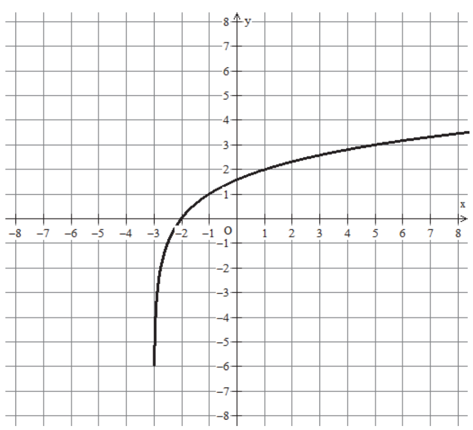
c. The graph of function *p* is the graph of $f(x) = \log_2 x$ reflected across the *x*-axis.
$p(x) =$ __________________

d. The graph of function *s* is the graph of $f(x) = \log_2 x$ reflected across the *y*-axis.

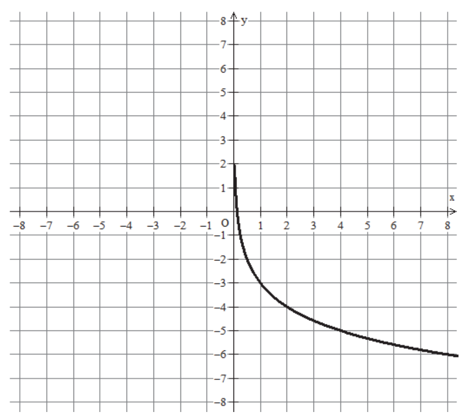
$s(x) =$ _______________________

e. The graph of function *w* is the graph of $f(x) = \log_2 x$ dilated by a factor of $\frac{1}{3}$ with respect to the *x*-axis. $w(x) =$ ___________________________

f. $t(x) =$ _____________



g. $z(x) =$ _________________



36. Let $f(x) = b^x$.

a. The value of $b$ must be _____________ than zero, but not equal to _____.

b. The function $f$ has a __________________ asymptote who equation is _______.

c. The domain of $f$ is _____________________________.

d. The range of $f$ is ______________________________.

e. The function is increasing when the value of $b$ is ____________________________.

f. The function is decreasing when the value of $b$ is ___________________________ .

g. The $y$-intercept is at the point _______.

37. Let $g(x) = \log_b x$.

a. The value of $b$ must be _____________ than zero, but not equal to _____.

b. The function $g$ has a __________________ asymptote who equation is _______.

c. The domain of $g$ is _____________________________.

d. The range of $g$ is ______________________________.

e. The function is increasing when the value of $b$ is ____________________________.

f. The function is decreasing when the value of $b$ is ___________________________ .

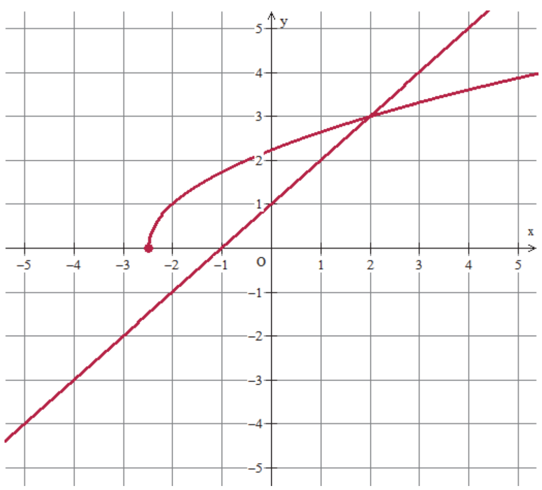
g. The $x$-intercept is at the point _______.

**Unit 2, Topic 1**

38. An Algebra 2 class is asked to solve the radical equation $\sqrt{2x+5} = x+1$

a. Jack decides to solve this equation symbolically. Solve the equation and determine which solutions, if any, are extraneous.

Your friend Sally decides to try a graphical solution. She graphs $y = \sqrt{2x+5}$ and $y = x+1$ on the same coordinate plane, as shown below.




b. How does Sally determine the solution to equation using the graph?

c. Since she only obtained one solution from the graph, she thinks there should be an extraneous solution, but does not know what to do to the graph to show the extraneous solution. Help Sally by adding a piece to her graph that would show the extraneous solution.

In the following problems, $i=\sqrt{-1}$ and $a+bi$ represents a complex number where $a$ and $b$ are real numbers.

39. What is the value of $i+i^2+i^3+i^4$?

40. Perform the following operations. Write your answer in the form $a+bi$.

    a. $(3+i)+(6-5i)$

    b. $(4+7i)-(2-6i)$

    c. $(3-2i)(4+9i)$

    d. $(4-3i)(4+3i)$

    e. $(7+5i)^2$

41. a. If $a$ is not equal to zero and $b$ is not equal to zero, explain why $(a+bi)^2$ can never be a real number.

    b. If $b$ is not equal to zero, explain why $(bi)^4$ is always a real number.

42. One of the solutions of a quadratic equation $q(x)=0$ is $x=-4$. Complete the following:

a. ___________________ is factor of $q(x)$

b. The point ______ is the location of an $x$-intercept of the graph of $q(x)$.

c. The second solution to the equation $q(x)=0$ is (always, sometimes, or never) real.

d. _______ is a zero of $q(x)$.

43. One solution of a quadratic equation $q(x)=0$ is $5-7i$. Complete the following:

a. ___________________ is factor of $q(x)$ .

b. The point ______ is the location of an $x$-intercept of the graph of $q(x)$.

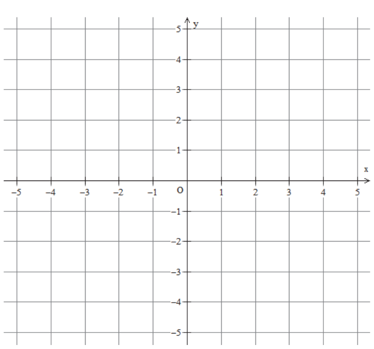
c. The second solution to the equation $q(x)=0$ is (always, sometimes, or never) real.

d. The second solution of $q(x)=0$ is $x=$ ____________________

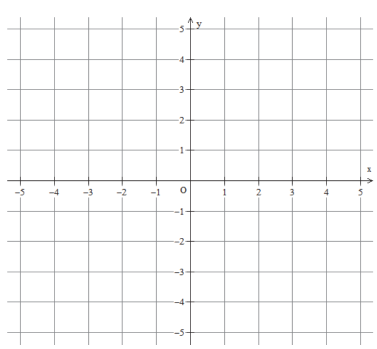
e. ___________and ___________ are the zeros of $q(x)$.

44. On the axes below, sketch the graph of a quadratic function with the stated roots.

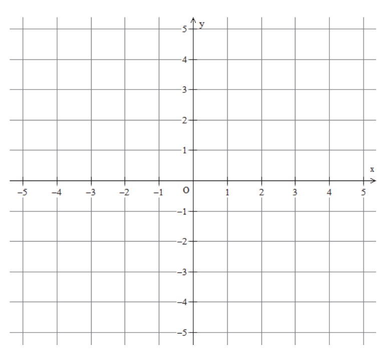
a. Two real roots



b. One double root



c. Two imaginary roots



45. Solve the following quadratic equations over the set of complex numbers. Show how you determined your solutions.

a. $(x-3)^2 = -49$

b. $(x+4)^2 = -11$

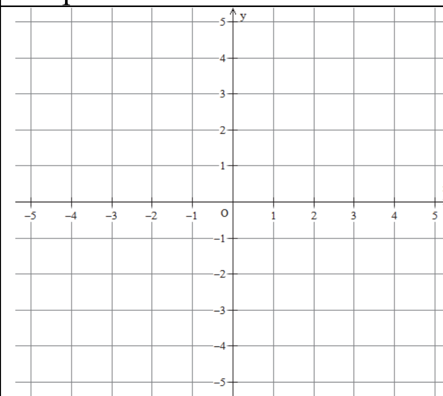
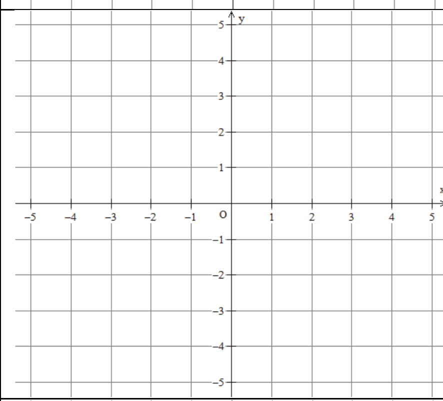
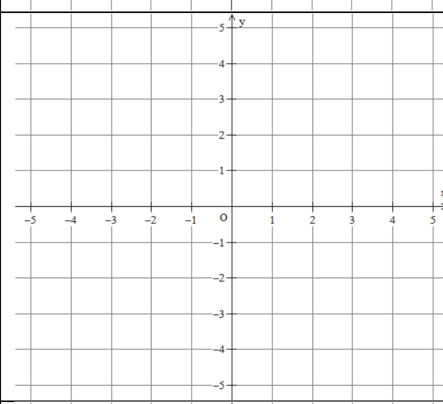
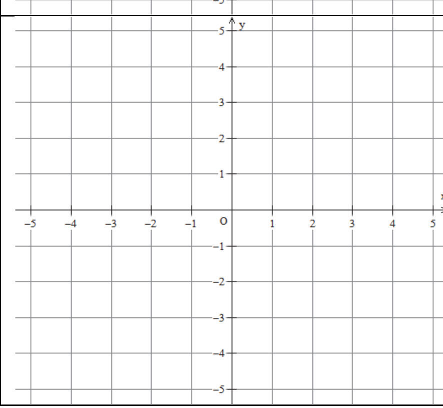
c. $x^2 + 8x = -30$

d. $3x^2 - 6x - 7 = -17$

e. $(x-2)(x+1) = (2x+1)^2$

**Unit 2, Topic 2**

46. Sketch a rough graph of a polynomial function that has each characteristic.

| Degree is: | Leading Coefficient is: | Graph |
|---|---|---|
| a. Even | Positive |  |
| b. Odd | Negative |  |
| c. Even | Negative |  |
| d. Odd | Positive |  |

47. Do the functions below have the same end behavior as $f(x)=x^4$, when $x\to-\infty$? Write yes or no.

a. ______ $g(x)=x^4-70x^3$

b. ______ $h(x)=x^3$

c. ______ $k(x)=2^x$

48. Do the functions below have the same end behavior as $f(x)=x^3$, when $x\to\infty$? Write yes or no.

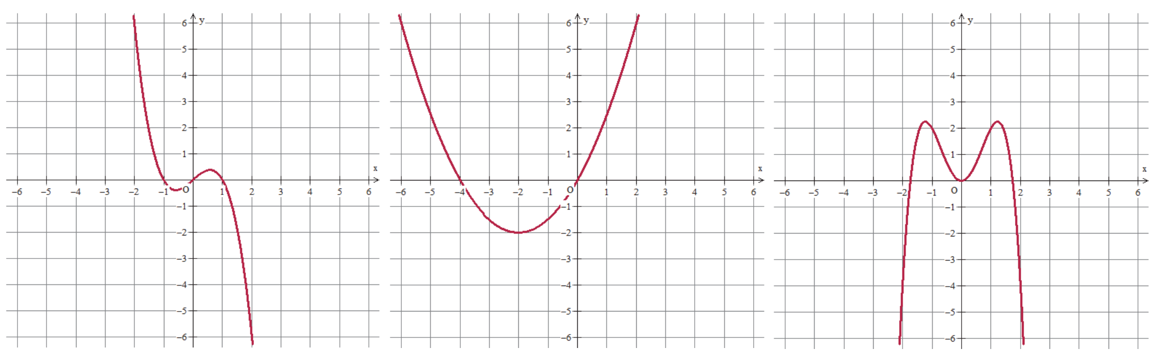
a. ______ $g(x)=x^3-200x^2-50-90$

b. ______ $h(x)=-x^4$

c. ______ $j(x)=\log x$

49. Classify each function represented by the graph as even, odd, or neither even nor odd.

a. ____________ b. ___________ c. ___________



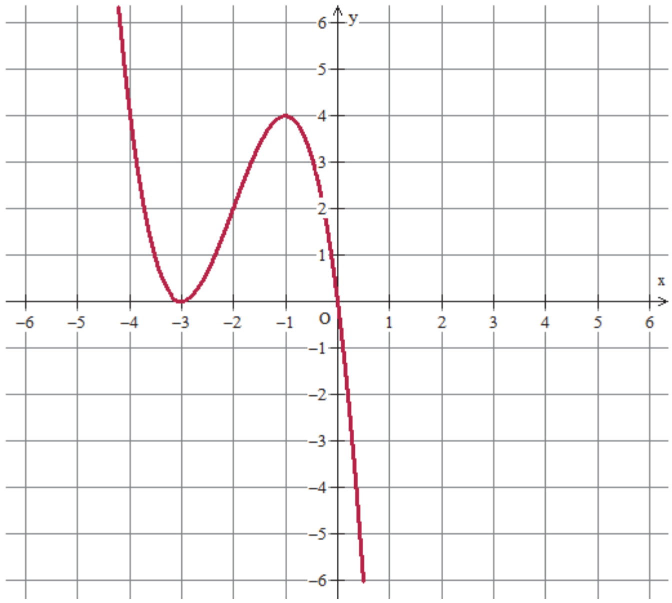
50. The point $(3,8)$ is on the graph of an even function. What are the coordinates of another point on its graph?

51. The graph of an even function has what symmetry?

52. The point $(5,7)$ is on the graph of an odd function. What are the coordinates of another point on its graph?

53. The graph of an odd function has what symmetry?

54. Look at the graph of the polynomial function $f(x) = -x(x+3)^2$




a. What is the degree of $f$? _______

b. As $x \to \infty, f(x) \to$ _______

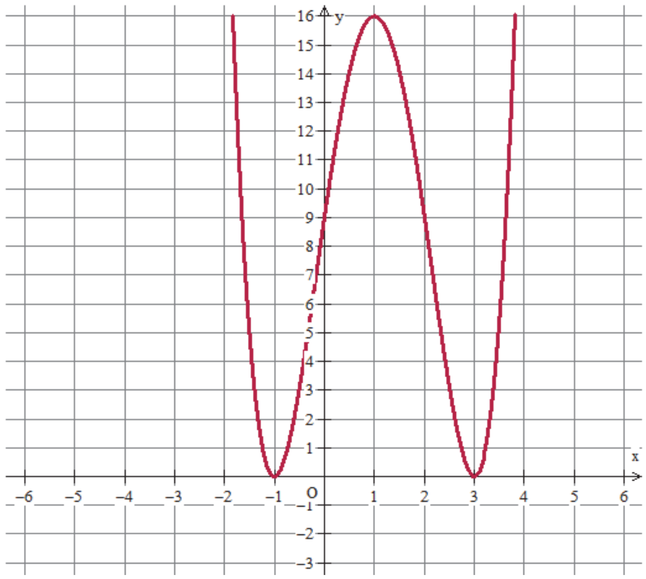
c. Write the zeros and their multiplicities.

d. Give coordinates of the point where a relative maximum occurs. _______

e. Give coordinates of the point where a relative minimum occurs. _______

f. On what interval(s) is the function increasing? ____________________

55. Look at the graph of $g(x)=(x+1)^2(x-3)^2$ below.




a. What is the degree of $g$? ________

b. As $x \to -\infty, g(x) \to$ ________

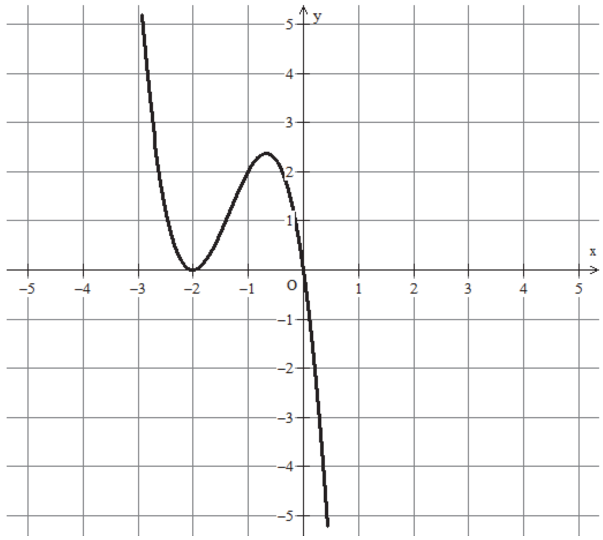
c. Write the zeros and their multiplicities.

d. Give coordinates of the point where a relative maximum occurs. ________

e. Give coordinates of the points where a relative minimum occurs.

f. On what interval(s) is the function decreasing? ______________________

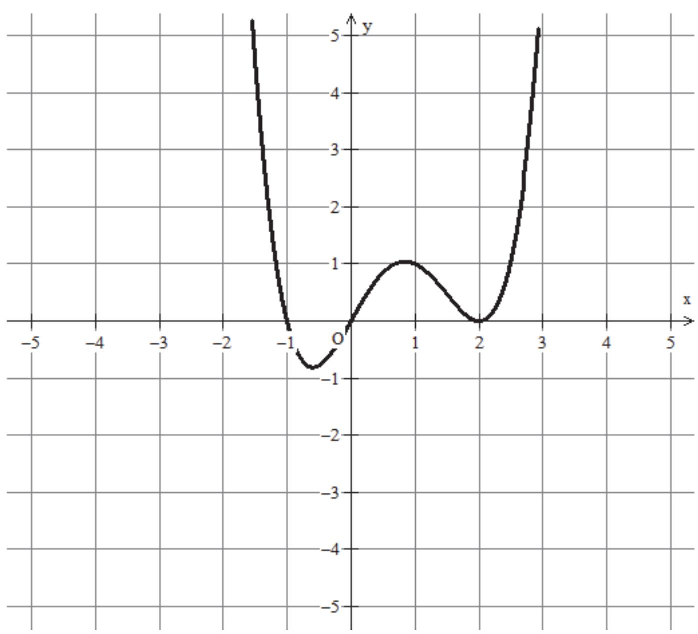
56. The graph of a polynomial function $f(x)$ with leading coefficient $-2$ is shown below.




Write a function equation represented by the graph.

$f(x) =$ ______________________

57. The graph of a polynomial function $g(x)$ with leading coefficient $\frac{1}{2}$ is shown below.




Write a function equation represented by the graph.

$g(x) =$ ______________________

58. Let $f(x) = x^2 + 2x - 7$, $g(x) = 2x^2 + 9$, and $h(x) = 5x + 3$.

a. Write $(f + g)(x)$ as a polynomial in standard form.

b. Write $(f - g)(x)$ as a polynomial in standard form.

c. What is the degree of the product $(f \cdot g)(x)$?

d. What is the degree of the product $(g \cdot h)(x)$?

e. Write $(f \cdot h)(x)$ as a polynomial in standard form.

f. Write $(f \cdot g)(x)$ as polynomial in standard form.

59. Jillian divides $x^3 - 8x^2 - 7x + 1$ by $x - 9$ and gets a quotient of $x^2 + x + 16$ with a remainder of 145.

a. Is $x - 9$ a factor of $x^3 - 8x^2 - 7x + 1$? Justify your answer.

b. If $f(x) = x^3 - 8x^2 - 7x + 1$, what is the value of $f(9)$?

60. Jake divides $x^3 - 3x^2 + 6x - 18$ by $x - 3$ and gets a quotient of $x^2 + 6$ with no remainder.

a. If $f(x) = x^3 - 3x^2 + 6x - 18$, what is the value of $f(3)$?

b. Why is $x - 3$ a factor of $x^3 - 3x^2 + 6x - 18$?

c. Write a factorization of $f(x) = x^3 - 3x^2 + 6x - 18$.

61. Let $f(x) = x^3 - 7x^2 - x - 56$.

a. Jessie looks at the function and believes that 8 is a zero of the function. He decides to test his value by finding $f(8)$. Confirm that 8 is a zero of the function.

b. What is the graphical interpretation of the fact that 8 is a zero of the function?

c. Since 8 is a zero of the function, then by the Factor Theorem, ___________ is a factor of $x^3 - 7x^2 - x - 56$.

d. If Jessie divides $x^3 - 7x^2 - x - 56$ by the factor in part c), what is the remainder?

e. When Jessie divides $x^3 - 7x^2 - x - 56$ by the factor in part c) he obtains a quotient of $x^2 + x + 7$. Use this quotient to determine whether or not the other zeros of $f$ are real.

62. A polynomial function of degree 4 has the following properties.

- A negative leading coefficient.
- Roots of $-3$ (multiplicity 1), 1 (multiplicity 1), and 4 (multiplicity 2).

Make a rough sketch of a function with these properties below.